# New flow observables

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**Abstract.** Event-by-event fluctuations of the initial transverse density profile result in a collective flow pattern which also fluctuates event by event. We propose a number of new correlation observables to characterize these fluctuations and discuss how they should be analyzed experimentally. We argue that most of these quantities can be measured at RHIC and LHC.

Thermalization of the matter produced in ultrarelativistic nucleus-nucleus collisions results in strong collective motion. The clearest experimental signature of such collective flow is obtained from azimuthal correlations between outgoing particles. There is now a growing consensus that correlations between particles emitted at large relative pseudorapidity $\Delta\eta$ are all due to flow [1]. Heavy-ion experiments at the CERN Large Hadron Collider (LHC) will be able to carry out correlation analyses with unprecedented accuracy due to large multiplicities, and the large detector coverage. We propose a number of new independent flow measurements and discuss their feasibility.

The flow hypothesis is that particles in a given event are emitted *independently* according to some azimuthal distribution. The most general distribution can be written as a sum of Fourier components,

$$\frac{dN}{d\varphi} = \frac{N}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n\varphi - n\Psi_n)\right), \tag{1}$$

where $v_n$ is the $n^{th}$ flow harmonic [2] and $\Psi_n$ the corresponding reference angle, all of which fluctuate event-by-event.

In practice, one cannot exactly reconstruct the underlying probability distribution from the finite sample of particles emitted in a given event. All known information about $v_n$ is inferred from azimuthal correlations. Generally, a $k$-particle correlation is of the type

$$v\{n_1, n_2, \ldots, n_k\} = \langle \cos\left(n_1\varphi_1 + \ldots + n_k\varphi_k\right)\rangle, \tag{2}$$

where $n_1, \ldots, n_k$ are integers, $\varphi_1, \ldots, \varphi_k$ are azimuthal angles of particles belonging to the same event, and angular brackets denote average over multiplets of particles and

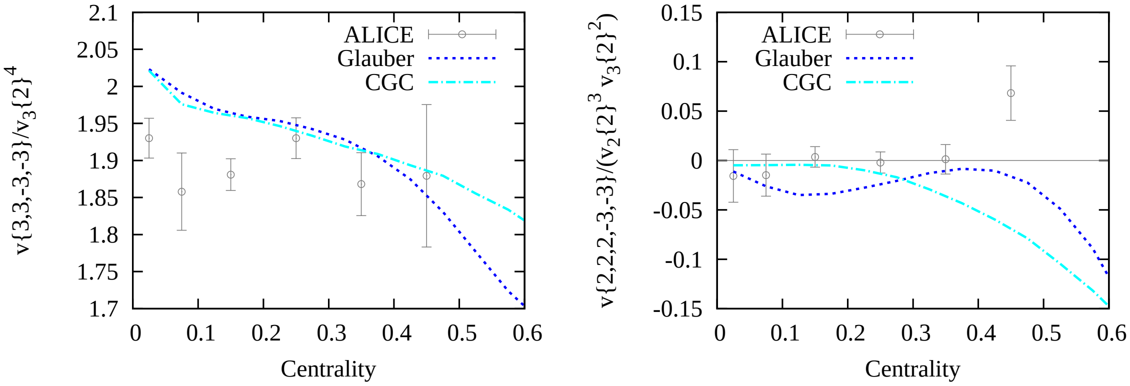


**Figure 1.** Curves are theoretical predictions for Pb-Pb collisions at 2.76 TeV [8] using the Monte-Carlo Glauber model [9] (dotted lines) and the Monte-Carlo KLN model [10, 11] (dashed lines). Data points are from ALICE [12].

events in a centrality class. Since the impact parameter orientation is uncontrolled, the only measurable correlations have azimuthal symmetry: $n_1 + \ldots + n_k = 0$.

Inserting Eq. (1) into Eq. (2) gives

$$v\{n_1, \ldots, n_k\} = \langle v_{n_1} \ldots v_{n_k} \cos(n_1\Psi_{n_1} + \ldots + n_k\Psi_{n_k})\rangle\,, \tag{3}$$

where the average is now only over events. To the extent that correlations are induced by collective flow, azimuthal correlations measure moments of the flow distribution.

In practice, the average over particles in Eq. (2) is a weighted average: in a given harmonic $n$, one gives more weight to particles which have larger $v_n$ in order to increase the resolution. Our goal here is to characterize initial-state fluctuations, which are approximately independent of rapidity [3]. Weights should therefore be chosen independent of (pseudo)rapidity, a nonstandard choice for odd harmonics [4].

The simplest $v_n$ measurement is the pair correlation [5], which corresponds to the event-averaged root-mean-square $v_n$

$$v_n\{2\} \equiv \sqrt{v\{n, -n\}} \simeq \sqrt{\langle {v_n}^2 \rangle}. \tag{4}$$

At this Conference, measurements of $v\{n, -n\}$ were presented for $n = 2, \cdots, 6$, but $v\{1, -1\}$ has not yet been directly analyzed with rapidity-independent weights [6].

Higher-order correlations yield higher moments of the $v_n$ distribution:

$$v\{n, n, -n, -n\} \equiv 2v_n\{2\}^4 - v_n\{4\}^4 \simeq \langle {v_n}^4 \rangle, \tag{5}$$

where we have used the standard notation $v_n\{4\}$ for the 4-particle cumulant [7]. Finally, one can construct correlations involving mixed harmonics. The first non-trivial correlations between $v_1$, $v_2$ and $v_3$ are [8]

$$\begin{aligned} v_{12} &\equiv v\{1, 1, -2\}, & v_{13} &\equiv v\{1, 1, 1, -3\}, \\ v_{23} &\equiv v\{2, 2, 2, -3, -3\}, & v_{123} &\equiv v\{1, 2, -3\}. \end{aligned} \tag{6}$$

These mixed correlations involve angular correlations between $\Psi_1$, $\Psi_2$ and $\Psi_3$. In order to single out these angular correlations, it is natural to scale $v\{n_1, \ldots, n_k\}$

by $v_{n_1}\{2\}\cdots v_{n_k}\{2\}$ in order to obtain a number of order unity. Figure 1 displays predictions for two such ratios [8] together with data recently released by the ALICE collaboration [12]. The measured $\langle v_3^4\rangle/\langle v_3^2\rangle^2$ (Figure 1, left) is compatible with theoretical prediction except for $0\% - 20\%$ central collisions, where the prediction is close to 2 (corresponding to Gaussian fluctuations [13]) while the measured value is close to 1.9. The second ratio (Figure 1, right) measures the correlation between the orientations of the ellipse ($\Psi_2$) and the triangle ($\Psi_3$). It is predicted to be small up to 50% centrality, in qualitative agreement with the measurement. Note, however, that neither model agrees quantitatively with data.

All correlations should be analyzed in such a way as to isolate the correlation induced by collective flow from other "nonflow" effects. An important nonflow effect is global momentum conservation. It contributes to $v\{1,-1\}$ [14] and also, to a lesser extent, to $v\{1,1,-2\}$ [15]. This nonflow effect can be suppressed [6] simply by using the weight $w = p_t - \langle p_t^2\rangle/\langle p_t\rangle$ for at least one of the particles with Fourier harmonic 1.

Other nonflow effects are correlations between a small number of particles — typically pairs of particles— and they are suppressed by putting rapidity gaps between particles [1]. In order to determine where rapidity gaps are important, we estimate nonflow effects by assuming that particles are emitted in collinear pairs. If $M$ particles are observed in each event, the probability that two random particles belong to the same pair is $1/(M-1) \simeq 1/M$. Consider the first correlation in Eq. (6), $v_{12}$, which involves three particles. There are different nonflow contributions to this correlation corresponding to the different pairings. Pairing the first two particles (with harmonic 1) gives a nonflow correlation of order $(v_2)^2/M$, while pairing 1 or 2 with 3 gives a correlation of order $(v_1)^2/M$. Since $v_2 \gg v_1$, it is important to put a rapidity gap between the first two particles. On the other hand, there is no restriction for the third particle. For $v_{13}$, a similar discussion shows that there must be rapidity gaps between the first three particles (again those with harmonic 1). For $v_{23}$, nonflow effects are small and rapidity gaps are not required. Finally, for $v_{123}$, the largest nonflow correlation is between the first and the third particle (harmonics 1 and 3) and is of order $v_2^2/M$. The next-to-largest is between the first two particles, of order $v_3^2/M$, which is much smaller, except for central collisions.

The limiting factor in the ability to measure these high-order correlations is statistics. The statistical error is

$$\frac{\delta v\{n_1,\cdots,n_k\}}{v_{n_1}\{2\}\cdots v_{n_k}\{2\}} = \sqrt{\frac{S}{2N_{\rm evts}}\left(1+\frac{1}{\chi_{n_1}^2}\right)\cdots\left(1+\frac{1}{\chi_{n_k}^2}\right)}, \tag{7}$$

where $\chi_n \equiv v_n\{2\}\sqrt{M}$ is the resolution parameter [16] in harmonic $n$, and $S$ is a symmetry factor which is given in Table 1 for the various correlations. Table 1 lists the number of events required in each experiment to measure the various ratios for central collisions. We have assumed $v_1\{2\} = 0.74\%$, $v_2\{2\} = 2.40\%$, $v_3\{2\} = 1.90\%$ and $M = 2000$, 6000 and 900 respectively for ALICE, CMS/ATLAS and STAR/PHENIX.

We have introduced new flow observables, most of which can be measured accurately

**Table 1.** Number of events needed in the 0% − 5% centrality class in order to measure various quantities within 5%.

| quantity | S | ALICE | ATLAS or CMS | STAR or PHENIX |
|---|---|---|---|---|
| $v\{2,-2\}$ | 2 | $3\times10^2$ | $2\times10^2$ | $9\times10^2$ |
| $v\{3,-3\}$ | 2 | $6\times10^2$ | $2\times10^2$ | $2\times10^3$ |
| $v\{1,-1\}$ | 2 | $1\times10^4$ | $2\times10^3$ | $5\times10^4$ |
| $v\{1,2,-3\}/(v_1v_2v_3)$ | 1 | $9\times10^3$ | $2\times10^3$ | $5\times10^4$ |
| $v\{2,2,-2,-2\}/v_2^4$ | 8 | $2\times10^4$ | $4\times10^3$ | $1\times10^5$ |
| $v\{3,3,-3,-3\}/v_3^4$ | 8 | $5\times10^4$ | $7\times10^3$ | $4\times10^5$ |
| $v\{1,1,-2\}/(v_1^2v_2)$ | 2 | $8\times10^4$ | $8\times10^3$ | $5\times10^5$ |
| $v\{2,2,2,-3,-3\}/(v_2^3v_3^2)$ | 12 | $9\times10^4$ | $1\times10^4$ | $1\times10^6$ |
| $v\{1,1,1,-3\}/(v_1^3v_3)$ | 6 | $3\times10^6$ | $1\times10^5$ | $5\times10^7$ |
| $v\{1,1,-1,-1\}/v_1^4$ | 8 | $2\times10^7$ | $4\times10^5$ | $3\times10^8$ |

at RHIC and LHC with a modest number of events. The most challenging measurements are $v\{1,1,1,-3\}$ and $v\{1,1,-1,-1\}$, which require wide pseudorapidity coverage or large statistics. These new observables will constrain models of initial-state fluctuations and deepen our knowledge of the collision dynamics.

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