# Momentum dependence of drag coefficients and heavy flavour suppression in quark gluon plasma

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The momentum dependence of the drag coefficient of heavy quarks propagating through quark gluon plasma (QGP) has been evaluated. The results have been used to estimate the nuclear suppression factor of charm and bottom quarks in QGP. We observe that the momentum dependence of the transport coefficients plays crucial role in the suppression of the heavy quarks and consequently in discerning the properties of QGP using heavy flavours as a probe. We show that the large suppression of the heavy quarks observed at RHIC and LHC is predominantly due to the radiative losses. The suppression of $D^0$ in Pb+Pb collisions at LHC energy - recently measured by the ALICE collaboration has also been studied.

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## I. INTRODUCTION

Simulations of QCD equation of state (EoS) on lattice show that at very high temperatures and/or densities the nuclear matter undergoes a phase transition to a new state of matter called Quark-Gluon Plasma(QGP). It is expected that QGP can be produced experimentally by colliding two nuclei at ultra-relativistic energies. Relativistic Heavy Ion Collider(RHIC) at BNL and Large Hadron Collider(LHC) at CERN provide two such experimental facilities. The depletion of hadrons with high transverse momentum ($p_T$) produced in Nucleus + Nucleus collisions with respect to those produced in proton + proton (pp) collisions has been considered as a signature of QGP formation. The two main processes which cause the depletion are (i) the elastic collisions and (ii) the radiative loss or the inelastic collisions of the high energy partons with the quarks, anti-quarks and gluons in the thermal bath. The importance of elastic energy loss in QGP diagnosis was pointed out first by Bjorken [1]. Later the calculations of elastic loss had been performed with improved techniques [2, 3] and its importance was highlighted subsequently [4, 5].

The abundance of charm and bottom quarks in the partonic plasma, in the expected range of temperature to be attained in the experiments, is small. Consequently, the bulk properties of the plasma is not decided by them and hence heavy quarks may act as an efficient probe for the diagnosis of QGP. The collisional energy loss of heavy quarks [6] has gained importance recently in view of the measured nuclear suppression in the $p_T$ spectra of non-photonic single electrons [7, 8]. In the present work we focus on the energy loss of heavy quarks in QGP in deducing the properties of the medium.

Several ingredients like inclusions of non-perturbative contributions from the quasi-hadronic bound state [9], 3-body scattering effects [10], the dissociation of heavy mesons due to its interaction with the partons in the thermal medium [11] and employment of running coupling constants and realistic Debye mass [12] have been proposed to improve the description of the experimental data. For mass dependence of energy loss due to radiative processes Dokshitzer and Kharzeev [13] argue that radiative energy loss of heavy quarks will be suppressed compared to that of light quarks due to dead cone effects [14]. However, Aurenche and Zakharov claim that the radiative process has an anomalous mass dependence [15] due to the finite size of the QGP which leads to small difference in energy loss between a heavy and a light quarks. Although the authors of [16] concluded that the suppression of radiative loss for heavy quarks is due to dead cone effects but it will be fair to state that the issue is not settled yet.

The other mechanism that can affect the radiative loss is the LPM effect [17] which depends on the relative magnitude of two time scales of the system [18]: the formation time ($\tau_F$) and the mean scattering time scale ($\tau_{sc}$) of the emitted gluons. If $\tau_F > \tau_{sc}$ then LPM suppression will be effective. The LPM effect is built-in in the expression for radiative energy loss of heavy quarks derived in [19–22]. In the present work the LPM and the dead cone effects are explicitly taken into account for heavy quark dissipation.

We assume here that the light quarks and gluons thermalize before heavy quarks. The charm and bottom quarks execute Brownian motion [23–33] in the heat bath of QGP. Therefore, the interaction of the heavy quarks with QGP may be treated as the interactions between equilibrium and non-equilibrium degrees of freedom. The Fokker-Planck (FP) equation provide an appropriate framework for the evolution of the heavy quark in the expanding QGP heat bath.

The system under study has two components. The equilibrium components, the QGP, is assumed to be formed at a temperature $T_i$ at an initial time $\tau_i$ after the nuclear collisions. The QGP, due to its high internal pressure expands, as a consequence it cools and reverts to hadronic phase at a temperature, $T_c$. The

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non-equilibrium component, the heavy quarks produced due to the collision of partons of the colliding nuclei has momentum distribution determined by the perturbative QCD (pQCD), which evolves due to their interaction with the expanding QGP background. The evolution of the heavy quark momentum distribution is governed by the FP equation. The interaction of the heavy quarks with the QGP is contained in the drag and diffusion coefficients. The drag and diffusion coefficients are provided as inputs, which are, in general, dependent on both temperature and momentum. The evolution of the temperature of the background QGP with time is governed by relativistic hydrodynamics. The solution of the FP equation at the (phase) transition point for the charm and bottom quarks gives the (quenched) momentum distribution of hadrons ($B$ and $D$ mesons) through fragmentation. The fragmentation of the initial momentum distribution of the heavy quarks results in the unquenched momentum distribution of the $B$ and $D$ mesons. The ratio of the quenched to the unquenched $p_T$ distribution is the nuclear suppression factor which is experimentally measured. The quenching is due to the dragging of the heavy quark by QGP. Hence the properties of the QGP can be extracted from the suppression factor. In contrast to earlier works where the momentum dependence of the drag coefficient were ignored (or considered its value at low momentum) in the present work we emphasize on its momentum dependence.

The paper is organized as follows: In the next section we briefly describe the FP equation, in section III the drag coefficients for collisional and radiative processes have been discussed. In section IV the initial conditions and the space time evolution of the system have been described. Results have been presented in section V and section VI has been dedicated to the summary and discussions.

## II. THE FOKKER PLANCK EQUATION

The Boltzmann transport equation describing non-equilibrium statistical system reads:

$$\left[\frac{\partial}{\partial t}+\frac{\mathbf{p}}{E}.\frac{\partial}{\partial \mathbf{x}}+\mathbf{F}.\frac{\partial}{\partial \mathbf{p}}\right]f(\mathbf{x},\mathbf{p},t)=\left[\frac{\partial f}{\partial t}\right]_{collisions} \quad (1)$$

$\mathbf{F}$ represents external forces acting on heavy quark. $\mathbf{p}$ and E denote three momentum and energy of the probe respectively. In the absence of any external force in a uniform plasma and defining

$$f(\mathbf{p},t)=\frac{1}{V}\int d^3\mathbf{x} f(\mathbf{x},\mathbf{p},t) \quad (2)$$

we have for the normalized probability distribution in momentum space,

$$\frac{\partial f(\mathbf{p},t)}{\partial t}=\left[\frac{\partial f}{\partial t}\right]_{collisions}. \quad (3)$$

For the problem under consideration we need to evaluate the collision integral for a situation where one of the colliding partners is in thermal equilibrium.

To proceed in this direction, we define the collision rate of the heavy quark with the thermal gluons as:

$$w^g(\mathbf{p},\mathbf{k})=\gamma_g\int\frac{d^3\mathbf{q}}{(2\pi)^3}f_g'(\mathbf{q})v_{q,p}\sigma^g_{\mathbf{p},\mathbf{q}\rightarrow\mathbf{p}-\mathbf{k},\mathbf{q}+\mathbf{k}} \quad . \quad (4)$$

where $f_g$ is the thermal gluon distribution, $v_{\mathbf{q},\mathbf{p}}$ is the relative velocity between the two collision partners, $\gamma_g$ is the degeneracy of gluon, and $\sigma^g$ is the cross section for the heavy quark-gluon elastic interaction. Similarly the collision rate of the heavy quarks with the light quarks and anti-quarks can be defined. In terms of the transition rates the collision integral of the Boltzmann transport equation can written as:

$$\left[\frac{\partial f}{\partial t}\right]_{collisions}=\int d^3\mathbf{k}[w(\mathbf{p}+\mathbf{k},\mathbf{k})f(\mathbf{p}+\mathbf{k})-w(\mathbf{p},\mathbf{k})f(\mathbf{p})]. \quad (5)$$

Using Landau approximation *i.e.* by expanding $w(\mathbf{p}+\mathbf{k},\mathbf{k})$ in powers of $\mathbf{k}$ and keeping upto quadratic term, the Boltzmann transport equation can be written as

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial p_i}\left[A_i(\mathbf{p})f+\frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p})f]\right] \quad , \quad (6)$$

where the kernels are defined as

$$A_i=\int d^3\mathbf{k}w(\mathbf{p},\mathbf{k})k_i \quad , \quad (7)$$

and

$$B_{ij}=\frac{1}{2}\int d^3\mathbf{k}w(\mathbf{p},\mathbf{k})k_ik_j. \quad (8)$$

Eq. 6 is a nonlinear integro-differential equation known as the Landau kinetic equation. For the problem under consideration one of the colliding partner is in equilibrium. In such situation the distribution function which appears in $w$ can be replaced by thermal distribution. As a consequence Eq.6 becomes a linear partial differential equation, known as Fokker-Planck equation. Assuming $A_i=p_i\gamma(p)$ and $B_{ij}=D(p)\delta_{ij}$, the FP equation can be written as:

$$\begin{aligned}\frac{\partial f}{\partial t} &= C_1(p_x,p_y,t)\frac{\partial^2 f}{\partial p_x^2}+C_2(p_x,p_y,t)\frac{\partial^2 f}{\partial p_y^2}\\ &+ C_3(p_x,p_y,t)\frac{\partial f}{\partial p_x}+C_4(p_x,p_y,t)\frac{\partial f}{\partial p_y}\\ &+ C_5(p_x,p_y,t)f+C_6(p_x,p_y,t). \quad . \end{aligned} \quad (9)$$

where,

$$C_1 = D \quad (10)$$

$$C_2 = D \quad (11)$$

$$C_3 = \gamma\, p_x + 2\,\frac{\partial D}{\partial p_T}\,\frac{p_x}{p_T} \quad (12)$$

$$C_4 \; = \; \gamma \, p_y \; + 2 \, \frac{\partial D}{\partial p_T} \, \frac{p_y}{p_T} \qquad (13)$$

$$C_5 \; = \; 2 \, \gamma \; + \frac{\partial \gamma}{\partial p_T} \, \frac{p_x^2}{p_T} \; + \frac{\partial \gamma}{\partial p_T} \, \frac{p_y^2}{p_T} \qquad (14)$$

$$C_6 \; = \; 0 \;\; . \qquad (15)$$

where the momentum, $\mathbf{p} = (\mathbf{p}_T, p_z) = (p_x, p_y, p_z)$. We numerically solve Eq. 9 [34] with the boundary conditions: $f(p_x, p_y, t) \to 0$ for $p_x$,$p_y \to \infty$ and the initial (at time $t = \tau_i$) momentum distribution of charm and bottom quarks are taken MNR code [35]. It is evident from Eq. 9 that with the momentum dependence transport coefficients the FP equation becomes complicated. It is possible to write down the solution of the FP equation in closed analytical form [27] in the special case of momentum independent drag and diffusion coefficients. The computer code used for the solution of Eq. 9 has been used to reproduce the closed form analytical solution of Ref. [27].

## III. DRAG COEFFICIENT

### A. Collisional process

The drag coefficient, $\gamma_{coll}$ due to collisional process can be written as [26]:

$$\begin{aligned}
\gamma_{coll} \; &= \; \frac{1}{2E_p} \int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_q} \int \frac{d^3\mathbf{q'}}{(2\pi)^3 2E_{q'}} \\
&\times \; \int \frac{d^3\mathbf{p'}}{(2\pi)^3 2E_{p'}} \, \frac{1}{\gamma_Q} \sum |M|^2 \\
&\times \; (2\pi)^4 \delta^4(p+q-p'-q') f'(\mathbf{q}) [1 - \frac{\mathbf{p}.\mathbf{p'}}{p^2}] \quad (16)
\end{aligned}$$

where $\mathbf{p'} = \mathbf{p} - \mathbf{k}$ and $\mathbf{q'} = \mathbf{q} + \mathbf{k}$. The scattering matrix elements are given explicitly in Ref. [36]. The integrations in Eq. 16 has been performed using the standard techniques [26, 32].

### B. Radiative process

The drag coefficient due to the radiative process, $\gamma_{rad}$ can be related to the energy loss as follows:

$$-\left[\frac{dE}{dx}\right]_{rad} = \gamma_{rad} \; p \quad , \qquad (17)$$

where $p$ is the momentum of the particle. We evaluate the radiative energy loss by using the following gluon spectrum [37]:

$$\frac{dn_g}{d\eta d^2k_\perp} = \frac{C_A \alpha_s}{\pi^2} \, \frac{q_\perp^2}{k_\perp^2 [(\mathbf{q}_\perp - \mathbf{k}_\perp)^2 + m_D^2]} \quad . \qquad (18)$$

where $k = (k_0, k_\perp, k_3)$ is the four momentum, $\eta = 1/2 \ln[(k_0 + k_3)/(k_0 - k_3)]$ is the rapidity of the emitted gluon and $q = (q_0, q_\perp, q_3)$ is the four momentum of the exchanged (propagator) gluon. $C_A = 3$ is the Casimir invariant of the $SU(3)$ adjoint representation, $\alpha_s = g^2/4\pi$ is the strong coupling and $m_D$ is the Debye mass.

The dead-cone effect is taken into account through the factor, F [13, 14]

$$F = \frac{k_\perp^2}{k_0^2 \theta_0^2 + k_\perp^2} \quad , \qquad (19)$$

where $\theta_0 = M/E$. The average energy per collision, $\epsilon$ is [38, 39]

$$\begin{aligned}
\epsilon \; &= \; \langle n_g k_0 \rangle = \int d\eta \; d^2k_\perp \; \frac{dn_g}{d\eta d^2 k_\perp} \\
&\times \; k_0 \; \Theta(\tau_{sc} - \tau_F) \; \Theta(E - k_\perp \cosh \eta) F^2 \quad , \quad (20)
\end{aligned}$$

where the formation time of the emitted gluon [40], $\tau_F = (C_A/2C_2) \, 2\cosh\eta/k_\perp$, and $C_A/2C_2 = N^2/(N^2-1)$ for quarks with $C_2 = C_F = 4/3$. We perform the integrations in Eq. 20 and substitute the value of $\mathbf{q}_\perp$ by its average value which is obtained as,

$$\langle q_\perp^2 \rangle = \frac{1}{\sigma} \int_{m_D^2}^{(q_\perp^{max})^2} dq_\perp^2 \, \frac{d\sigma}{dq_\perp^2} \, q_\perp^2 \quad , \qquad (21)$$

where,

$$(q_\perp^{max})^2 = \frac{s}{4} - \frac{M^2}{4} + \frac{M^4}{48pT} \, \ln\left[\frac{M^2 + 6ET + 6pT}{M^2 + 6ET - 6pT}\right], \qquad (22)$$

and $s \approx 6ET$, is the centre of mass energy squared for the scattering of a heavy quark with energy $E$ off the thermal partons at temperature $T$.

The LPM effect is taken into account by introducing non-zero formation time of the emitted gluon through the first theta function in Eq. 20. The scattering time scale, $\tau_{sc}$ can be estimated from the scattering rate ($\Lambda$) by using the relation, $\tau_{sc} = \Lambda^{-1}$, where $\Lambda$ is given by:

$$\begin{aligned}
\Lambda \; &= \; \frac{1}{2E_p} \int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_q} \int \frac{d^3\mathbf{q'}}{(2\pi)^3 2E_{q'}} \\
&\times \; \int \frac{d^3\mathbf{p'}}{(2\pi)^3 2E_{p'}} \, \frac{1}{\gamma_Q} \sum |M|^2 \\
&\times \; (2\pi)^4 \delta^4(p+q-p'-q') f(q') \qquad (23)
\end{aligned}$$

Note that deletion of $k_i$ from the expression for $A_i$ (Eq. 7) gives the scattering rate, $\Lambda$. Knowing $\epsilon$ and $\Lambda$ the energy loss of heavy quark can be expressed as:

$$-\left[\frac{dE}{dx}\right]_{rad} = \Lambda \epsilon \quad . \qquad (24)$$

The drag due to radiative loss can now be estimated using Eqs. 16 and 24. The effective drag due to collisional and radiative processes is obtained as: $\gamma = \gamma_{coll} + \gamma_{rad}$. It should be mentioned that the radiative process is not fully independent of the collisional one. Therefore, the collisional and radiative transport coefficient may not be added to obtain the effective one. In absence of any rigorous method, however, we add them up. Since the radiative loss is larger than the collisional one, therefore, this approximation may not be treated as unreasonable. The diffusion coefficient has been obtained from the drag coefficients by using the Einsteins relation, $D = \gamma MT$, where $M$ is the mass of the heavy quark. In our calculations the temperature dependence of $\alpha_s$ [41] has been taken into account.

## IV. INITIAL CONDITIONS

In order to solve Eq. 9 we supply the initial distribution functions, $f_{in}(p_T, t)$ for charm and bottom quarks from the well known MNR code [35]. The initial temperature,$T_i$ and the initial thermalization time, $\tau_i$ for the background QGP expected to be formed at RHIC and LHC can be constrained to the total multiplicity as follows:

$$T_i^3 \tau_i \approx \frac{2\pi^4}{45\zeta(3)} \frac{1}{4a_{eff}} \frac{1}{\pi R_A^2} \frac{dN}{dy} \quad , \tag{25}$$

where $R_A$ is the radius of the system, $\zeta(3)$ is the Riemann zeta function, $a_{eff} = \pi^2 g_{eff}/90$, and $g_{eff}(= 2 \times 8 + 7 \times 2 \times 2 \times 3 \times N_F/8)$ is the degeneracy of quarks and gluons in QGP and $N_F$ is the number of flavours. The value of the transition temperature, $T_c$ has been taken to be 175 MeV. We have used the boost invariant model of relativistic hydrodynamics proposed by Bjorken [42] for the space time evolution of the expanding QGP back ground. The geometry of the collision and the space time evolution has been described in detail earlier, therefore, we avoid further discussions here and refer to our previous work [32] for details. The value of $T_i$ and $\tau_i$ for the QGP fireball are taken as $T_i = 300$ MeV and $\tau_i = 0.5$ fm/c. The corresponding quantities for LHC are $T_i = 550$ MeV and $\tau_i = 0.1$ fm/c. The pressure ($P$)-energy density($\epsilon$) relation for the QGP has been taken as $P = \epsilon/3$.

## V. RESULTS

With the formalism described and the inputs mentioned above, we present the results now. The momentum dependence of the drag coefficient ($\gamma$) of charm quark propagating through QGP has been displayed in Fig. 1 for $T = 300$ MeV. The variation of $\gamma$ with $p$ is non-negligible both for radiative as well as collisional processes. The value of $\gamma$ due to collisional processes at $p = 5$ GeV is about 0.036 fm$^{-1}$ which reduces to a value of 0.018 fm$^{-1}$ at $p = 10$ GeV. Such a variation, which was neglected earlier, will have crucial consequences on the nuclear suppression factor, $R_{\rm AA}$ for the charm and bottom quarks. In the inset of Fig. 1 we display the drag coefficient for collisional process for the low momentum

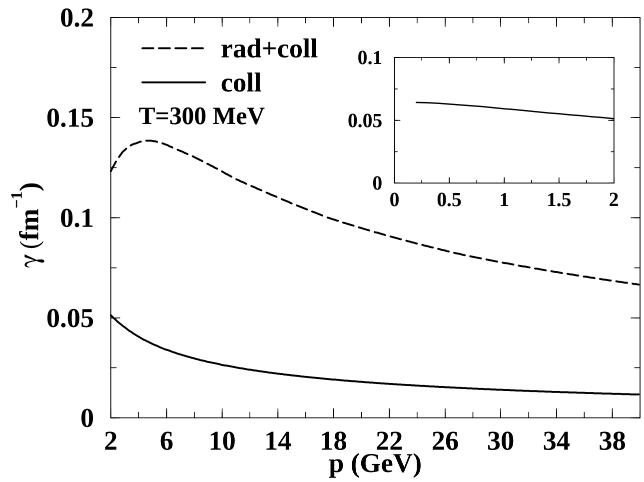


FIG. 1: Drag coefficients of charm assuming running strong coupling, $\alpha_s(T)$ and temperature dependent Debye screening mass, $m_D(T)$ for gluon, quark, and antiquark scattering. Inset: The variation of drag with $p$ in the low domain.

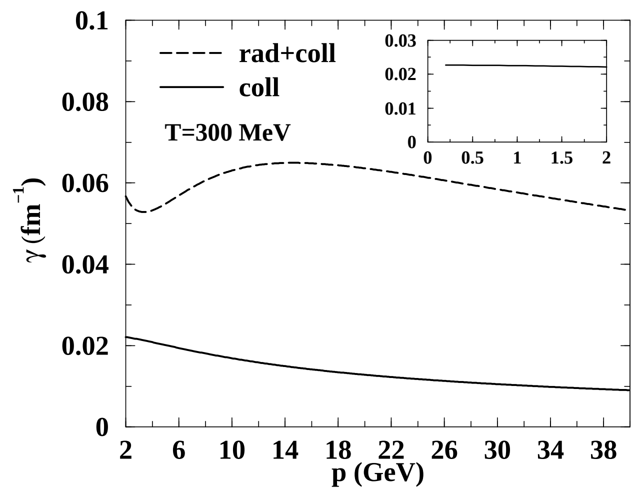


FIG. 2: Drag coefficients of bottom assuming running strong coupling, $\alpha_s(T)$ and temperature dependent Debye screening mass, $m_D(T)$ for gluon, quark, and antiquark scattering. Inset: The variation of drag with $p$ in the low domain.

domain. In the limited momentum range the drag remains almost independent of momenta. This constant values of drag has been used earlier in the FP equation and subsequently the solution was used in estimating the nuclear suppression factor. Since the value of $\gamma$ reduces with $p$ one will over estimate the suppression by taking its value at low $p$. For the low momentum region we use $\gamma = \gamma_{coll}$. In Fig. 2 we depict the drag coefficients of a bottom quark which shows slower (compared to charm) variation with momentum. The drag coefficient for collisional loss has been shown in the inset of Fig. 2 for low momentum domain.

The suppression of both charm and bottom quarks (before fragmentation to hadrons) are plotted against $p_T$ in Figs. 3 and 4 respectively. We note that if one takes the drag to be momentum independent (or more precisely takes the value of $\gamma$ at low $p$ and extends it upto very high $p$) then the drag due to collisional process causes about 50% suppression (dashed line). However, if we take into account the variation of $\gamma$ with $p$ obtained from pQCD calculation then about 20% of suppression can be achieved, *i.e.* the contribution from the collisional loss becomes smaller with the momentum dependent drag. Therefore, the observed large suppression of the heavy quarks at RHIC is predominantly due to radiative loss. In fact, the inclusion of the radiative processes increases the suppression to about 75%. This can be understood from the fact that the drag due to the radiative loss is large. The suppression of the bottom quark is much less because of the smaller values of drag and initial harder momentum distribution (Fig. 4).

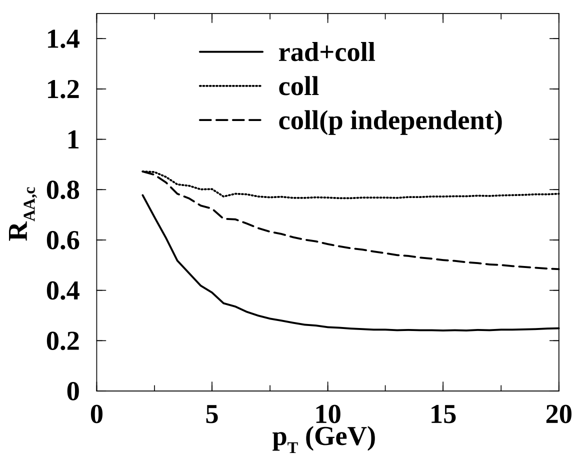


FIG. 3: Suppression of momentum of charm quarks in QGP as a function $p_T$

The hadronization of charm and bottom quarks to $D$ and $B$ mesons respectively are done by using Peterson fragmentation function [43]. The variation of $R_{AA}$ with $p_T$ of the $D$ and $B$ mesons has been displayed in Fig. 5 for RHIC initial condition ($T_i = 300$ MeV). The suppression for bottom is much less for the reasons mentioned earlier. The theoretical results show a slight upward trend for $p_T$ above 10 GeV both for mesons containing charm and bottom quarks. Similar trend has recently been experimentally observed for light mesons at LHC energy [44]. This may originate from the fact that the drag (and hence the quenching) for charm and bottom quarks are less at higher momentum.

The same formalism is extended to evaluate the nuclear suppression factor, $R_{AA}$ both for charm and bottom at LHC energy. Result has been compared with the recent ALICE data(Ref. [45]) in Fig. 6. The data is reproduced well by assuming formation of QGP at an initial temperature $\sim 550$ MeV after Pb+Pb collisions at $\sqrt{s_{\rm NN}} = 2.76$ TeV.

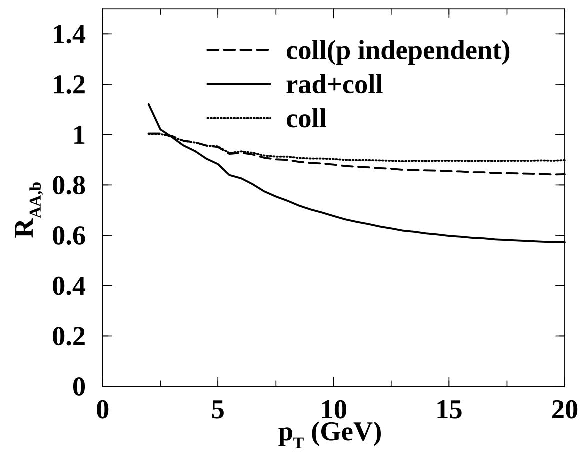


FIG. 4: Suppression of momentum of bottom quarks in QGP as a function $p_T$

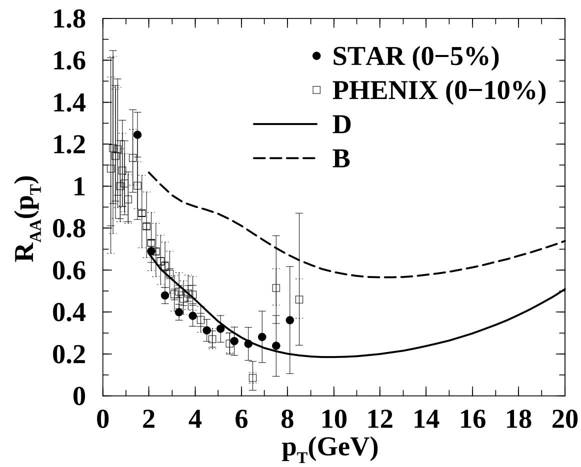


FIG. 5: $R_{AA}$ as a function of $p_T$ for $D$ and $B$ mesons at RHIC. Experimental data taken from [7] and [8].

## VI. SUMMARY AND DISCUSSIONS

The temperature and momentum dependence of drag and diffusion coefficients of heavy quarks interacting with the thermal partonic medium have been evaluated for both the elastic and inelastic interactions. We have employed both the dead-cone and the LPM effects in the calculation for the inelastic processes. The initial $p_T$ distributions for charm and bottom are taken from MNR code [35]. The FP equation has been solved with the momentum dependent transport coefficients and subsequently the nuclear suppression factors, $R_{AA}$ have been calculated for $D$ and $B$ mesons for RHIC and LHC conditions. The momentum dependence of the drag coefficient

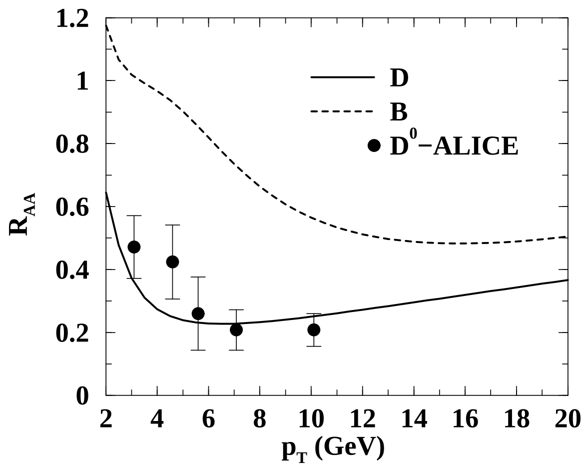


FIG. 6: $R_{AA}$ as a function of $p_T$ for $D$ and $B$ mesons at LHC. Experimental data taken from [45].

is found to be crucial in reproducing the trend in the $p_T$ dependence of the experimental data. It has been seen that the radiative loss plays more dominant role than the collisional process.

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[1] J. D. Bjorken, Fermi-lab preprint 82/59-THY, 1982(unpublished)
[2] M. H. Thoma and M. Gyulassy, Nucl. Phys. B **351**, 491(1991)
[3] A. Peshier, Phys. Rev. Lett. **97**, 212301(2006)
[4] A. K. Dutt-Mazumder, J. Alam, P. Roy and B. Sinha, Phys. Rev. D **71** 094016 (2005).
[5] M. G. Mustafa and M. H. Thoma, Acta Phys. Hung. A **22**, 93 (2005).
[6] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 2625 (1991).
[7] B. I. Abeleb *et al.* (STAR Collaboration), Phys. Rev. Lett. **98**, 192301 (2007).
[8] A. Adare *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **98**, 172301 (2007).
[9] H. van Hees, M. Mannarelli, V. Greco and R. Rapp, Phys. Rev. Lett. **100**, 192301 (2008).
[10] C. M. Ko and W. Liu, Nucl. Phys. A **783**, 23c (2007).
[11] A. Adil and I. Vitev, Phys. Lett. B **649**, 139 (2007).
[12] P. B. Gossiaux and J. Aichelin, Phys. Rev. C **78**, 014904 (2008).
[13] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B, **519**, 199 (2001).
[14] R. K. Ellis, W. J. Stirling and B. R. Webber, QCD and Collider Physics, Cambridge University Press, Cambridge, 1996.
[15] B. G Zakharov, JETP Lett. **86**, 444(2007); P. Aurenche and B. G. Zakharov, JETP Lett. **90**, 237 (2009).
[16] Roy A. Lacey *et al.*, Phys. Rev. Lett.**103**,142302(2009)
[17] M. Gyulassy and X.-N. Wang, Nucl. Phys. B **420**, 583 (1994).
[18] S. Klein, Rev. Mod. Phys. **71**, 1501 (1999)
[19] N. Armesto, C. A. Salgado and U. A. Wiedemann, Phys. Rev. D **69**, 114003 (2004).
[20] B. W. Zhang, E. Wang and X.-N. Wang, Phys. Rev. Lett. **93**, 072301 (2004).
[21] N. Armesto, A. Dainese, C. A. Salgado and U. A. Wiedemann, Phys. Rev. D **71**, 054027 (2005).
[22] M. Djordjevic and M. Gyulassy, Nucl. Phys. A **733**, 265 (2004).
[23] G. D. Moore and D. Teaney, Phys. Rev. C **71**, 064904 (2005).
[24] J. Alam, S. Raha and B. Sinha, Phys. Rev. Lett. **73**, 1895 (1994).
[25] S. Chakraborty and D. Syam, Lett. Nuovo Cim. **41**, 381 (1984).
[26] B. Svetitsky, Phys. Rev. D **37**, 2484( 1988).
[27] H. van Hees, R. Rapp, Phys. Rev. C,**71**, 034907 (2005).
[28] S. Turbide, C. Gale, S. Jeon and G. D. Moore, Phys. Rev. C **72**, 014906 (2005).
[29] J. Bjoraker and R. Venugopalan, Phys. Rev. C **63**, 024609 (2001).
[30] P. Roy, J. Alam, S. Sarkar, B. Sinha and S. Raha, Nucl. Phys. A **624**, 687 (1997).
[31] P. Roy, A. K. Dutt-Mazumder and J. Alam, Phys. Rev. C **73**, 044911 (2006).
[32] S. K. Das, J. Alam and P. Mohanty, Phys. Rev. C **82**, 014908 (2010); S. K. Das, J. Alam and P. Mohanty, Phys. Rev. C **80**, 054916 (2009); S. K. Das, J. Alam, P. Mohant and B. Sinha, Phys. Rev. C **81**, 044912 (2010).
[33] W. M. Alberico *et al.*, arXiv:1101.6008 [hep-ph].
[34] H. M. Antia, Numerical Methods for Scientists and Engineers, Tata McGraw-Hill, 1991.
[35] M. L. Mangano, P. Nason and G. Ridolfi, Nucl. Phys. B **373**, 295 (1992).
[36] B. L. Combridge, Nucl. Phys.B **151**, 429(1979)
[37] J.F. Gunion, G.Bertsch, Phys. Rev. D, **25**,746(1982).
[38] X. W. Chang *et al.*, Chin. Phys. Lett **22**, 72(2005)
[39] M. G. Mustafa, D. Pal, D. K. Srivastava, and M. H. Thoma, Phys. Lett. B **428**, 234(1998)
[40] X. N. Wang, M. Gyulassy and M. Plümer, Phys. Rev. D **51**, 3436 (1995).
[41] O. Kaczmarek and F. Zantow, Phys. Rev. D, **71**, 114510(2005)
[42] J. D. Bjorken, Phys. Rev. D **27** , 140(1983).
[43] C. Peterson *et al.*, Phys. Rev. D **27**, 105 (1983).

[44] Y. J. Lee (for CMS collaboration), Quark Matter, 2011, May 23-28, Annecy, France.
[45] A. Rossi (for ALICE collaboration), Quark Matter, 2011, May 23-28, Annecy, France.