P_{11} Resonance Extracted from πN Data and Its Stability

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Abstract. We study the stability of resonance poles in πN P_{11} partial wave, particularly the Roper resonance, by varying parameters significantly within the EBAC dynamical coupled-channels model, keeping a good fit to the empirical amplitude. We find that two Roper poles are stable against the variation. However, for higher energies, the number of poles can change depending on how the parameters are fitted within error bars. We also developed a model with a bare nucleon which forms the physical nucleon by being dressed by the meson-cloud. We still find a good stability of the Roper poles.

Keywords: πN scattering, amplitude analysis, Roper resonance
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INTRODUCTION

For extracting N^* information, first, one needs to construct a reaction model through a comprehensive analysis of data. Then, pole positions and vertex form factors are extracted from the model with the use of the analytic continuation. Therefore, the N^* information extracted in this manner is inevitably model-dependent. Thus, commonly asked questions are how much model-dependent the extracted resonance parameters are, and how precise data have to be for a stable resonance extraction. These are the questions we address at Excited Baryon Analysis Center (EBAC) at JLab[1], within a dynamical coupled-channels model (EBAC-DCC) [2]. We focus on the πN P_{11} partial wave and the stability of its pole positions, particularly those corresponding to the Roper resonance. In the region near Roper $N(1440)$, two poles close to the $\pi\Delta$ threshold were found in our recent extraction [3] from the JLMS model [4] (JLMS is one of EBAC-DCC model), while only one pole in the similar energy region was reported in some other analyses. We examine the stability of this two-pole structure against the following variation, keeping a good reproduction of SAID single-energy (SAID-SES) solution [5].

- Large variation of the parameters of the meson-baryon and bare N^* parameters of the EBAC-DCC model.
- Inclusion of a bare nucleon state: The analytic structure of this model is different from the EBAC-DCC model in the region near the nucleon pole [6], .

DYNAMICAL COUPLED-CHANNELS MODELS

Here, we briefly describe the EBAC-DCC model and the bare nucleon model. The EBAC-DCC model contains πN, ηN and $\pi\pi N$ channels and the $\pi\pi N$ channel has $\pi\Delta$, ρN and σN components. These meson-baryon (MB) channels are connected with each other by meson-baryon interactions ($v_{MB,M'B'}$), or excited to bare N^* states by vertex interactions ($\Gamma_{MB\leftrightarrow N^*}$). With these interactions, The partial-wave amplitude for the $M(\vec{k})+B(-\vec{k})\rightarrow M'(\vec{k}')+B'(-\vec{k}')$ reaction can be conveniently decomposed into two parts as $T_{MB,M'B'}(k,k',E)=t_{MB,M'B'}(k,k',E)+t^R_{MB,M'B'}(k,k',E)$. The first term is obtained by solving the coupled-channels Lippmann-Schwinger equation with $v_{MB,M'B'}$ only. The second term is associated with the bare N^* states, and given by

$$t^R_{MB,M'B'}(k,k',E) \;=\; \sum_{i,j}\bar{\Gamma}_{MB\rightarrow N^*_i}(k,E)[D(E)]_{i,j}\bar{\Gamma}_{N^*_j\rightarrow M'B'}(k',E), \qquad (1)$$

where the dressed vertex function $\bar{\Gamma}_{N^*_j\rightarrow M'B'}(k,E)$ is calculated by convoluting the bare vertex $\Gamma_{N^*_j\rightarrow M'B'}(k)$ with the amplitudes $t_{MB,M'B'}(k,k',E)$. The inverse of the propagator of dressed N^* states in Eq. (1) is

$$[D^{-1}(E)]_{i,j}=(E-m^0_{N^*_i})\delta_{i,j}-\Sigma_{i,j}(E), \qquad (2)$$

where $m^0_{N^*_i}$ is the bare mass of the i-th N^* state, and the N^* self-energy is defined by

$$\Sigma_{i,j}(E)=\sum_{MB}\int_{C_{MB}}q^2dq\bar{\Gamma}_{N^*_j\rightarrow MB}(q,E)G_{MB}(q,E)\Gamma_{MB\rightarrow N^*_i}(q,E)\;, \qquad (3)$$

where G_{MB} is the meson-baryon propagator, and C_{MB} is the integration contour in the complex-q plane used for the channel MB.

To examine further the model dependence of resonance extractions, it is useful to also perform analysis using models with a bare nucleon, as developed in Ref. [8]. Within the formulation of EBAC-DCC model, such a model can be obtained by adding a bare nucleon (N_0) state with mass m^0_N and $N_0\rightarrow MB$ vertices and removing the direct $MB\rightarrow N\rightarrow M'B'$ in the meson-baryon interactions $v_{MB,M'B'}$. All numerical procedures for this model are identical to that used for the EBAC-DCC model, except that the resulting amplitude must satisfy the nucleon pole condition:

$$t^R_{\pi N,\pi N}(k\rightarrow k_{\rm on},k\rightarrow k_{\rm on},E\rightarrow m_N)=-\frac{[F_{\pi NN}(k_{\rm on})]^2}{E-m^0_N-\tilde{\Sigma}(m_N)}. \qquad (4)$$

with

$$m_N=m^0_N+\tilde{\Sigma}(m_N) \qquad \text{and} \qquad F_{\pi NN}(k_{\rm on})=F^{\rm phys.}_{\pi NN}(k_{\rm on})\;. \qquad (5)$$

Here we have used the on-shell momentum defined by $E=\sqrt{m_N^2+k_{\rm on}^2}+\sqrt{m_\pi^2+k_{\rm on}^2}$. Also, $\tilde{\Sigma}(m_N)$ is the self-energy for the nucleon. More details for the calculational procedure following Afnan and Pearce is found in Refs. [1, 8].

RESULTS

Now we show our numerical results to examine the stability of the P_{11} poles. We present results from various fits by varying the dynamical content of the EBAC-DCC model, and by using a model with a bare nucleon. We show in figures the quality of fits of these models, and in Table 1 the pole positions from the models as well as χ^2 per data point (χ^2_{pd}). We find the poles with the method of analytic continuation discussed in detail in Refs. [3, 9]. In Table 1, we also present pole positions from JLMS[4] and SAID-EDS (energy-dependent)[5].

First we varied both the parameters for the meson-baryon interactions ($v_{MB,M'B'}$) and parameters associated with bare N^* states ($m^0_{N^*}$, $\Gamma_{N^*\leftrightarrow MB}$) within EBAC-DCC model. The obtained meson-baryon interactions are quite different from those of JLMS. We obtained several fits which are different in how the oscillatory behavior of SAID-SES amplitude for higher W is fitted. The results from the $2N^*$-3p (dotted curves) and $2N^*$-4p (dashed curves) fits are compared with the JLMS fit (solid curves) in Fig. 1. The resulting resonance poles are listed in the 3th and 4th rows of Table 1. Here we see the first two poles near the $\pi\Delta$ threshold from both fits agree well with the poles from the JLMS fit. This seems to further support the conjecture that these two poles are mainly sensitive to the data below $W \sim 1.5$ GeV where the SAID-SES has rather small errors. However, the $2N^*$-4p fit has one more pole at $M_R = 1630 - i45$ MeV. This is perhaps related to its oscillating structure near $W \sim 1.6$ GeV (dashed curves), as shown in the Figs. 1. On the other hand, this resonance pole could be fictitious since the fit $2N^*$-3p (dotted curve) with only three poles are equally acceptable within the fluctuating experimental errors. Our result suggests that it is important to have more accurate data in the high W region for a high precision resonance extraction.

Next, we show our results obtained with the bare nucleon model, and then address the question whether difference in the analytic structure of the πN amplitude below πN threshold strongly affects the resonance extractions. The bare nucleon model is fitted to SAID-SES, and at the same time, to the nucleon pole conditions Eq. (5). Meanwhile, the original EBAC-DCC model has different singular structure below the πN threshold. The question is whether such differences can lead to very different resonance poles. Our fit

TABLE 1. The resonance pole positions M_R for P_{11} [listed as ($\mathrm{ReM_R}$, $-\mathrm{ImM_R}$) in the unit of MeV] extracted from various parameter sets. The location of the pole is specified by, e.g., $(s_{\pi N}, s_{\eta N}, s_{\pi\pi N}, s_{\pi\Delta}, s_{\rho N}, s_{\sigma N}) = (upuupp)$, where p and u denote the physical and unphysical sheets for a given reaction channel, respectively. χ^2_{pd} is χ^2 per data point.

Model	*upuupp*	*upuppp*	*uuuupp*	*uuuuup*	χ^2_{pd}
SAID-EDS	(1359, 81)	(1388, 83)	—	—	2.94
JLMS	(1357, 76)	(1364, 105)	—	(1820, 248)	3.55
$2N^*$-3p	(1368, 82)	(1375, 110)	—	(1810, 82)	3.28
$2N^*$-4p	(1372, 80)	(1385, 114)	(1636, 67)	(1960, 215)	3.36
$1N_0 1N^*$-3p	(1363, 81)	(1377, 128)	—	(1764, 137)	2.51

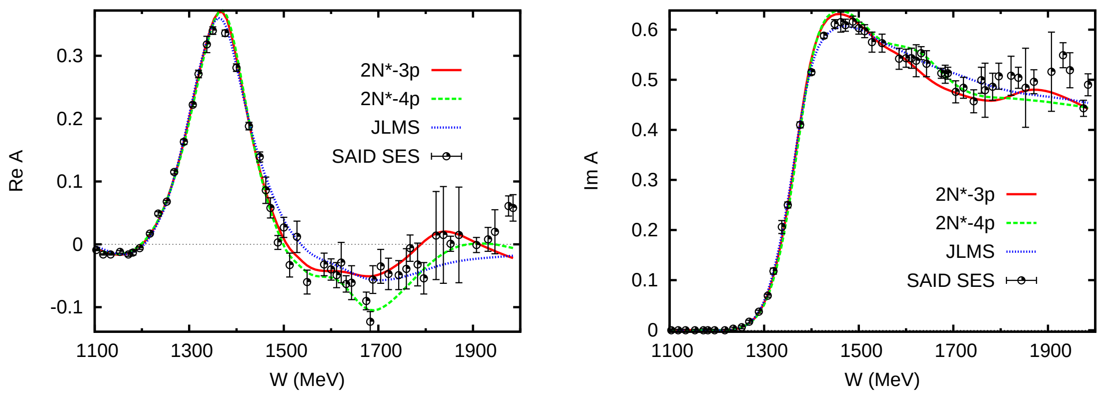

FIGURE 1. The real (left) and imaginary (right) parts of the on-shell P_{11} amplitudes as a function of the πN invariant mass W (MeV). A is unitless in the convention of Ref. [5].

of the bare nucleon model agree very well with JLMS below $W = 1.5$ GeV, while their differences are significant in the high W region. The corresponding resonance poles are given in Table 1. We also see here that the first two poles near the $\pi\Delta$ threshold are close to those of JLMS. Our results seem to indicate that these two poles are rather insensitive to the analytic structure of the amplitude in the region below πN threshold, and are mainly determined by the data in the region $m_N + m_\pi \leq W \leq 1.6$ GeV.

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