

Multifragmentation at the balance energy of mass asymmetric colliding nuclei.

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(Dated: June 21, 2011)

Using the quantum molecular dynamics model, we study the role of mass asymmetry of colliding nuclei on the fragmentation at the balance energy and on its mass dependence. The study is done by keeping the total mass of the system fixed as 40, 80, 160, and 240 and by varying the mass asymmetry of the ($\eta = \frac{A_T - A_P}{A_T + A_P}$; where A_T and A_P are the masses of the target and projectile, respectively) reaction from 0.1 to 0.7. Our results clearly indicate a sizeable effect of the mass asymmetry on the multiplicity of various fragments. The mass asymmetry dependence of various fragments is found to increase with increase in total system mass (except for heavy mass fragments). Similar to symmetric reactions, a power law system mass dependence of various fragment multiplicities is also found to exist for large asymmetries.

PACS numbers: 24.10.Cn, 24.10.Lx, 25.70.-z, 25.75.Ld

I. INTRODUCTION

The main goals in the study of heavy-ion collisions at the intermediate energies are the determination of the bulk properties of nuclear matter, or the nuclear equation of state, and the understanding of the collision processes, which vary over the large range of energies available today. These goals are related to each other and an improved insight into one can lead to a better understanding of the other. The study of multifragmentation in the intermediate energy range gives us a possibility to understand the properties of nuclear matter at extreme conditions of temperature and density. The detailed experimental and theoretical studies clearly point towards the dependence of the reaction dynamics on entrance channel parameters such as incident energy, impact parameter as well as mass asymmetry of the colliding nuclei [1–6].

It is well known that the reaction dynamics for symmetric and asymmetric reactions are entirely different. The former leads to higher compression whereas the latter has a large share as thermal energy [7]. In a recent study by Puri and collaboration, a detailed analysis is presented on the effect of mass asymmetry of colliding nuclei on the collective flow and its disappearance, nuclear stopping, elliptical flow, multifragmentation (at fixed energies), and nuclear dynamics (at the balance energy E_{bal} ; i.e. energy at which collective flow disappears) by keeping the total mass of the system fixed and at different impact parameters [8, 9]. A sizeable role of mass asymmetry has been found in all the cases. Unfortunately, the role of mass asymmetry of the colliding nuclei on the fragment structure at the balance energy is not presented in the literature. A similar attempt was made by Dhawan and Puri [10], but it was limited for symmetric colliding nuclei only. Therefore, in the present

work, we aim to see the effect of mass asymmetry of the colliding nuclei on the fragment structure and its mass dependence by simulating the reactions at their corresponding balance energies. The mass asymmetry of the reaction is varied by keeping the total mass of the system fixed. The quantum molecular dynamics (QMD) model [1–3, 5–12] is used for the present study and is explained in section II. Section III is devoted to the results and discussion followed by summary in section IV.

II. THE MODEL

In quantum molecular dynamics model [1–3, 5–12], nucleons (represented by Gaussian wave packets) interact via mutual two- and three-body interactions. Here each nucleon is represented by a coherent state of the form:

$$\phi_i(\vec{r}, \vec{p}, t) = \frac{1}{(2\pi L)^{3/4}} e^{[-\{\vec{r}-\vec{r}_i(t)\}^2/4L]} e^{[i\vec{p}_i(t)\cdot\vec{r}/\hbar]}. \quad (1)$$

The Wigner distribution of a system with $A_T + A_P$ nucleons is given by

$$f(\vec{r}, \vec{p}, t) = \sum_{i=1}^{A_T+A_P} \frac{1}{(\pi\hbar)^3} e^{[-\{\vec{r}-\vec{r}_i(t)\}^2/2L]} e^{[-\{\vec{p}-\vec{p}_i(t)\}^2 2L/\hbar^2]}, \quad (2)$$

with $L = 1.08 fm^2$.

The center of each Gaussian (in the coordinate and momentum space) is chosen by the Monte Carlo procedure. The momentum of nucleons (in each nucleus) is chosen between zero and local Fermi momentum $[\equiv \sqrt{2m_i V_i(\vec{r})}]$; $V_i(\vec{r})$ is the potential energy of nucleon i . Naturally, one has to take care that the nuclei, thus generated, have right binding energy and proper root mean square radii.

The centroid of each wave packet is propagated using the classical equations of motion:

$$\frac{d\vec{r}_i}{dt} = \frac{dH}{d\vec{p}_i}, \quad (3)$$

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$$\frac{d\vec{p}_i}{dt} = -\frac{dH}{d\vec{r}_i}, \quad (4)$$

where the Hamiltonian is given by

$$H = \sum_i \frac{\vec{p}_i^2}{2m_i} + V^{tot}. \quad (5)$$

Our total interaction potential V^{tot} reads as

$$V^{tot} = V^{Loc} + V^{Yuk} + V^{Coul} + V^{MDI}, \quad (6)$$

with

$$V^{Loc} = t_1\delta(\vec{r}_i - \vec{r}_j) + t_2\delta(\vec{r}_i - \vec{r}_j)\delta(\vec{r}_i - \vec{r}_k), \quad (7)$$

$$V^{Yuk} = t_3e^{-|\vec{r}_i - \vec{r}_j|/m} / (|\vec{r}_i - \vec{r}_j|/m), \quad (8)$$

with $m = 1.5$ fm and $t_3 = -6.66$ MeV.

The static (local) Skyrme interaction [13] can further be parametrized as:

$$U^{Loc} = \alpha \left(\frac{\rho}{\rho_o} \right) + \beta \left(\frac{\rho}{\rho_o} \right)^\gamma. \quad (9)$$

Here α, β and γ are the parameters that define equation of state. The momentum dependent interaction is obtained by parameterizing the momentum dependence of the real part of the optical potential. The final form of the potential reads as

$$U^{MDI} \approx t_4 \ln^2 [t_5 (\tilde{p}_i - \tilde{p}_j)^2 + 1] \delta(\tilde{r}_i - \tilde{r}_j). \quad (10)$$

Here $t_4 = 1.57$ MeV and $t_5 = 5 \times 10^{-4} MeV^{-2}$. A parameterized form of the local plus momentum dependent interaction (MDI) potential (at zero temperature) is given by

$$U = \alpha \left(\frac{\rho}{\rho_0} \right) + \beta \left(\frac{\rho}{\rho_0} \right) + \delta \ln^2 [\epsilon (\rho/\rho_0)^{2/3} + 1] \rho/\rho_0. \quad (11)$$

The parameters α, β , and γ in above Eq. (11) must be readjusted in the presence of momentum dependent interactions so as to reproduce the ground state properties of the nuclear matter. The set of parameters corresponding to different equations of state can be found in Ref. [1].

III. RESULTS AND DISCUSSION

For the present work, we simulated the central reactions of $^{17}_8O + ^{23}_{11}Na$ ($\eta = 0.1$), $^{14}_7N + ^{26}_{12}Mg$ ($\eta = 0.3$), $^{10}_5B + ^{30}_{14}Si$ ($\eta = 0.5$), and $^6_3Li + ^{34}_{16}S$ ($\eta = 0.7$) for $A_{TOT} = 40$, $^{36}_{18}Ar + ^{44}_{20}Ca$ ($\eta = 0.1$), $^{28}_{14}Si + ^{52}_{24}Cr$ ($\eta = 0.3$), $^{20}_{10}Ne + ^{60}_{28}Ni$ ($\eta = 0.5$), and $^{10}_5B + ^{70}_{32}Ge$ ($\eta = 0.7$) for $A_{TOT} = 80$, $^{70}_{32}Ge + ^{90}_{40}Zr$ ($\eta = 0.1$), $^{54}_{26}Fe + ^{106}_{48}Cd$ ($\eta = 0.3$), $^{40}_{20}Ca + ^{120}_{52}Te$ ($\eta = 0.5$), and $^{24}_{12}Mg + ^{136}_{58}Ce$ ($\eta = 0.7$) for $A_{TOT} = 160$, and $^{108}_{48}Cd + ^{132}_{56}Ba$ ($\eta = 0.1$), $^{84}_{38}Sr + ^{156}_{66}Dy$ ($\eta = 0.3$), $^{60}_{28}Ni + ^{180}_{74}W$ ($\eta = 0.5$), and $^{36}_{18}Ar + ^{204}_{82}Pb$

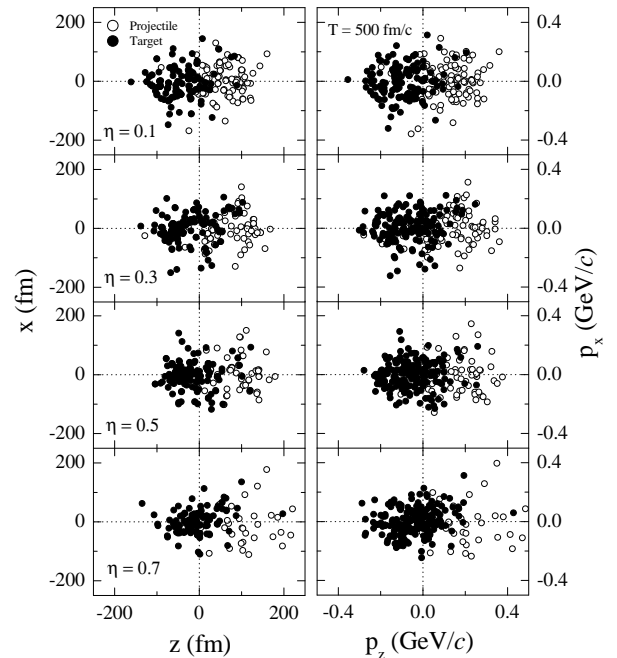


FIG. 1: Snapshots of a single event in the phase space (x, z), left side, and (p_x, p_z), right side, for fixed system mass $A_{TOT} = 240$ and $\eta = 0.1, 0.3, 0.5$, and 0.7 at their corresponding balance energies.

($\eta = 0.7$) for $A_{TOT} = 240$, at their corresponding theoretical balance energies (taken from Ref. [8]). The balance energies at which these reactions are simulated were calculated using a momentum dependent soft equation of state with standard energy dependent cugnon cross-section. The reactions are followed uniformly up to 500 fm/c. A simple spatial clusterization algorithm dubbed as minimum spanning tree (MST) method is used to clusterize the phase space [1].

In Fig. 1, we display the snapshots of the final phase-space (i.e.; $x-z$ (left column) and p_x-p_z (right column)) of a single event at the balance energy for $\eta = 0.1-0.7$ by keeping the total mass of the system fixed as $A_{TOT} = 240$. We see a isotropic emission of nucleons for nearly symmetric colliding nuclei whereas a binary character starts emerging out as η increases. One can say that phase-space is less homogenous for large asymmetries. The behavior is similar in spatial and momentum spaces. The above picture is quite similar for large number of different events indicating a uniform distribution.

In Fig. 2, we display the time evolution of the largest fragment survived A^{max} , free nucleons, the light charged particles (LCP's) $2 \leq A \leq 4$, the medium mass fragments (MMF's) $5 \leq A \leq 9$, the heavy mass fragments (HMF's) $15\% \leq A \leq 30\%$ as well as the intermediate mass fragments (IMF's) $4 \leq A \leq 30\%$ (of the largest between target and projectile). The results are displayed for different mass asymmetries by keeping the total mass fixed as $A_{TOT} = 240$. In order to avoid unwanted and artificial heavy fragments for large asymmetries and lighter collid-

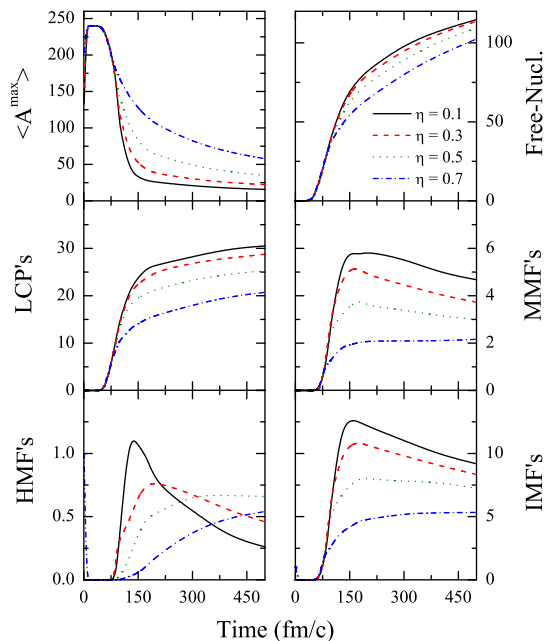


FIG. 2: (Color Online) Time evolution of the largest fragment A^{max} , free nucleons, LCP's ($2 \leq A \leq 4$), MMF's ($5 \leq A \leq 9$), HMF's ($15\% \leq A \leq 30\%$) and IMF's ($4 \leq A \leq 30\%$) for fixed system mass $A_{TOT} = 240$ and $\eta = 0.1, 0.3, 0.5$, and 0.7 at their corresponding balance energies. The solid, dashed, dotted, and dashed-dotted lines, correspond to $\eta = 0.1, 0.3, 0.5$, and 0.7 , respectively.

ing nuclei, the percentages are taken in HMF and IMF definitions. As expected, A^{max} has a peak around 20-80 fm/c for all η . The excited compound nucleus formed in the early stage is independent of η and decays by the emission of nucleons and fragments. Therefore, the free nucleons and LCP's display a constant rise in their multiplicity. The MMF's, HMF's, and IMF's are unstable and decay at a later time. The size of largest fragment at final stage increases with increase in η whereas a reverse trend is seen for free nucleons, LCP's, MMF's, and IMF's. This is because of a decrease in the participant zone and hence net nucleon-nucleon collisions, with increase in η . The trend that HMF's follows with η is different. It is clear from the figure that the multiplicity of HMF's increases with time for larger asymmetries. Also, since balance energy for large asymmetric colliding nuclei is larger than that in small asymmetric nuclei, it takes small time for large asymmetric colliding nuclei to saturate.

In Fig. 3, we display the time evolution of average binding energy per nucleon of LCP's and IMF's for $\eta = 0.1-0.7$ by keeping the total mass fixed as 240. It is clear from the figure that independent of η , all fragments are reasonably bound. The average binding energy/nucleon is around -4 MeV/nucleon for LCP's, whereas it increases to around -8 MeV/nucleon for IMF's.

In Fig. 4, we display the normalized rapidity distri-

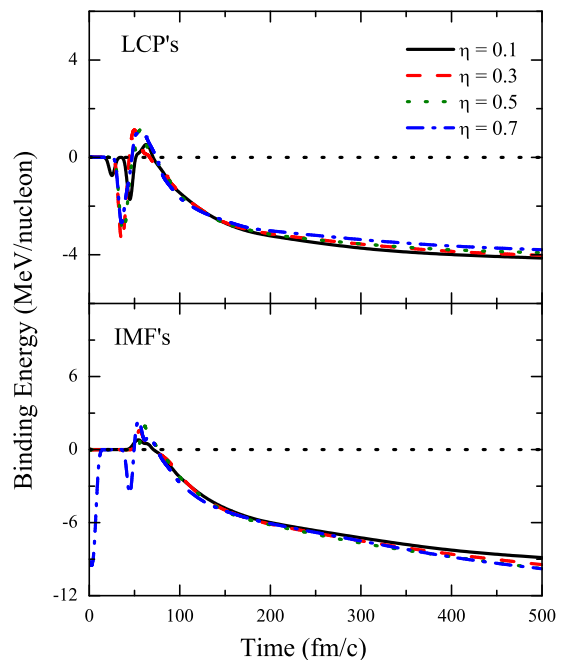


FIG. 3: (Color Online) The average binding energy per nucleon of LCP's and IMF's as a function of time for fixed system mass $A_{TOT} = 240$ and $\eta = 0.1, 0.3, 0.5$, and 0.7 at their corresponding balance energies. Lines have same meaning as in Fig. 2.

bution ($(dN/dY)_{norm}$) as a function of scaled rapidity ($Y_{c.m.}/Y_{beam}$) for $\eta = 0.1-0.7$ by keeping the system mass fixed as $A_{TOT} = 40-240$. The rapidity is defined as:

$$Y(j) = \frac{1}{2} \ln \frac{\mathbf{E}(j) + \mathbf{p}_z(j)}{\mathbf{E}(j) - \mathbf{p}_z(j)}. \quad (12)$$

Here $\mathbf{E}(j)$ and $\mathbf{p}_z(j)$ are, respectively, the total energy (nucleon) and longitudinal momentum per nucleon for the j th nucleon. The parameter $Y_{c.m.}/Y_{beam} = 0$ corresponds to the mid-rapidity (participant) zone and, hence, is responsible for the hot and compressed zone. On the other hand, $Y_{c.m.}/Y_{beam} \neq 0$ corresponds to the spectator zone, ($Y_{c.m.}/Y_{beam} < -1$ corresponds to target-like (TL) and $Y_{c.m.}/Y_{beam} > 1$ corresponds to projectile-like (PL) distributions). We see that the rapidities of nucleons emitted for $\eta = 0.1-0.7$ are not similar. Due to large balance energy for larger asymmetries, single broader Gaussian is observed that is peaked around the target rapidity, as the major contribution is due to the target in all cases. As mass asymmetry decreases, the balance energy decreases, therefore, one find peaks at target and projectile rapidities indicating a non-equilibrium situation. However, if the reactions would have been simulated at a fixed incident energy, the peak shifts toward the mid-rapidity with the decrease of the mass asymmetry and a greater thermalization would have been observed in the case of a nearly symmetric collision compared to an asymmetric collision. The same trend is seen for all fixed

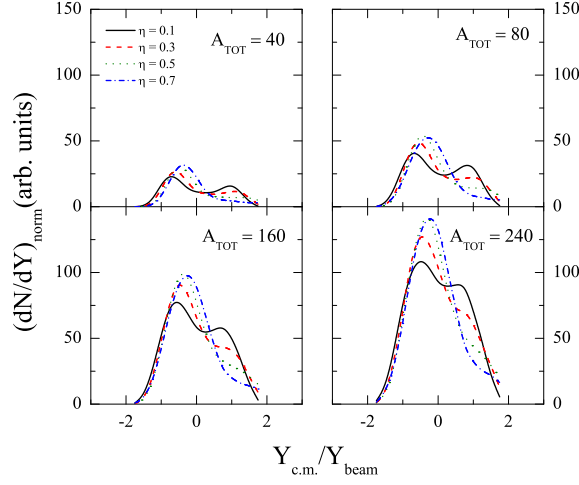


FIG. 4: (Color Online) Normalized rapidity distribution $\frac{1}{A_P+A_T} \frac{dN}{dY}$ as a function of scaled rapidity $Y_{c.m.}/Y_{beam}$ for $\eta = 0.1-0.7$ by keeping the system mass fixed as $A_{TOT} = 40-240$. Lines have same meaning as in Fig. 2.

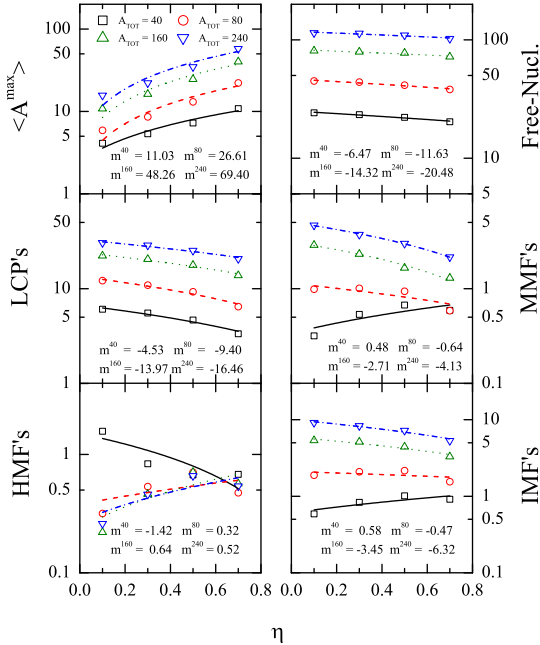


FIG. 5: (Color Online) The multiplicities of A^{max} , free nucleons, LCP's, MMF's, HMF's and IMF's as a function of mass asymmetry of colliding nuclei. The results for different system masses $A_{TOT} = 40, 80, 160,$ and 240 are represented, respectively, by the open squares, circles, triangles and inverted triangles. Lines are the linear fits ($\propto m\eta$); m values without errors are displayed.

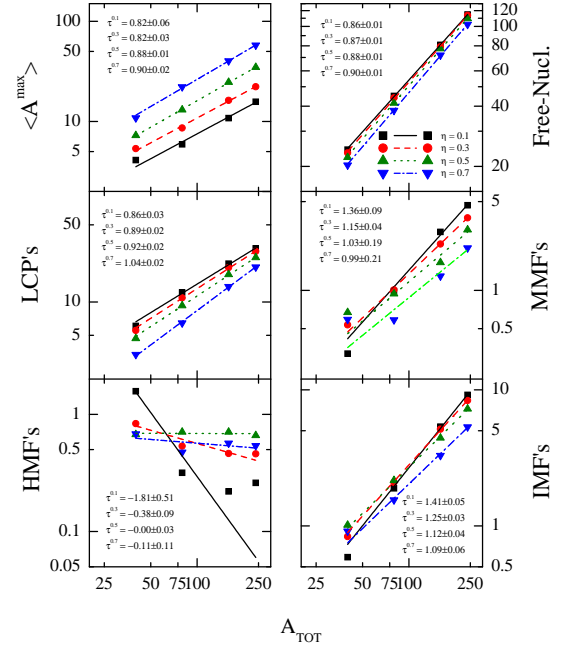


FIG. 6: (Color Online) Same as Fig. 5, but as a function of total mass of the system. The results for different asymmetries $\eta = 0.1, 0.3, 0.5,$ and 0.7 are represented, respectively, by the solid squares, circles, triangles and inverted triangles. The lines are power law ($\propto A_{TOT}^\tau$) fits to the calculated results. The values of the power factor τ are displayed in the figure for various quantities.

system masses.

In Fig. 5, we display the mass asymmetry dependence of different fragments shown in Fig. 2 for $A_{TOT} = 40-240$. Lines are the linear fits ($\propto m\eta$). The values of m are displayed in figure. The mass of the largest fragment increases with increase in η for each A_{TOT} , whereas an opposite trend is seen for free nucleons, LCP's, MMF's (except $A_{TOT} = 40$), and IMF's (except $A_{TOT} = 40$). The multiplicity of HMF's show entirely different behavior. It is clear from the figure that η dependence increases with increase in system mass. This is because of decrease in balance energy with increase in A_{TOT} . At low incident energies, the Pauli-principal hinders the nucleon-nucleon collisions and the increase of mass asymmetry further adds the same effect. While at large incident energies for smaller A_{TOT} , the role of η decreases compared to large system masses.

Similar to Fig. 5, we display the mass dependence of various fragments in Fig. 6. The mass asymmetry of the reaction is varied from 0.1 to 0.7. Lines are power law fits ($\propto A_{TOT}^\tau$); where values of power factor τ are displayed in the figure. Similar to mass symmetric reactions [10], a power law system mass dependence for various fragment multiplicities exists for larger asymmetries. All the quantities except HMF's show increasing trends for each η . It is clear from the values of τ that, for A^{max} , free nucleons, and LCP's; the mass dependence increases with increase

in η , whereas opposite trend is seen for MMF's, HMF's, and IMF's. The trend of MMF's, HMF's and IMF's with change in η from 0.1 to 0.7 in the lighter mass range gets reversed as one goes to higher mass range. This is because for lighter system mass, the incident energy is large compared to heavier system mass, therefore, large mass asymmetric colliding nuclei will produce more heavy fragments. The situation is entirely opposite for heavier system mass.

IV. SUMMARY

We presented the study of role of mass asymmetry of colliding nuclei on the fragmentation at the balance energy and on its mass dependence using quantum

molecular dynamics model. The analysis was done by keeping the total mass of the system fixed as 40, 80, 160, and 240 and by varying the mass asymmetry of the colliding nuclei from 0.1 to 0.7. We find a sizeable effect of the mass asymmetry on the multiplicity of various fragments. Our finding at the balance energy clearly point towards a power law system mass dependence of different fragment multiplicities for each mass asymmetric colliding nuclei.

Author is thankful to Dr. Rajeev K. Puri for interesting and constructive discussions. This work is supported by a research grant from the Council of Scientific and Industrial Research (CSIR), Govt. of India, vide grant no. 09/135(0563)/2009-EMR-1.

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