α_s with GAPP

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The FORTRAN package GAPP [1] (Global Analysis of Particle Properties) computes so-called pseudo-observables and performs least-χ^2 fits in the $\overline{\rm MS}$ scheme. Fit parameters besides α_s and M_H include the heavy quark masses which are determined from QCD sum rule constraints thus affecting and being affected by α_s. When possible, analytical expressions (or expansions) are used to capture the full dependence on α_s and the other fit parameters.

Z-pole observables from LEP 1 and SLC include the Z-width, Γ_Z, hadronic-to-leptonic partial Z-width ratios, R_ℓ, and the hadronic peak cross section, $\sigma_{\rm had}$. These are most sensitive to α_s by far, but the weak angle enters and needs to be known independently. Thus, the extracted α_s depends on the set of other, purely electroweak (EW) measurements employed in the fits, such as various asymmetries and experiments exploiting parity violation. The statistical and systematic experimental correlations of Γ_Z, $\sigma_{\rm had}$ and the R_ℓ are known, small and included. The parametric uncertainties (such as from M_H) are non-Gaussian but treated exactly. The theoretical errors in Γ_Z, $\sigma_{\rm had}$, and the R_ℓ are identical, and induce a negligibly small uncertainty in $\Delta\alpha_s(M_Z) = \pm 0.00009$, dominated ($\pm 0.00007$) by the axial-vector singlet contribution [2] which is unknown at $\mathcal{O}(\alpha_s^4)$. As in the case of τ decays, one may opt for either fixed-order perturbation theory (FOPT) or contour-improved perturbation theory (CIPT) [3], and we take the difference[1] as the massless non-singlet uncertainty (± 0.00005). The W-width also features a strong α_s dependence, but it is currently not competitive and usually interpreted rather as a measurement of a combination of CKM matrix elements.

The global EW fit excluding τ decays (the Z-pole alone) yields $\alpha_s(M_Z) = 0.1203 \pm 0.0027$ (0.1198 ± 0.0028). These results are expected to be stronger affected by physics beyond the Standard Model than other α_s determinations which is the primary reason to include another α_s constraint in the fits as a control. If the new physics affects only the gauge boson propagators (oblique corrections) the resulting $\alpha_s(M_Z) = 0.1199^{+0.0027}_{-0.0030}$ hardly changes, while allowing new physics corrections to the $Zb\bar{b}$-vertex gives the lower $\alpha_s(M_Z) = 0.1167 \pm 0.0038$.

As the aforementioned α_s control we choose the τ lifetime, τ_τ, not least because of its transparent (even if controversial) theory uncertainty. Our master formula [4] reads,

$$\tau^{\rm expt} \equiv \tau[\mathcal{B}^{\rm expt}_{e,\mu}, \tau^{\rm expt}_{\rm direct}] = \hbar \frac{1 - \mathcal{B}^{\rm expt}_s}{\Gamma^{\rm theo}_e + \Gamma^{\rm theo}_\mu + \Gamma^{\rm theo}_{ud}} = 291.09 \pm 0.48 \text{ fs}\,, \tag{1}$$

where $\tau^{\rm expt}_{\rm direct} = 290.6\ (1.0)$ fs is the directly measured τ lifetime [5]. $\tau[\mathcal{B}^{\rm expt}_{e,\mu}] = 291.24\ (0.55)$ fs is the combination of indirect determinations, using $\tau[\mathcal{B}_{e,\mu}] = \hbar\,\mathcal{B}^{\rm expt}_{e,\mu}/\Gamma^{\rm theo}_{e,\mu}$ and the experimental branching ratios, $\mathcal{B}^{\rm expt}_e = 0.1785\ (5)$ and $\mathcal{B}^{\rm expt}_\mu = 0.1736\ (5)$, together with their 13% anti-correlation [5]. Decays into net strangeness, S, are plagued by the uncertainty in the $\overline{\rm MS}$ strange mass, $\hat{m}_s(m_\tau)$, and a poorly converging QCD series proportional to $\hat{m}_s^2$, so that in Eq. (1) we employ the measured $\Delta S = -1$ branching ratio, $\mathcal{B}^{\rm expt}_s = 0.0286\ (7)$ [5].

[1] This difference has the opposite sign from τ decays indicating that their theory errors are uncorrelated.

The partial τ-width into light quarks contains logarithmically enhanced EW corrections, $S(m_\tau, M_Z) = 1.01907 \pm 0.0003$ [6], and reads (employing FOPT as advocated in Ref. [7]),

$$\Gamma_{ud}^{\rm theo} = \frac{G_F^2 m_\tau^5 |V_{ud}|^2}{64\pi^3} S(m_\tau, M_Z) \left(1 + \frac{3}{5}\frac{m_\tau^2}{M_W^2}\right) \times \left(1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.202\frac{\alpha_s^2}{\pi^2} + 26.37\frac{\alpha_s^3}{\pi^3} + 127.1\frac{\alpha_s^4}{\pi^4} - 1.393\frac{\alpha(m_\tau)}{\pi} + \delta_q\right), \tag{2}$$

where δ_q collects quark condensate, $\delta_{\rm NP}$ [8], as well as heavy and light quark mass effects. The dominant experimental and theoretical errors are given in the following tables, respectively:

source	uncertainty	$\Delta\alpha_s(M_Z)$
$\Delta\tau^{\rm expt}$	± 0.48 fs	∓ 0.00039
$\Delta\mathcal{B}_s^{\rm expt}$	± 0.0007	∓ 0.00017
ΔV_{ud}	± 0.00022	∓ 0.00007
Δm_τ	± 0.17 MeV	∓ 0.00002
total		0.00043

source	uncertainty	based on	$\Delta\alpha_s(M_Z)$
PQCD	∓ 0.0119	α_s^4-term	${}^{+0.00167}_{-0.00137}$
RGE	$\beta_4 = \mp 579$	[1]	${}^{+0.00038}_{-0.00034}$
$\delta_{\rm NP}$	± 0.0038	[8]	∓ 0.00048
~~OPE~~	± 0.0008	[9] & [10]	∓ 0.00012
total			${}^{+0.00178}_{-0.00150}$

The perturbative QCD (PQCD) error dominates and is estimated as the α_s^4-term in Eq. (2). It is re-calculated in each call in the fits to access its α_s-dependence and features asymmetric. It basically covers the range from the higher values favored by CITP down to the lower ones one obtains from assuming that the roughly geometric form of FOPT continues. Note that if CIPT is used, the error from the renormalization group evolution (RGE) parametrized by the unknown 5-loop β-function coefficient, β_4, and part of the PQCD error are correlated. Effects breaking the operator product expansion, ~~OPE~~, are estimated by assuming the instanton motivated functional form [9], $A\,\alpha_s^{-6}\exp[-2\pi/\alpha_s(s_0)]$, and adjusting A to the difference between the OPE and data curves in Fig. 22 of Ref. [10]. Our result is $\alpha_s[\tau_\tau] = 0.1174^{+0.0018}_{-0.0015}$.

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