

**DISCUSSION OF: A STATISTICAL ANALYSIS OF MULTIPLE
TEMPERATURE PROXIES: ARE RECONSTRUCTIONS OF
SURFACE TEMPERATURES OVER THE LAST
1000 YEARS RELIABLE?¹**

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This article (MW) has stimulated much valuable discussion and helped to focus attention on an important area for the application of statistics. Given the short amount of space, however, we reluctantly comment only on the second and last sections.

Excursions in the history of science. Although Section 2 of this paper is lively reading, we feel that the viewpoint is not balanced and emphasizes statistical correctness over the broader issues of scientific understanding. Recounting a controversy that has both a political dimension and involves scientific issues from several disciplines is perhaps better left to a historian of science. Wegman’s quote on page 9 of the article is actually from a later written response to Representative Stupak, not from the original testimony [see Questions surrounding the hockey stick (2006)]. We encourage readers to also read the transcript of the congressional hearings and the contemporaneous report by the National Academies, NRC (2006) to follow this debate.

Paleoclimate reconstructions. The Wegman committee’s original report stopped short of redoing the temperature reconstruction with Mann’s data and with the correct centering of the principal components. Although this exercise was beyond the report’s charge, it is sound statistical practice to evaluate changes in intermediate methodology by their influence on the final statistical inference. The string of references that are cited by MW on page

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10 beginning with Mann and Rutherford (2002) established the robustness of the reconstruction with respect to centered versus noncentered methods if several PCs are included. This is a finding that might have been uncovered by the Wegman committee as well. In this context, we applaud MW for carrying through to a reconstruction to assess the impact of methodological choices. We term the model used in Section 5 a *direct* approach because it builds a predictive regression model for temperature directly from the proxies. To complement this article, we discuss an *indirect* approach that takes advantage of some current work in Bayesian statistics.

A Bayesian hierarchical model (BHM). Although a direct approach may be useful for comparison with previous work, we hold that a BHM provides a better solution to the reconstruction problem. A BHM can be described as *indirect* in that one models the dependence of the proxies conditional on temperature. Bayes’ theorem is then used to invert the relationship to arrive at a posterior predictive distribution of temperature given the data. We sketch this approach below using seminal ideas from Tingley and Huybers (2010) and some features from Li, Nychka and Ammann (2010). Let \mathbf{T}_t be the true temperatures on a grid at time t and let Northern Hemisphere (NH) temperature, y_t , be a linear combination of the \mathbf{T}_t . A possible HBM for this problem is:

<i>Data level:</i>	Proxies	$x_{t,i} = \gamma_i \mathbf{h}_i \mathbf{T}_t + u_{t,i}$
<i>Process level:</i>	Space–time process: NH mean process:	$\mathbf{T}_t = y_t \mathbf{1} + \mathbf{v}_t; \mathbf{v}_t = A \mathbf{v}_{t-1} + \mathbf{e}_t; \mathbf{e}_t \sim N(0, \Sigma)$ $y_t = \mu + S_t \omega_S + V_t \omega_V + C_t \omega_C + w_t$
<i>Prior level:</i>		$[\boldsymbol{\gamma}, \boldsymbol{\omega}, A, \Sigma, \dots]$

The data equation asserts that the i th proxy at time t is a linear combination of the true temperature field plus noise. \mathbf{h}_i is a known row vector of weights and γ_i an unknown parameter to “calibrate” each proxy. The errors, $u_{t,i}$, for each proxy (i) may have autocorrelation but we will assume that between proxies the noise time series are independent—the goal is to explain correlation among proxies by the temperature field (or other geophysical variables). At the Process level the temperature field evolves as a space–time process with the variation in the NH average prescribing its mean level. Here \mathbf{v}_t is assumed to be a first-order vector autoregressive process with A and Σ determining the spatial dependence. The NH mean level reflects the basic energy balance of the Earth’s climate system. The external series of solar radiance (S), volcanic dust (V) and carbon dioxide concentrations (C) are large scale drivers of temperature. A low-order autoregressive process, w_t , reflects additional interannual variation. Finally, the prior level favors diffuse priors on unknown statistical parameters but can also consolidate information across many similar parameters. Specifically, priors for the

regression parameters, $\{\gamma_i\}$ can borrow strength across proxies and control overfitting. Given this hierarchy, one samples the predictive distribution for y_t using Bayes' theorem and Markov chain Monte Carlo.

Benefits of the hierarchical approach. It is better to model how a proxy depends on observed climate rather than formulating a prediction model through a direct relationship. Climate scientists working on a particular proxy spend much effort in understanding and quantifying this forward relationship: conditional on the climate what would be the response of the proxy? Thus, the data level is a useful framework to incorporate their expert knowledge into the reconstruction. The process level is attractive to geoscientists as well because it builds in constraints in the reconstruction that are reasonable and well accepted. One strategy for formulating this process model is to use the output from high resolution climate system models to identify the form of the vector autoregression (A) and the spatial correlations among the innovations (Σ). The BHM addresses missing and irregular proxy information and temperatures in a consistent way. The posterior distribution can be sampled when proxies are missing over different time periods and so a single statistical model is used to derive the reconstruction at all times. This is in contrast to the direct approach where one has to use a separate model for each different subset of proxies that are available for a given reconstruction period.

The hierarchical model and the indirect approach avoid the problem of proxy centering that was first encountered by Mann. Direct approaches, even using the Lasso, can suffer from the attenuation effects caused by measurement errors in proxies [Ammann, Genton and Li (2010)]. If this effect is not corrected, the RMSE could be misleading. For example, the in-sample mean could have smaller RMSE than a biased reconstruction due to attenuation. However, the biased reconstruction could capture the basic structure of the temperature process while the in-sample mean contains no information. In contrast, the hierarchical models with the indirect approach are free of this concern. Overall, we believe that HBM are a success in transferring mainstream statistical ideas to a substantial application in the geosciences and we thank the authors again for initiating this discussion.

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