Estimation and Hedging Effectiveness of Time-Varying Hedge Ratio: Flexible Bivariate GARCH Approaches*

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Abstract

Bollerslev's (1990) constant conditional correlation (CCC) and Engle's (2002) dynamic conditional correlation (DCC) bivariate generalized autoregressive conditional heteroskedasticity (BGARCH) models are usually used to estimate time-varying hedge ratios. In this paper, we extend the above model to more flexible ones to analyze the behavior of the optimal conditional hedge ratio based on two BGARCH models: (i) adapting more flexible bivariate density functions such as a bivariate skewed-t density function; (ii) considering asymmetric individual conditional variance equations; and (iii) incorporating asymmetry in the conditional correlation equation for the DCC based model. Hedging performance in terms of variance reduction and also value at risk and expected shortfall of the hedged portfolio are also conducted. Using daily data of the spot and futures returns of corn and soybeans we find asymmetric and flexible density specifications help increase the goodness-of-fit of the estimated models, but does not guarantee higher hedging performance. We also find that there is an inverse relationship between the variance of hedge ratios and hedging effectiveness.

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1 Introduction

There have been a number of studies for the estimation of optimal hedge ratios using the futures in empirical finance literature. One classical way is to use the ordinary least squares (OLS) for the slope parameter in the linear regression of the spot returns on the futures returns. However, if the joint distribution of the spot and futures prices is changing over time, the classical constant hedge ratio might be inappropriate. Indeed, it is well known in terms of autoregressive conditional heteroskedastic (ARCH) (Engle, 1982) and generalized ARCH (GARCH)(Bollerslev, 1986) models that financial time series have conditional heteroskedasticity, i.e., a time-varying second moment. Recently, various bivariate GARCH (BGARCH) type models have been used to estimate time-varying hedge ratios (Cecchetti, Cummby & Figlewski, 1988; Baillie & Myers, 1991; Kroner & Sultan, 1993; Bera, Garcia & Roh, 1997; Lien, Tse & Tsui, 2002; Brook, Henry & Persand, 2002; Miffrre, 2004). Time-varying hedge ratios are often referred to as the conditional hedge ratio because they are conditioned on the information set available at the previous time period.

There are some interesting issues that arise from the above studies. First, financial asset returns are known to have leverage effects, i.e., negative shocks usually generate higher volatilities than positive shocks in the next time period. Thus, it is interesting to analyze whether imposing asymmetries in the covariance structures help to reduce risk of the hedged portfolio. Second, most of the above studies use the BGARCH model under the conditional bivariate normality assumption. However, it is well known in the finance literature that conditional normality is not enough to explain unconditional skewness and kurtosis of financial time-series data. The usefulness of flexible conditional distributions for reducing risk of the hedged portfolio could be another issue. Third, there have been large controversies among various studies whether the conditional hedging strategy can outperform the unconditional (OLS) hedging strategy. The conditional hedge ratio, for example Baillie and Myers (1991), Park and Switzer (1995), Garcia, Roh, and Leuthold (1995), and Bera, Garcia, and Roh

(1997) generates a higher variance reduction than a conservative OLS hedge ratio, while recent studies, for example, Collins (2000) and Lien, Tse and Tsui (2002), found that adapting a conditional hedging scenario does not provide any advantage for the futures hedge compared to an OLS hedging scenario. We investigate the situation under which the conditional hedging strategy outperforms the naive hedging strategy by comparing the degree of fluctuation of the conditional variances.

In this paper, the spot and futures returns series for corn and soybeans are examined using Engle's (2002) dynamic conditional correlation (DCC) BGARCH model. For our data, the DCC specification is chosen against the constant conditional correlation (CCC) specification by the constant conditional correlation test proposed by Bera and Kim (2002). Using the test results we modify these BGARCH models to more flexible ones to estimate the conditional hedge ratios: (i) adapting more flexible bivariate density functions such as bivariate Student's t and skewed-t density functions; (ii) considering asymmetric individual conditional variance equations; and (iii) incorporating asymmetries in the conditional correlation equation for the DCC case. By doing this we can investigate the benefits of considering both the asymmetric conditional covariance structures and flexible conditional bivariate densities. Moreover, we analyze empirical linkages between the variability of hedge ratios and hedging performance to investigate under which cases conditional hedging strategy does not perform well. The variability of the forecasted hedge ratios, (i.e., standard deviation of the forecasted hedge ratios), can be estimated using the bootstrap method. For the hedging effectiveness many studies use variance reduction, i.e., the reduction in the conditional variance of the portfolio returns relative to the no hedging scheme, as a measure of the hedging effectiveness. Although variance reduction is a widely used measure for the hedging effectiveness, it may not represent an appropriate risk when the portfolio return distribution deviates from a normal distribution. To incorporate such a problem of variance reduction measure we consider tail risk measures, such as the Value at Risk (Jorion, 2000) and the Expected Shortfall

(Artzner, Delnaen, Eber & Heath, 1999), to evaluate hedging performance in addition to the variance reduction measure.

It is well known that commodity markets have different structural characteristics such as seasonality, time-to-maturity and convenience yields which can influence model specifications (see Sørensen, 2002; Richter & Sørensen, 2002). However, many papers, for example, Sørensen (2002) and Richter and Sørensen (2002), did not consider the estimation of hedge ratios but agriculture commodity futures. Our objective is estimating and evaluating conditional hedge ratios, using various BGARCH model specification, in the agriculture commodity market. In particular, we mainly focus on model specification and empirical comparison of various BGARCH models with different types of conditional variance-covariance specifications and distributions. There are numerous articles which consider various BGARCH models in the context of the conditional hedge ratio estimation without considering the structural characteristics of agriculture commodity market. Thus, our model is consistent with the literature and makes our findings more comparable to previous studies.

In the next section, the model specification and estimation method are presented. The data used in this study and their descriptive statistics are described in Section 3. The results from the estimation and examination of the hedging effectiveness of various BGARCH specifications, along with a conventional OLS hedging scheme, are reported in Section 4. Finally, Section 5 offers some concluding remarks.

2 Model specification

The unconditional optimal hedge ratio can be derived by maximizing quadratic expected utility assuming the futures price follows a martingale process. This unconditional optimal hedge ratio can be estimated using ordinary least squares (OLS) by regressing the spot returns on the futures returns. In a similar way, the conditional optimal hedge ratio, β_{t-1}, at time t can be written as

$$\beta_{t-1} = \frac{-\mathrm{Cov}(R_t^s, R_t^f | \mathcal{F}_{t-1})}{\mathrm{Var}(R_t^f | \mathcal{F}_{t-1})}, \tag{1}$$

where $R_t^s = P_t^s - P_{t-1}^s$ and $R_t^f = P_t^f - P_{t-1}^f$ are the returns of spot and futures prices denoting the natural logarithms of spot and futures prices at time t as P_t^s and P_t^f, respectively, and $\mathcal{F}_{t-1}$ is the σ-algebra generated by all the available information up to time $t-1$. Since β_{t-1} is conditioned on the information set, $\mathcal{F}_{t-1}$, the optimal hedge ratio is time-varying. A natural and widely used model for estimating (1) is a bivariate GARCH (BGARCH) model. In general, BGARCH model can be written as

$$R_t = m_t(\zeta) + \epsilon_t \tag{2}$$

$$\epsilon_t | \mathcal{F}_{t-1} \sim F(0, H_t), \tag{3}$$

where $R_t = (R_t^s, R_t^f)'$, $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$, $m_t(\cdot) = (m_{1t}(\cdot), m_{2t}(\cdot))'$ denotes a vector-valued conditional mean function, $\zeta = (\zeta_1, \zeta_2)'$ is $p \times 2$ conditional mean parameters, F denotes a bivariate distribution, and H_t is a time-varying 2×2 positive definite conditional covariance matrix. We consider two types of BGARCH models. Bollerslev (1990) suggested a simplified version of H_t in (3) such that the conditional correlation between ϵ_{1t} and ϵ_{2t} is constant over time. H_t for the constant conditional correlation (CCC) model can be written as

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix} \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix}, \tag{4}$$

where ρ_{12} is the constant correlation coefficient. The individual variance $h_{11,t}$ and $h_{22,t}$ are assumed to follow a standard GARCH process (Bollerslev, 1986), for example,

$$h_{ii,t} = \omega_i + \beta_i h_{ii,t-1} + \gamma_i \epsilon_{i,t-1}^2, \quad \text{for} \quad i = 1, 2. \tag{5}$$

If we assume conditional normality, the number of parameters to be estimated is only 7 in a CCC-BGARCH(1,1) model. The positive definiteness of H_t is assured if $h_{11,t} > 0$ and $h_{22,t} > 0$. This conditions can be immediately satisfied by usual parameter constraints

($\omega_i > 0$, $\gamma_i > 0$, $\beta_i > 0$, and $\gamma_i + \beta_i < 1$) of an individual GARCH(1,1) process. These nice features lead to wide usages of a CCC-BGARCH model as a tool for estimating time-varying hedge ratios (see, Park & Switzer, 1995; Lien, Tse & Tsui, 2002). However, the constancy of the correlation coefficient over the time horizon is a strong assumption and purely an empirical question. Hence, we adopt Bera and Kim's (2002) procedure to test the constancy of the conditional correlation assumption in a BGARCH model. There are two attractive aspects of their test procedures as compared to other test procedures of the constancy of conditional correlation (e.g. Tse, 2000): (i) it does not depend on the functional form of the individual conditional variance equation; and (ii) they suggest a studentized version of the test statistic when error distributions are not normal. Since we deal with more flexible BGARCH models in the sense that we extend to an individual asymmetric GARCH model with more general error distributions, we expect their test statistic to be useful in our case. Denoting the standardized disturbances $u_{it} = \epsilon_{it}/\sqrt{h_{ii,t}}$, $i = 1, 2$, the test statistic is written by

$$IM_e = \frac{\left[\sum_{t=1}^{T} (\hat{v^*}_{1t}^2 \hat{v^*}_{2t}^2 - 1 - 2\hat{\rho}^2)\right]^2}{4T(1 + 4\hat{\rho}^2 + \hat{\rho}^4)}, \tag{6}$$

where $v_t^* = (v_{1t}^*, v_{2t}^*)' = \left(\frac{u_{1t}-\rho u_{2t}}{\sqrt{1-\rho^2}}, \frac{u_{2t}-\rho u_{1t}}{\sqrt{1-\rho^2}}\right)'$, and ρ might be estimated consistently by $\hat{\rho} = \sum_{t=1}^{T} \hat{u}_{1t}\hat{u}_{2t}/T$. It is well known that using the standard normal conditional error distribution in the GARCH model is not enough to explain unconditional heavy tail behaviors of financial time-series data. Since (6) is derived from the conditional normality assumption of the error distribution the test statistic can be misspecified if the error distribution follows a non-normal distribution. This leads to an over-rejection of the null hypothesis. In such cases, Bera and Kim (2002) suggest a studentized version of the test statistic. If we denote $\eta_t = \hat{v^*}_{1t}^2 \hat{v^*}_{2t}^2 - 1 - 2\hat{\rho}^2$, a studentized version of (6) can be written as

$$IM_s = \frac{\left[\sum_{t=1}^{T} \eta_t\right]^2}{\sum_{t=1}^{T} (\eta_t - \bar{\eta})^2}. \tag{7}$$

When we reject the null hypothesis, i.e. there is a time-varying conditional correlation, the CCC-BGARCH model is misspecified. Thus, one has to deal with time-varying conditional correlation models. In such cases, we used the dynamic conditional correlation (DCC) BGARCH model proposed by Engle (2002). The DCC-BGARCH model can be represented as

$$H_t = D_t \Gamma_t D_t, \tag{8}$$

$$D_t = \mathrm{diag}\{h_{11,t}^{1/2}, h_{22,t}^{1/2}\}, \tag{9}$$

$$h_{ii,t} = \omega_i + \beta_i h_{ii,t-1} + \gamma_i \epsilon_{i,t-1}^2, \quad i = 1, 2, \tag{10}$$

$$\Gamma_t = (\mathrm{diag}\{Q_t\})^{-1/2} Q_t (\mathrm{diag}\{Q_t\})^{-1/2}, \tag{11}$$

$$Q_t = (1 - \delta_1 - \delta_2)\bar{Q} + \delta_1 u_{t-1} u'_{t-1} + \delta_2 Q_{t-1}, \tag{12}$$

where ϵ_t denotes a vector of unexpected returns and $u_{i,t} = (u_{1t}, u_{2t}) = \epsilon_{i,t}/\sqrt{h_{ii,t}}$ denotes a vector of standardized unexpected returns. $h_{ii,t}$ can be defined as a standard GARCH process, and Q_t denotes a 2×2 symmetric positive definite matrix. $\bar{Q} = E[u_t u'_t]$ is a 2×2 unconditional variance matrix of u_t. δ_1 and δ_2 are scalar parameters, and $\delta_1 \geq 0$, $\delta_2 \geq 0$ and $\delta_1 + \delta_2 < 1$ guarantee positive definiteness of the conditional correlation matrix during the optimization. There are other alternative specifications such as the diagonal vech and positive definitive variance specifications (Engle & Kroner, 1995). Positive definiteness of the conditional variance-covariance matrix for the vech model is not assured without imposing a nonlinear parametric restriction. Moreover, the number of parameters to be estimated is 21 for the vech model. Thus we use the simplified version of the DCC model (Engle (2002)). Given the bivariate model of the spot and futures prices changes, the time-varying hedge ratio can be expressed with the variance-covariance estimates from (4) or (8) for the CCC and DCC models, respectively, as

$$\hat{\beta}_{t-1} = \frac{\hat{h}_{12,t}}{\hat{h}_{22,t}} = \frac{\hat{h}_{sf,t}}{\hat{h}_{f,t}}. \tag{13}$$

2.1 Conditional mean specifications

It has been shown by many empirical studies that the spot and futures prices for the same commodity have very similar behaviors (move up and down together) in the long-run. However, there could be short-run deviations from the stable long-run relationship due to mispricing of either the spot or futures price, transition costs or market microstructure effects among others. These leads many studies to incorporate vector error correction models (VECM) to estimate the optimal hedge ratio (Brooks, Henry, & Persand, 2002; Yang and Awokus, 2003; Lien & Yang, 2004). In the commodity market, harvest patterns, convenience yields and storage cost have been considered to explain spot prices behavior (Sørenson, 2002; Moschini & Myers, 2002). Because of these characteristics of the commodity market, some studies tried to include such variables in VECM. Moreover, by considering the stochastic interest rate in the conditional mean equation, Fortenbery and Zapata (1996), and Yang, Bessler, and Leathan (2001) showed the existence of a cointegrating relationship between the spot and futures prices. However, we do not consider the above characteristics in the conditional mean specification. In Section 3, we show that the cointegrating relationships between the spot and futures prices of corn and soybeans exist without considering such variables for our data. Hence, we consider a bivariate VECM as

$$R_t = C + \sum_{i=1}^{p} \Gamma_i R_{t-1} + \Pi v_{t-1} + \epsilon_t, \tag{14}$$

where $v_{t-1} = (P_{s,t-1} - \omega_0 - \omega_1 P_{f,t-1})$ denotes a cointegrating equation, and

$$C = \begin{bmatrix} C_s \\ C_f \end{bmatrix}, \quad \Gamma_i = \begin{bmatrix} \Gamma_{i,s}^s & \Gamma_{i,f}^s \\ \Gamma_{i,s}^f & \Gamma_{i,f}^f \end{bmatrix}, \quad \Pi = \begin{bmatrix} \Pi_s \\ \Pi_f \end{bmatrix}.$$

When the spot price exceeds the long-run relationship of the spot and futures prices at time $t-k$ for some k, (i.e., $v_{t-k} > 0$), Π_s and Π_f are supposed to have negative and positive values, respectively, in order to maintain the long-run relationship. In a similar manner, when the spot price falls below the long-run relationship, (i.e, $v_{t-k} < 0$), Π_s and Π_f are expected to have negative and positive signs, respectively.

2.2 Asymmetry specifications

It is worthwhile to note that only the magnitude of past return innovations are taken into consideration to determine the variance and covariance at time t in the CCC and DCC BGARCH models. However, the positive and negative values of the past return innovations may affect the present variance and correlation differently in the real world. In order to incorporate effects of previous positive and negative shocks separately in the conditional covariance matrix, we use the GJR (Glosten, Jagannathan & Runkle, 1993) specification in the individual GARCH process (for CCC and DCC) and the conditional correlation equation (for DCC). The GJR specification of an individual GARCH process can be written as

$$h_{ii,t} = \omega_i + \beta_i h_{ii,t-1} + \gamma_i \epsilon_{i,t-1}^2 + \tau_i I_{t-1} \epsilon_{i,t-1}^2, \quad i = 1, 2, \tag{15}$$

where $I_t = 1$ if $\epsilon_t < 0$ and otherwise 0. When $\tau_i > 0$, previous negative shocks generate higher volatility than positive shocks. This effect is called the leverage effect. In a similar way, asymmetric specification of the conditional correlation equation (12) could be represented by

$$Q_t = (1 - \delta_1 - \delta_2)\bar{Q} - \delta_3 \bar{S} + \delta_1 u_{t-1} u_{t-1}' + \delta_2 Q_{t-1} + \delta_3 s_{t-1} s_{t-1}', \tag{16}$$

where $s_t = I[u_t < 0] \bigodot u_t$ and $\bar{S} = E[s_t s_t']$. $I[\cdot]$ is the indicating function and $\bigodot$ denotes the element-by-element multiplication operator. The asymmetry representation of the conditional correlation equation is only possible for the DCC model because the CCC model has the constant conditional correlation by its construction. We replace $\bar{Q}$ and $\bar{S}$ by their empirical counterparts, $T^{-1} \sum_{t=1}^{T} u_t u_t'$ and $T^{-1} \sum_{t=1}^{T} s_t s_t'$, respectively, which make estimation more simple, as suggested by Engle and Sheppard (2001) and Engle (2002).

2.3 Distribution specifications

Most applications of BGARCH models with the estimation of the optimal hedge ratio are based on the bivariate normality of conditional distribution. Bollerslev and Wooldridge

(1992) showed consistency and asymptotic normality of the quasi-maximum likelihood estimator (QMLE) of the GARCH model. Because of these attractive features of QMLE, conditional normality is usually adopted in many applications. However, Engle and González-Rivera (1991) showed that there is a large efficiency loss of QMLE when the underlying conditional distribution is non-normal. Thus, the forecasting ability of the GARCH model based on QMLE might be affected by this efficiency loss. For example, forecasting intervals could be much wider than the true intervals. In many applications of the GARCH model it has been found that conditional normality is not enough to explain excess kurtosis in the financial data. The unconditional distribution of financial returns is often skewed so that the conditional distribution which can capture skewness is also needed. With the above reasons we consider two conditional distributions: (i) we assume the underlying disturbance follows bivariate Student's t distribution; and (ii) we assume a skewed-t distribution to capture skewness in addition to high excess kurtosis.

The standardized bivariate Student's t density can be defined as

$$g(z|\nu) = \frac{\Gamma((\nu+2)/2)}{\Gamma(\nu/2)(\pi(\nu-2))}\left[1+\frac{z'z}{\nu-2}\right]^{-(2+\nu)/2}, \tag{17}$$

where Γ is the gamma function and ν denotes the degree of freedom and is restricted to be larger than 2 to ensure the covariance matrix exists. To incorporate skewness, Bauwens and Laurent (2005) suggest a multivariate skewed-t density which is based on Fernández and Steel's (1998) skewed filter to multivariate Student's t. A standardized bivariate skewed-t density is

$$g(z|\xi,\nu) = \left(\frac{2}{\sqrt{\pi}}\right)^2\left(\prod_{i=1}^{2}\frac{\xi_i s_i}{1+\xi_i^2}\right)\frac{\Gamma((\nu+2)/2)}{\Gamma(\nu/2)(\nu-2)}\left(1+\frac{\kappa'\kappa}{\nu-2}\right)^{-(2+\nu)/2}, \tag{18}$$

where

$$\begin{aligned}\kappa &= (\kappa_1,\kappa_2)', \\ \kappa_i &= (s_i z_i + m_i)\xi_i^{-I_i},\end{aligned}$$

$$
\begin{aligned}
m_i &= \frac{\Gamma((\nu-1)/2)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma((\nu/2))}\left(\xi_i - \frac{1}{\xi_i}\right), \\
s_i^2 &= \left(\xi_i^2 + \frac{1}{\xi_i^2} - 1\right) - m_i^2, \\
I_i &= \begin{cases} 1 & \text{if} \quad z_i \geq -\frac{m_i}{s_i} \\ -1 & \text{if} \quad z_i < -\frac{m_i}{s_i}. \end{cases}
\end{aligned}
$$

$m_i(\xi_i, \nu)$ and $s_i(\xi_i, \nu)$ are the mean and standard deviation of the non-standardized univariate skewed-t of Fernández and Steel (1998). Note that the sign of $\ln \xi_i$ indicates the direction of the skewness: when $\ln \xi_i > 0$ (< 0), the third moment is positive (negative), and density is skewed to the right (left).

2.4 Model types

For the CCC representation we consider two types of individual GARCH processes: (i) a symmetric individual GARCH model (a standard GARCH process); and (ii) an asymmetric individual GARCH model (the GJR representation). For DCC case we can also add asymmetric terms in the conditional correlation equations in addition to (i) and (ii) given in the CCC model. Thus, two more types of models can be considered for the DCC model: (iii) asymmetric terms are imposed only to the conditional correlation equations; and (iv) asymmetric terms are added to all equations (individual GARCH equations and conditional correlation equations). For convenience we will write (i), (ii), (iii) and (iv) as T1, T2, T3 and T4, respectively, throughout the paper. Since there is one conditional mean and three distributional specifications we have 6 and 12 models for the CCC and DCC cases, respectively.

3 Data and descriptive statistics

We investigate the time-series behavior of two daily cash and nearby futures prices for corn, and soybeans traded on the Chicago Board of Trade (CBT). The nearby futures price series were constructed as follows. First, we specified the nearby futures contract as the contract with the nearest active trading delivery month to the day of trading. Prices for the nearby futures contract are used until the contract reaches the first day of the delivery month or its expiry date. Then, prices for the next nearby contract are used. The daily cash closing prices for corn and soybean are No.2 yellow corn cash prices in Chicago and No.1 yellow soybean cash prices in Chicago, respectively. Datastream provided all data[1].

The data cover the period from January 1, 1997, to January 23, 2001, for a total of 1060 observations. Samples are split into two periods. The first period covers from January 1, 1997, to October 31, 2000 (1000 observations) and is used for the in-sample model estimation to evaluate various models and statistical tests. The second period covers the next 60 days and is used for the out-of-sample evaluation of the estimated models. The spot and futures returns, R_t^s and R_t^f, are calculated by $P_t^s - P_{t-1}^s$ and $P_t^f - P_{t-1}^f$, respectively, where P_t^s and P_t^f are the logarithms of spot and futures prices at time t. We should note, as we addressed in Introduction, our main focus is the model specification and empirical comparison of various types of BGARCH models for conditional hedge ratio estimation and evaluation. Thus, even though the analyzed data set is limited, this paper can provide informative findings comparable to previous studies.

We perform the Augmented Dickey-Fuller (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests with including a constant and linear trend in the regression equation. The results for these two tests are reported in Table 1. For all prices data, we fail to reject the null hypothesis of the ADF test while we reject the null hypothesis of the KPSS test. These

[1] The information from Datastream shows that cash and futures prices are recorded at the end of the trading day.

indicate that all price series are non-stationary. However, for the returns series, as expected, we reject the null hypothesis of the ADF test while we fail to reject the null hypothesis of the KPSS test. Thus, the returns series are stationary.

[TABLE 1]

The results of Johansen's cointegration test are presented in Table 2. The Johansen test statistics, for both the maximum eigenvalue form and the trace form, clearly reject the null of no cointegrating vector at a 5% level for both the corn and soybean series. However, those cannot reject the null of one cointegrating vector. These results suggest the existence of a cointegrating relationship between the spot and futures prices in the corn and soybean series. These results show there exists a cointegrating relationship between the spot and futures prices without considering the stochastic interest rate and time-to-maturity effects in the conditional mean equation.

[TABLE 2]

The summary statistics for the in-sample return data are presented in Table 3. The sample kurtosis indicate that all returns have excess kurtosis (> 3), and there is also some degree of skewness in the returns. The futures returns for soybeans exhibit not only high excess kurtosis but also distinct negative skewness. All the Jarque-Bera statistics are greater than $\chi^2_{2,0.99} = 9.21$, and, thus, the null hypothesis of a normality assumption is strongly rejected. This indicates that all returns series are highly non-normal. The Ljung-Box (LB) test statistics at lags 12 day,Q and Q^2, are calculated using the series and its squares, respectively. Higher values of Q and Q^2 indicate that there are first order dependence and second order nonlinear dependence in the data, respectively. This implies there is a significant presence of conditional heteroskedasticity.

The reported cointegration test results and descriptive statistics justify the usage of VEC and BGARCH models as the conditional mean and conditional variance, respectively. Moreover, highly non-normal behaviors of the spot and futures returns might be well explained by the conditional student's t and skewed-t density functions.

[TABLE 3]

4 Empirical results

4.1 Model estimation results

We adapt a two-stages estimation method to lessen the computational burden for the conditional variance-covariance equations. In the first stage, conditional mean equations for the spot and futures returns are estimated using the VECM. In the second stage, various BGARCH models are fitted to the estimated residuals from the first stage using the maximum likelihood estimation method.

Table 4 summarizes the results of the VEC models. The optimal truncation lag p in (14) is selected based on the Schwartz information criteria for each model.

[TABLE 4]

The estimated VECM coefficients ($\hat{\Pi}_s$ and $\hat{\Pi}_f$) are all correctly signed, i.e., negative and positive for the spot and futures returns equations, respectively. Based on the estimated residuals, we conduct the Ljung-Box Q, Q^2 and normality tests. None of the Q statistics are significant. This indicates that the VECM takes account of the first order dependence presence in the data very well. High values of Q^2 statistics implies there still remain the second order dependence in the residuals. Jarque-Bera (JB) test statistics show that the residuals are highly non-normal. High Q^2 and JB test statistics indicate that constructing non-normal BGARCH models can help to explain behaviors of the underlying residuals series.

To determine whether the conditional correlation is time-varying, we perform the constancy of the conditional correlation test proposed by Bera and Kim (2002). Table 5 shows the results of the tests for the spot and futures residuals of corn and soybean models.

[TABLE 5]

As mentioned before, IM_e test statistics seem to over-reject the null hypothesis of the constancy of conditional correlations. This over-rejection might be due to the highly non-normal characteristics of the estimated residuals from the first stage estimation. Thus, we use the studentized version of IM_e, IM_s, with two specifications: the individual conditional variance functions are symmetric (T1) and asymmetric (T2) GARCH processes. IM_s test statistics, for both T1 and T2, clearly reject the null of the constancy of conditional correlation at a 5% level. Thus, we adapt the DCC specification of Engle (2002) to estimate BGARCH models for both corn and soybean series.

The estimates for the corn and soybean models along with the values of log-likelihood functions and model selection criteria (Akaike information criteria (AIC) and Schwartz information criteria (SIC)) are reported in Tables 6 and 7, respectively.[2] The standard errors, calculated using a robust covariance matrix formula, are given in parentheses.

[TABLE 6]

[TABLE 7]

For the corn model (Table 6), all $\hat{\tau}_1$'s are significant and their signs are positive. These imply that there are leverage effects for the spot series. For the futures series $\hat{\tau}_2$ are significant and positive except for the case of the conditional normal model. For the conditional normal models $\hat{\delta}_1$'s are significant but $\hat{\delta}_2$'s are not. This implies that time-varying conditional

[2] AIC and SIC are calculated, respectively, by $-2loglik + 2(\kappa/T)$ and $-2loglik + \kappa(ln(T)/T)$, where κ denotes dimension of parameter vector, T is sample size, and $loglik$ is log-likelihood value.

correlations are present but not persistent. When we estimate models under conditional Student's t and skewed-t distribution, $\hat{\delta}_2$ are high and, moreover, significant. $\hat{\delta}_3$'s which represent asymmetry effects in the conditional correlation equations are not significant except in the T3-skewed-t model. The estimated distribution parameters are highly significant for the Student's t and skewed-t models. The degree of freedom, $\hat{\nu}$ is around 3.2 for both Student's t and skewed-t models. $\hat{\xi}_1 < 1$ and $\hat{\xi}_2 > 1$ stand for negative and positive skewness for the standardized residuals of the spot and futures equations, respectively.

We can consider the difference of log-likelihood values between Student's t and normal models, 347.4, 315.5, 351.3 and 332.6, as increments for incorporating excess kurtosis for T1, T2, T3 and T4 models, respectively. For example, a low value of the estimated degree of freedom, 3.0429 for T4, also supports that assertion. Gains in the log-likelihood values of skewed-t from those of Student's t are 14.8, 23.0, 14.6 and 15.0 for T1, T2, T3 and T4, respectively. These values could be thought of as contributions due to distribution asymmetry. We could perform the likelihood ratio (LR) test for the null hypothesis $\xi_1 = \xi_2 = 0$, such that the test statistic can be calculated by $2(loglik_{T_j} - loglik_{T_i}) \sim \chi^2_2$ where $i = j = 1, 2, 3, 4$, and i and j denote the Student's t and skewed-t models, respectively. The LR values are 29.6, 46.0, 29.2 and 30.0 for $i = j = 1, 2, 3, 4$, respectively, and clearly high enough to reject the null hypothesis ($\chi^2_{2,0.99} = 9.21$). Based on the model selection criteria of AIC and SIC, T2, T4 and T4 are more attractive than other models for normal, Student's t and skewed-t models, respectively. Overall, the T4-skewed-t model has the lowest AIC and SIC values.

For the soybean models (Table 7) all $\hat{\tau}_1$ and $\hat{\tau}_2$ are insignificant. Moreover, all $\hat{\delta}_3$ are also insignificant, indicating the absence of asymmetry effects in the soybean series. This means that previous positive or negative shocks have the same influence on the volatility of soybean returns. This contrast with the corn series findings in which the effect of negative shocks on volatility is larger than positive shocks. On the other hand, almost all $\hat{\gamma}_1$, $\hat{\gamma}_2$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\delta}_1$ and $\hat{\delta}_2$ are very significant. The difference of log-likelihood values between the Student's t and normal models are 380.6, 365.0, 368.8 and 376.8 for T1, T2, T3 and T4, respectively. Similar to the corn case, these values explain excess kurtosis of the standardized residual series. The difference of log-likelihood values between the skewed-t and Student's t models are 0.8, 0.7, 0.7 and 0.7 for T1, T2, T3 and T4, respectively.

Incorporating skewness does not increase the goodness-of-fit since the LR test statistics for $H_0 : \xi_1 = \xi_2 = 0$ (1.6, 1.4, 1.4 and 1.4 for T1, T2, T3 and T4, respectively) are less than $\chi^2_{2,0.99} = 9.21$. Overall, the T1-Student's t model has the lowest AIC and SIC values.

Based on the standardized residuals from each BGARCH model, we conduct the Ljung-Box Q and Q^2. None of the Q and Q^2 statistics are significant. This indicates that the proposed models take account of first and second order dependence presented in the corn and soybean series very well.

The time-varying hedge ratios of corn and soybeans calculated by (13) are plotted in Figure 1. We only present the estimated hedge ratios of the best and worst cases in terms of AIC values for each distribution specification. To conserve space we do not report all the results but these can be obtained from us on request. In the Figure 1, the unconditional hedge ratio estimates of 0.858 and 0.851 based on the OLS method, for corn and soybeans, respectively, are plotted with the horizontal lines. In case of corn, the estimated time-varying hedge ratios from the conditional normal specifications demonstrate similar movements around the unconditional hedge ratios. Unlike cases of the conditional normal specification, the Student's t and skewed-t specifications yield that the mean of the estimated conditional hedge ratios are strictly higher than the unconditional hedge ratios. Especially, for the T4-Student's t and T4-skewed-t specifications, the hedge ratios move along with the OLS hedge ratios without heavy local fluctuations. In the case of soybeans, the estimated conditional hedge ratios for the Student's t and skewed-t specifications seem to be very similar, but the estimated hedge ratios for the conditional Student's t models are slightly different from those of the conditional normal model. This difference might be due to the excess kurtosis. Similar to the models of conditional Student's t and skewed-t specifications for corn, the means of the estimated time-varying hedge ratios for all models are higher than the unconditional hedge ratio.

[FIGURE 1]

4.2 Hedging performance of BGARCH models

The results of the model estimation for several BGARCH specifications provided us that there are significant leverage effects ($\hat{\tau}_1$ and $\hat{\tau}_2 > 0$) in the corn series while the soybean series do not have such effects. Moreover, AIC and SIC show us that the T4-skewed-t and T1-Student's t models are the best for the corn and soybean models, respectively. However, these distinct characteristics of certain models may not lead to better hedging performance against other competing models which might have an even lower degree of the goodness-of-fit. Thus, empirical investigations of the in- and out-of-sample hedging performance of the estimated BGARCH models are needed.

The most frequently used hedging performance measure is variance reduction. Variance reduction in the conditional variance of the portfolio returns relative to the no hedging scenario can be expressed as

$$VR = 1 - \frac{Var(R^p)}{Var(R^s)}, \tag{19}$$

where $Var(R^p)$ and $Var(R^s)$ denote the variances of the hedged and unhedged portfolio, respectively. The hedged portfolio at time t is represented by

$$R_t^p = R_t^s - \beta_{t-1} R_t^f.$$

However, taking the variance as a risk measure has some drawbacks: (i) the variance takes care of not only bad events but good events; and (ii) the variance does not represent the proper scale if the underlying distribution is deviated from normal. Thus, we evaluate the hedging performance by Value at Risk (VaR) and Expected Shortfall (ES) along with variance reduction. VaR is very a popular risk measure such that it is an estimate of how much a certain portfolio can lose at a given confidence interval and is used by many banks and financial institutions as a key measure of market risk (Jorion, 2000). VaR is defined as

$$VaR_q(R^p) \equiv -\inf_{\nu}\{\nu : P_t(R^s \leq \nu) \geq q\} = Q_{R^p}(q), \tag{20}$$

where q is a prescribed probability and Q_{R^p} denotes the quantile function of R^p. VaR gives only an upper bound on the losses that occur with a given frequency. However, ES is defined as expectation of all events less than VaR_q and can be written by

$$ES = -\frac{1}{q}\int_0^q Q_{R^p}(p)dp. \tag{21}$$

ES is known as a coherent risk measure satisfying certain conditions (Artzner, Delnaen, Eber & Heath, 1999). We use the bootstrap method to forecast the future hedge ratio and calculate the aforementioned three risk measures for 60 days horizons based on twelve different types of BGARCH models. For example, the following recursion procedures can be used for the estimated DCC-T4 model to obtain the bootstrap replicates $\{\epsilon^*_{1,b}, \cdots, \epsilon^*_{T,b}\}$, $b = 1, \cdots, B$, which has similar behavior to the original ϵ_t:

$$\begin{aligned}
\hat{h}^*_{ii,t} &= \hat{\omega}_i + \hat{\beta}_i \hat{h}^*_{ii,t-1} + \hat{\gamma}_i \epsilon^{*2}_{i,t-1} + \hat{\tau}_i I_{t-1}\epsilon^{*2}_{i,t-1}, \quad \text{for} \quad i = 1, 2, \\
\hat{Q}^*_t &= (1 - \hat{\delta}_1 - \hat{\delta}_2)\bar{Q}^* - \hat{\delta}_3 \bar{S}^* + \hat{\delta}_1 \hat{u}^*_{t-1}\hat{u}'^*_{t-1} + \hat{\delta}_2 \hat{Q}^*_{t-1} + \hat{\delta}_3 \hat{s}^*_{t-1}\hat{s}'^*_{t-1}, \\
\epsilon^*_t &= \hat{H}^{*1/2}_t \eta^*_t,
\end{aligned}$$

where $(\hat{\omega}_i, \hat{\beta}_i, \hat{\gamma}_i, \hat{\tau}_i, \hat{\delta}_1, \hat{\delta}_2, \hat{\delta}_3)'$, $i = 1, 2$, are the maximum likelihood (ML) estimates under the presumed conditional distribution such as the Student's t or skewed-t distribution, the initial value for H^*_t is given by $H^*_1 = \hat{H}_1$, and η^*_t are random draws from the empirical distribution, say, $(\hat{F}_T)$, of the standardized residuals. In the next step, $(\hat{\omega}_{i,b}, \hat{\beta}_{i,b}, \hat{\gamma}_{i,b}, \hat{\tau}_{i,b}, \hat{\delta}_{1,b}, \hat{\delta}_{2,b}, \hat{\delta}_{3,b})'$ are estimated by the ML method using the bootstrap replicates $\epsilon^*_{t,b}$ for $t = 1, 2, \cdots, T$

and $b = 1, 2, \cdots, B$. Thus, our bootstrap method incorporates the estimation uncertainty. Bootstrap forecasts for future values of H_{T+k} are obtained by the following recursions:

$$\begin{aligned}
\hat{h}^*_{ii,T+k} &= \hat{\omega}_{i,b} + \hat{\beta}_{i,b}\hat{h}^*_{ii,T+k-1} + \hat{\gamma}_{i,b}\epsilon^{*2}_{i,T+k-1} + \hat{\tau}_{i,b}I_{T+k-1}\epsilon^{*2}_{i,T+k-1}, \\
\hat{Q}^*_{T+k} &= (1 - \hat{\delta}_{1,b} - \hat{\delta}_{2,b})\bar{Q}^* - \hat{\delta}_{3,b}\bar{S}^* + \hat{\delta}_{1,b}\hat{u}^*_{T+k-1}\hat{u}'^*_{T+k-1} \\
&\quad + \hat{\delta}_{2,b}\hat{Q}^*_{T+k-1} + \hat{\delta}_{3,b}\hat{s}^*_{T+k-1}\hat{s}'^*_{T+k-1}, \\
\epsilon^*_{T+k} &= \hat{H}^{*1/2}_{T+k}\eta^*_{T+k}, \quad \text{for} \quad i = 1, 2 \quad k = 1, 2, \cdots, 60,
\end{aligned}$$

where the initial values for ϵ^*_T and $\hat{H}^*_T$ are given by ϵ_T and $\hat{H}_T$, respectively, and η^*_{T+k} is random draws from $\hat{F}_T$. As we mentioned before, the DCC-T4 model includes DCC-T1, -T2 and -T3, and CCC-T1 and -T2 as special cases. Thus, presented bootstrap procedures are immediately applied to other model types. Since there are B numbers of the hedged portfolio returns at each time $T + k$, $k = 1, 2, \cdots, 60$, $60 \times B$ numbers of samples over the whole forecasting horizons can be used to calculate hedging effectiveness measures. If we consider an empirical distribution, say, $\hat{G}(R^p)$, of the hedged portfolio returns, VaR at 5% and 1% are calculated by the sample quantiles at $q = 0.05$ and 0.01, respectively, as

$$VaR_q(R^p) = \hat{Q}_{R^p}(q), \tag{22}$$

where $\hat{Q}_{R^p}$ denotes sample quantile function of $\hat{G}(R^p)$. Since ES is given by the expected returns lower than a given level, say, $\tilde{R}$ which, of course, depends on the quantile, q. Equation (21) can be approximated by the Riemann sum

$$ES \simeq -\sum_{i=1}^{m} c_i[\hat{G}(c_i) - \hat{G}(c_{i-1})], \tag{23}$$

where c_i is a sufficient fine grid of the interval $[\min\{(R^p)\}, \tilde{R}]$. For computational convenience, (23) can be represented with the sample quantile function in a form of the Riemann sum by

$$ES \simeq -\sum_{q_i \le \bar{q}} \hat{Q}_{R^p}(q_i)[q_i - q_{i-1}], \tag{24}$$

where

$$\bar{q} = \sup\{q_j : \hat{Q}_{R^p}(q_j) \leq \tilde{R}\}.$$

Tables 8 and 9 summarizes the in- and out-of-sample performance of the hedge ratios derived from the VECM-BGARCH models compared with zero, naive (unit) and OLS hedges, respectively. For the in-sample hedging performance, variance reduction is used to evaluate hedging performance. On the other hand, for the out-of-sample hedging performance, variance reduction as well as VaR and ES of the portfolio returns are used to evaluate hedging performance. VaR and ES of the portfolio returns can be calculated by (22) and (24) using B numbers of the variance-covariance matrices at each time $T + k$, $k = 1, 2, \cdots, 60$.

[TABLE 8]

[TABLE 9]

The bootstrap sample size is set to $B = 500$. The in-sample results are based on the $1,000$ observations used in the estimation process, while the next 60 observations are used for the out-of-sample hedging performance.

The results of the in-sample hedging performance are reported in Table 8. For corn, all the BGARCH specifications produce higher variance reduction than the naive hedging strategy ($\beta = 1$), 0.6049. However, only two BGARCH representations, the T2-normal and T4-normal, have higher variance reduction than the OLS hedge strategy. Note that the best model for the corn is the T4-skewed-t model. Variance reduction of the T4-skewed-t, 0.6152, is less than the OLS hedging strategy, 0.6220, by 0.68%. For soybeans, all BGARCH specifications lead to higher variance reduction than the OLS hedging strategy except for the T4 cases. Also note that the best model in terms of AIC and SIC for soybeans is the T1-Student's t model. The variance reduction of the T1-Student's t, 0.7607, is relatively low compared to other BGARCH representations. For both corn and soybeans, the normal models outperform Student's t and skewed-t models in terms of VR on average. It is worth noting that the best model for corn and soybeans does not lead to better hedging performance compared to the OLS hedging strategy in the in-sample period.

Table 9 shows the results for the out-of-sample hedging performance. For corn, three BGARCH specifications, i.e., the T4-Student's t, T3-skewed-t and T4-skewed-t, outperform the OLS hedging strategy. This finding might be reasonable since the three mentioned models showed high goodness-of-fit in the model estimation stage. However, the difference of variance reduction between those three and the OLS scheme are relatively small. For each distributional specification we would say that there are improvements of variance reduction by considering the leverage effects in the conditional covariance matrix. Note that $\hat{\tau}_1$'s and $\hat{\tau}_2$'s are very significant for the corn models. The contribution of asymmetric effects to the improvement of hedging performance is also shown empirically by Brooks, Henry and Persand (2002). They show that asymmetries help to reduce portfolio risk at the shortest forecasting horizons, and these distinct benefits decrease as the duration of the hedge is increased. Our results support their findings. However, we find such benefits are relatively small. For example, the difference of the average variance reduction percentage between the T4-skewed-t and the T1-skewed-t is 1.16% (67.21% − 66.05%). VaR and ES at 5% also provide similar results in which those three models have relatively low risk. However, if we consider VaR and ES at 1%, T1- and T2-normal perform better than the T3- and T4-skewed-t models. This implies that distributions of the portfolio returns are not symmetric ones so that near-extreme tail risk measures, say VaR and ES at 1%, yield different results from a conservative risk measure, say, variance. For soybeans no BGARCH specification dominates the OLS hedging scheme in terms of variance reduction. Variance reduction of the T1-Student's t which has the highest SIC among all considered models is not outstanding, i.e., it is ranked as the 4th lowest value. The contributions of asymmetric effects are negligible for the soybean case. This can be resulted from insignificance of $\hat{\tau}_1$, $\hat{\tau}_2$ and $\hat{\delta}_3$ in Table 7. VaR and ES at 5% and

1% show roughly similar patterns to associated variance reductions. This implies estimated hedged portfolios do not seem to have asymmetric distributions.

Many previous studies compared the performance of the conditional hedge ratios to the OLS hedge ratio and provided conflicting results from each other. For example, Baillie and Myers (1991) and Park and Switzer (1995) found that the time-varying hedge ratios computed from BGARCH models showed better out-of-sample performance in terms of variance reduction than the OLS hedge ratio. On the other hand, Lien, Tse and Tsui (2002) found that the CCC-BGARCH hedge strategy provided no benefits in variance reduction over the OLS hedge. So far, there is no definite conclusion concerning the benefits of using BGARCH models in optimal hedging performance. These compelling questions can be revisited by examining the variation of the estimated conditional hedge ratio. Simply speaking, if the conditional hedge ratios are too volatile, the performance of these hedge ratios is likely to be deteriorated. On the other hand, if the conditional hedge ratios are constant, i.e., the OLS hedge ratio, they lack flexibility. Thus it can be useful to compare the variance of the estimated conditional hedge ratio among various conditional and unconditional hedging models to see the relationship between the hedging performance and the variance of the estimated time-varying hedge ratio. Since we use the bootstrap method to forecast a future time-varying variance-covariance matrix, standard deviation of the estimated hedge ratios at each forecasting time $T+k$, $k = 1, 2, \cdots, 60$ can be calculated. Figures 2 displays the estimated standard deviation of the bootstrapped hedge ratios at $T+k$ for corn and soybeans. For corn, the standard deviation of the T4-Student's t and T4-skewed-t are very smooth and relatively low compared to other specifications. These smooth and low standard deviation of hedge ratios could make those two models perform well in terms of variance reduction. This relatively low variability can also be confirmed in Figure 1. In Figure 1, the hedge ratios of the T4-Student's t and T4-skewed-t show little local fluctuations compared to other models. The difference between the T1-normal and T4-skewed-t models is worth mentioning. Even though the values of the standard deviations of the T1-normal and T4-skewed-t models are very similar, the variance reduction of the T4-skewed-t model is much higher than that of the T1-normal. However, when we compare VaR and ES of T1-normal to T4-skewed-t, we get the reverse results. These results are due to the high fluctuation of standard deviation of the T1-normal model. Since the standard deviation of the T4-skewed-t has little variability after a few periods and, moreover, is higher than that of the T1-normal, the tail behaviors of such hedge ratios might be highly affected. For soybeans, the T2-normal has the lowest standard deviation and the highest variance reduction, 0.7109, but it is dominated by the OLS hedging strategy. In contrast to the corn case, it is hard to detect the differences of the estimated hedge ratios from Figure 1.

[**FIGURE 2**]

5 Concluding Remarks

We examined the in-sample and out-of-sample hedging performance of the vector error correction model with various bivariate GARCH specifications. Specifications of asymmetric conditional variance equations and leptokurtic and skewed conditional distribution, such as bivariate skewed-t, help to improve the goodness-of-fit of the model estimation for the corn series. On the other hand, imposing asymmetric effects to the conditional covariance structures and the conditional densities provides little benefits for soybeans. If the estimated coefficients representing leverage effects are significant, then there are improvements of the out-of-sample hedging performance in terms of variance reduction but such benefits are not distinct. There are no systematic results that selecting the best model in terms of the model selection criteria such as AIC and SIC may not lead to the optimal hedging strategy which minimizes the variance of the portfolio returns. Moreover, when other tail risk measures, such as Value at Risk or Expected Shortfall, are considered to evaluate hedging performance, the results are affected by distributional forms of the portfolio returns.

We estimate standard deviation of the forecasted hedge ratios using the bootstrap method and show empirically that the out-of-sample hedging performance is highly related with the variance of the estimated hedge ratios. When hedge ratios are too volatile, i.e., standard deviation of the estimated hedge ratios are high, corresponding hedging performance measured by variance reduction, value at risk or expected shortfall become worse. The evidence suggests that some BGARCH hedging strategies may have modest improvements when their standard deviations are stable and low enough. However, the improvement is not big enough to guarantee a BGARCH hedging strategy is superior to OLS hedging strategy. Moreover, if one considers transactions cost, the benefits of conditional hedging strategy with BGARCH specifications could be shrunken.

There are a number of issues that require further attention. In particular, imposing a structure which decreases volatility of the estimated hedge ratios is of an interesting issue. In a simple formation, one can restrict hedge ratios within a certain interval. For example, this can be done in an extremely simple way by letting $[-2\sigma_h, 2\sigma_h]$, where σ_h is an unconditional standard deviation of a hedge ratio. Recently, some authors proposed shrinkage estimation for the covariance matrix (Ledoit & Wolf, 2004a, 2004b). In view of the results in this paper, it would be very interesting to see if an alternative hedge ratio which shrinks conditional hedge ratios to an unconditional (OLS) hedge ratio can improve the hedging performance over BGARCH or OLS hedging schemes. This will be our future study.

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Table 1. Unit-root and Stationarity tests

		ADF		KPSS		ADF		KPSS
		stat.	Lag	stat		stat	Lag	stat
Corn	P_t^s	-1.8971	8	2.9270	R_t^s	-8.9374	6	0.1034
	P_t^f	-2.0618	7	2.9999	R_t^f	-10.1271	5	0.0890
Soybeans	P_t^s	-1.1487	8	3.3763	R_t^s	-7.5351	7	0.1174
	P_t^f	-1.1852	13	3.3504	R_t^f	-7.8438	9	0.1101

Notes: R_t^s is the log price of the spot and R_t^f is the log price of futures. Similarly, ΔR_t^s is the spot returns and ΔR_t^f is the futures returns. ADF is the augmented Dicky-Fuller statistic for the hypothesis that the price series have a unit root. KPSS is the Kwiatkowski-Phillips-Schmidt-Shin statistic for the hypothesis that the price series is stationary. The critical value for ADF and KPSS are -3.4352 and -2.8636, and 0.7390 and 0.4630 at 1% and 5%, respectively. Lag is the optimal lag truncation chosen by SIC criteria. For KPSS bartlett kernel is used and the bandwidth is chosen by Newey-West procedures.

Table 2. Johansen cointegration tests

	Corn		Soybeans	
	λ_{Max}	λ_{Trace}	λ_{Max}	λ_{Trace}
$\gamma = 0$	22.335	25.747	22.836	24.348
$\gamma = 1$	3.4116	3.4116	1.5118	1.5118
	Cointegrating Equation			
ω_0	-1.8462		-0.5464	
ω_1	-1.3252		-1.0829	
	(0.0549)		(0.0269)	

Notes: 5% critical values of test statistics for $\gamma = 0$ and $\gamma = 1$ are $\lambda_{Max} = 14.07, 3.76$, $\lambda_{Trace} = 15.41, 3.76$, respectively. Standard errors are in the parenthesis.

Table 3. Descriptive statistics

	Corn		Soybean	
	R_t^s	R_t^f	R_t^s	R_t^f
Mean	-0.0003	-0.0002	-0.0004	-0.0003
Median	0.0000	0.0000	0.0000	0.0000
Maximum	0.0799	0.0866	0.0665	0.0754
Minimum	-0.1020	-0.0624	-0.0854	-0.1241
Std.Dev.	0.0162	0.0149	0.0141	0.0143
SK	-0.2447	0.6323	-0.2949	-1.0806
KUR	7.0302	7.0887	6.3723	14.271
JB	686.05**	762.43**	487.85**	5482.6**
$Q(12)$	29.13**	10.66	14.21	16.55
$Q^2(12)$	138.3**	90.5**	131.9**	95.2**

Notes: Standard errors are given in the parentheses. SK and KUR are coefficient of skewness and kurtosis ($E[\Delta R_t - \mu]^2/\sigma$ and $E[\Delta R_t - \mu]^4/\sigma$, respectively, where μ is the mean and σ is the standard deviation). JB test denotes the Jarque and Bera test for normality defined as $T[SK^2/6 + (KUR - 3)^2/24]$ which is asymptotically distributed as $\chi^2(2)$. Q denotes the Ljung-Box test statistic. * and ** denote statistical significant at the 5% and 1% level, respectively.

Table 4. Estimation results for the VEC model

	Corn		Soybean	
	P_t^s	P_t^f	P_t^s	P_t^f
Π_s, Π_f	-0.0098	0.0415**	-0.0329*	0.0067
	(0.015)	(0.014)	(0.016)	(0.016)
$\Gamma_{1,s}^s, \Gamma_{1,f}^f$	-0.1725**	0.0731	-0.0585	-0.0267
	(0.053)	(0.049)	(0.063)	(0.064)
$\Gamma_{2,s}^s, \Gamma_{2,f}^f$	-0.1738**	-0.0777	0.0796*	0.1134**
	(0.055)	(0.051)	(0.039)	(0.064)
$\Gamma_{3,s}^s, \Gamma_{3,f}^f$	0.0306	0.0296		
	(0.055)	(0.051)		
$\Gamma_{4,s}^s, \Gamma_{4,f}^f$	-0.0639	-0.1065**		
	(0.053)	(0.049)		
$\Gamma_{1,f}^s, \Gamma_{1,s}^f$	0.2017**	-0.0197	0.0633	0.0345
	(0.058)	(0.054)	(0.062)	(0.063)
$\Gamma_{2,f}^s, \Gamma_{2,s}^f$	0.1752**	0.0759	- 0.0491	-0.0490
	(0.059)	(0.055)	(0.062)	(0.063)
$\Gamma_{3,f}^s, \Gamma_{3,s}^f$	-0.0814	-0.0500		
	(0.059)	(0.055)		
$\Gamma_{4,f}^s, \Gamma_{4,s}^f$	0.1136*	0.1089*		
	(0.057)	(0.053)		
SK	-0.3379	0.6211	-0.2676	-1.0218
KUR	6.3954	6.8274	6.2053	13.763
JB	496.91**	671.28**	437.82**	4976.1**
$Q(10)$	11.063	6.2257	12.269	12.865
$Q^2(10)$	122.22**	65.623**	140.50**	118.24**

Notes: Since we use demeaned series for R_t^s and R_t^f there are no constant terms. Truncation lag p is selected based on SIC for each model. p are 4 and 2 for corn and soybeans, respectively. Standard errors are given in the parentheses. * and ** denote statistical significant at the 5% and 1% level, respectively.

Table 5. Results for the constancy of correlation test

	Corn		Soybean	
	T1	T2	T1	T2
IM_e	194.19	329.64	211.73	300.38
p-value	0.0000	0.0000	0.0000	0.0000
IM_s	4.2631	10.552	5.2441	9.2765
p-value	0.0389	0.0011	0.0220	0.0023

Notes: IM_e and IM_s are calculated by (6) and (7), respectively. These test statistics follow χ^2 with degree of freedom one. T1 and T2 denote symmetric and asymmetric individual conditional variance specifications of bivariate GARCH models, respectively.

Table 6. DCC BGARCH Estimates for corn series

	Normal				Student's t				Skewed-t			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
γ_1	0.1473**	0.0270	0.1348**	0.0254	0.2333**	0.0699	0.1908**	-0.0096	0.2374**	-0.0210	0.1964**	-0.0118
	(0.048)	(0.028)	(0.042)	(0.028)	(0.060)	(0.157)	(0.059)	(0.018)	(0.059)	(0.017)	(0.062)	(0.017)
τ_1		0.1605**		0.1825**		0.1600*		0.0700**		0.0737**		0.0692**
		(0.075)		(0.081)		(0.079)		(0.025)		(0.020)		(0.024)
β_1	0.3747**	0.5617**	0.3793**	0.5601**	0.2926	0.6016	0.2966	0.9638**	0.2897	0.9753**	0.2561	0.9671**
	(0.137)	(0.157)	(0.144)	(0.154)	(0.209)	(0.548)	(0.238)	(0.015)	(0.191)	(0.013)	(0.257)	(0.014)
γ_2	0.0809	-0.0124	0.0665	-0.0127	0.1854**	0.0465	0.1457**	-0.0054	0.1953**	-0.0119	0.1554**	-0.0065
	(0.056)	(0.009)	(0.044)	(0.008)	(0.049)	(0.146)	(0.048)	(0.010)	(0.052)	(0.008)	(0.059)	(0.009)
τ_2		0.1401		0.1571		0.1538**		0.0864**		0.0828**		0.0857**
		(0.114)		(0.118)		(0.059)		(0.027)		(0.025)		(0.026)
β_2	0.5625**	0.8419**	0.5690**	0.8384**	0.4536**	0.7195**	0.4572*	0.9465**	0.4278**	0.9584**	0.3782	0.9479**
	(0.079)	(0.132)	(0.075)	(0.109)	(0.119)	(0.394)	(0.187)	(0.023)	(0.122)	(0.019)	(0.319)	(0.020)
δ_1	0.1046*	0.1306*	0.2312**	0.0880*	0.2033**	0.1886**	0.0050	0.0077	0.2098**	0.1741**	0.0042	0.0067
	(0.052)	(0.068)	(0.087)	(0.045)	(0.070)	(0.076)	(0.008)	(0.013)	(0.058)	(0.052)	(0.007)	(0.009)
δ_2	0.0294	0.0000	0.2665	0.0000	0.7305**	0.7491**	0.9114**	0.8862**	0.7227**	0.7667**	0.9157**	0.9009**
	(0.308)	(0.000)	(0.263)	(0.000)	(0.113)	(0.123)	(0.059)	(0.111)	(0.093)	(0.085)	(0.041)	(0.073)
δ_3			-0.2181	0.1166			0.0760	0.0928			0.0732*	0.0819
			(0.103)	(0.305)			(0.046)	(0.080)			(0.032)	(0.054)
ν					3.2813**	3.3125**	3.2005**	3.0429**	3.3177**	3.0065**	3.2146**	3.0380**
					(0.307)	(0.327)	(0.300)	(0.346)	(0.303)	(0.299)	(0.295)	(0.333)
ξ_1									0.9203**	0.9274**	0.9129**	0.9239**
									(0.033)	(0.038)	(0.031)	(0.035)
ξ_2									1.1883**	1.1887**	1.1820**	1.1895**
									(0.044)	(0.044)	(0.047)	(0.047)
Loglik	6067.5	6101.2	6071.9	6102.1	6414.9	6416.7	6423.2	6434.7	6429.7	6439.7	6437.8	6449.7
AIC	-12.179	-12.243	-12.187	-12.243	-12.876	-12.875	-12.891	-12.909	-12.902	-12.918	-12.916	-12.936
SIC	-12.141	-12.194	-12.142	-12.189	-12.831	-12.821	-12.841	-12.851	-12.847	-12.854	-12.857	-12.867
$Q_{\epsilon_1}(10)$	10.733	11.852	10.452	11.945	7.5421	7.2627	8.4030	7.9923	7.4350	7.2982	8.5286	8.4189
$Q^2_{\epsilon_1}(10)$	10.599	11.535	11.054	11.363	7.115	14.532	11.546	12.689	16.752	11.058	11.827	14.615
$Q_{\epsilon_2}(10)$	9.4611	10.1613	9.6131	10.1922	6.9046	7.9038	8.0258	8.6554	7.0131	8.6233	8.0298	8.5784
$Q^2_{\epsilon_2}(10)$	9.7221	11.386	10.189	11.346	8.9445	7.5676	6.6623	8.1656	9.1599	10.8703	6.8430	8.3096

Notes: Robust standard errors are given in the parentheses. Q_{ϵ_i} and $Q^2_{\epsilon_i}$ denote the Ljung- Box test statistic for level and squared variables of ϵ_i, respectively. * and ** denote statistical significance at the 5% and 1% level, respectively.

Table 7. DCC BGARCH Estimates for soybeans series

	Normal				Student's t				Skewed-t			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
γ_1	0.0943**	0.1048**	0.0947**	0.2200	0.0793**	0.0870*	0.0803**	0.0853**	0.0784**	0.0862**	0.0793**	0.0845**
	(0.021)	(0.036)	(0.027)	(0.109)	(0.023)	(0.034)	(0.023)	(0.031)	(0.022)	(0.027)	(0.022)	(0.030)
τ_1		-0.0305		-0.0873		-0.0166		-0.0114		-0.0157		-0.0112
		(0.040)		(0.072)		(0.041)		(0.037)		(0.027)		(0.039)
β_1	0.8805**	0.8805**	0.8782**	0.7692**	0.8930**	0.8960**	0.8928**	0.8954**	0.8920**	0.8944**	0.8917**	0.8939**
	(0.018)	(0.022)	(0.022)	(0.086)	(0.030)	(0.030)	(0.030)	(0.030)	(0.030)	(0.030)	(0.030)	(0.030)
γ_2	0.0800**	0.0598*	0.0743**	0.2373	0.0623**	0.0639*	0.0631**	0.0621**	0.0615**	0.0632*	0.0622**	0.0615*
	(0.028)	(0.026)	(0.023)	(0.186)	(0.020)	(0.032)	(0.020)	(0.028)	(0.020)	(0.024)	(0.020)	(0.029)
τ_2		0.0419		0.0356		-0.0012		0.0034		-0.0008		0.0032
		(0.036)		(0.099)		(0.043)		(0.036)		(0.028)		(0.041)
β_2	0.9136**	0.9099**	0.9174**	0.6787**	0.9118**	0.9127**	0.9114**	0.9126**	0.9112**	0.9117**	0.9107**	0.9116**
	(0.018)	(0.027)	(0.018)	(0.172)	(0.027)	(0.028)	(0.027)	(0.027)	(0.027)	(0.028)	(0.027)	(0.027)
δ_1	0.1280*	0.1267*	0.5996*	0.4298	0.1955**	0.1940**	0.1556*	0.1578*	0.1949**	0.1936**	0.1589**	0.1613**
	(0.063)	(0.059)	(0.324)	(0.362)	(0.028)	(0.028)	(0.066)	(0.070)	(0.028)	(0.028)	(0.066)	(0.069)
δ_2	0.8009**	0.8083**	0.6970**	0.7738**	0.7893**	0.7911**	0.7886**	0.7903**	0.7900**	0.7916**	0.7893**	0.7909**
	(0.120)	(0.110)	(0.220)	(0.141)	(0.031)	(0.031)	(0.031)	(0.031)	(0.031)	(0.031)	(0.031)	(0.031)
δ_3			-0.4725	-0.2839			0.0419	0.0380			0.0378	0.0338
			(0.338)	(0.313)			(0.064)	(0.068)			(0.063)	(0.067)
ν					3.1072**	3.1055**	3.0837**	3.0841**	3.1506**	3.1479**	3.1280**	3.1278**
					(0.322)	(0.321)	(0.322)	(0.321)	(0.335)	(0.335)	(0.335)	(0.335)
ξ_1									1.0059**	1.0038**	1.0061**	1.0043**
									(0.039)	(0.039)	(0.039)	(0.039)
ξ_2									0.9581**	0.9584**	0.9599**	0.9599**
									(0.036)	(0.036)	(0.037)	(0.037)
Loglik	6666.2	6682.5	6678.4	6670.9	7046.8	7047.5	7047.2	7047.7	7047.6	7048.2	7047.9	7048.4
AIC	-13.383	-13.412	-13.406	-13.387	-14.146	-14.144	-14.145	-14.142	-14.144	-14.141	-14.142	-14.140
SIC	-13.344	-13.363	-13.361	-13.333	-14.102	-14.089	-14.096	-14.083	-14.090	-14.077	-14.083	-14.071
$Q_{\epsilon_1}(10)$	13.449	12.396	10.856	9.894	15.647	15.686	16.230	16.177	15.609	15.626	16.133	16.065
$Q^2_{\epsilon_1}(10)$	15.777	15.670	11.302	9.760	16.052	16.183	16.799	16.908	15.902	15.966	16.561	16.601
$Q^2_{\epsilon_2}(10)$	3.8292	4.0077	4.4952	3.8527	4.3051	4.1919	4.3033	4.2285	4.3091	4.1998	4.3056	4.2306
$Q^2_{\epsilon_2}(10)$	6.6293	7.1764	6.1165	9.0648	5.3727	5.4119	5.5277	5.4880	5.3533	5.3837	5.4895	5.4490

Notes: Robust standard errors are given in the parentheses. Q_{ϵ_i} and $Q^2_{\epsilon_i}$ denote the Ljung- Box test statistic for level and squared variables of ϵ_i, respectively. * and ** denote statistical significance at the 5% and 1% level, respectively.

Table 8. In-sample comparisons of hedge performance

	Unhedged position		Naive hedge ($\beta = 1$)			OLS conventional hedge		
	Mean	Variance	Mean	variance	VR	mean	variance	VR
Corn	-0.0302	0.0262	-0.0058	0.0103	0.6049	-0.0093	0.0099	0.6220
Soybeans	-0.0245	0.0221	-0.0031	0.0056	0.7215	-0.0092	0.0051	0.7445

	Normal				Student's t				Skewed-t			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
Corn												
Mean	-0.070	-0.012	-0.081	0.009	-0.042	-0.012	0.000	0.022	-0.045	-0.068	-0.004	0.021
Variance	0.0098	0.0094	0.0099	0.0095	0.0107	0.0102	0.0104	0.0101	0.0106	0.0100	0.0104	0.0102
VR	0.6215	0.6298	0.6198	0.6278	0.6059	0.6150	0.6097	0.6156	0.6061	0.6190	0.6104	0.6152
Soybean												
Mean	-0.172	-0.169	-0.204	-0.201	-0.161	-0.158	-0.155	-0.154	-0.161	-0.159	-0.156	-0.155
Variance	0.0022	0.0021	0.0022	0.0055	0.0023	0.0022	0.0023	0.0059	0.0023	0.0022	0.0023	0.0059
VR	0.7668	0.7731	0.7629	0.6278	0.7607	0.7629	0.7608	0.6156	0.7607	0.7628	0.7608	0.6152

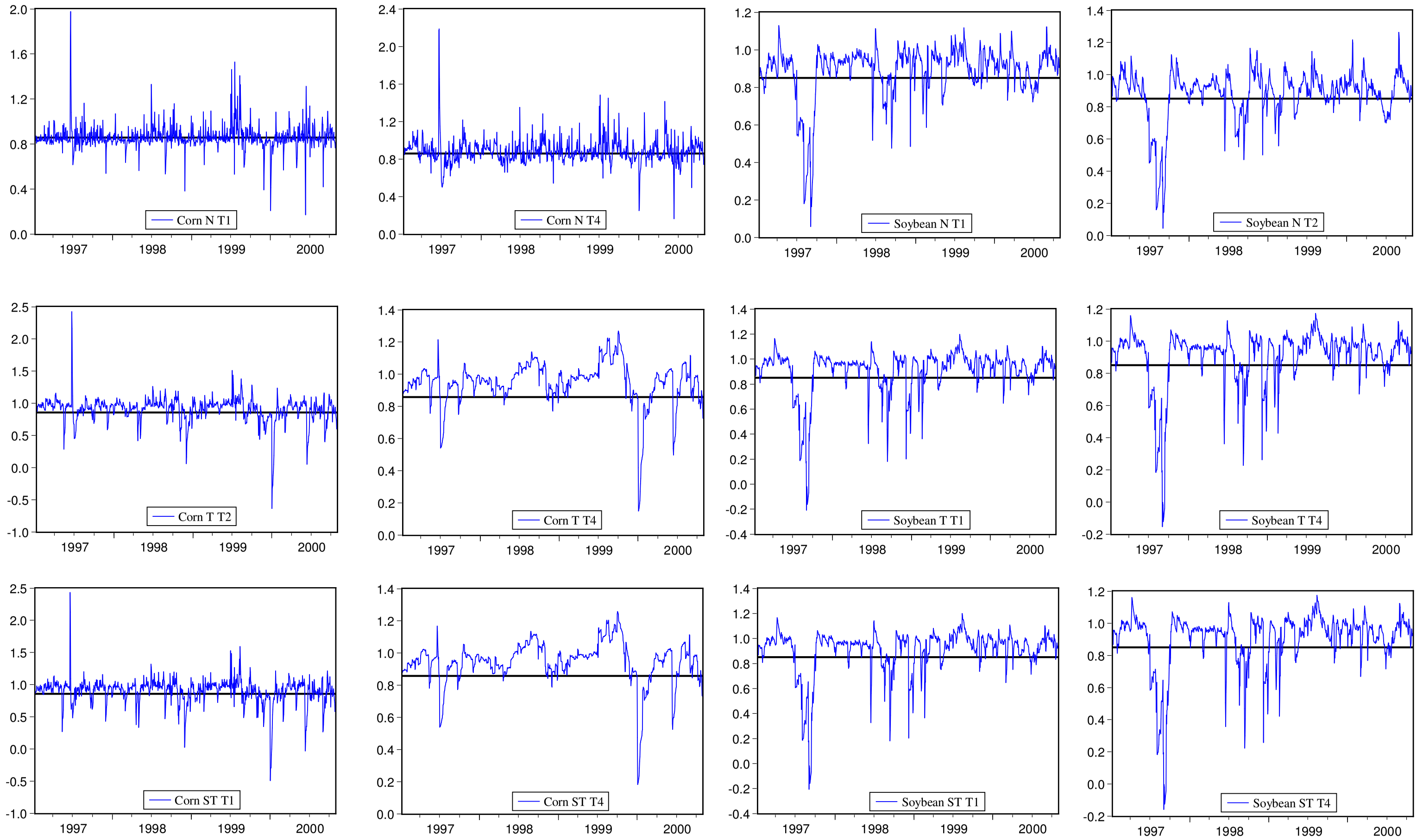
Notes: VR denotes variance reduction calculated by (19).

TABLE 9. OUT-OF-SAMPLE COMPARISONS OF HEDGE PERFORMANCE

	Unhedged position				Naive hedge ($\beta = 1$)				OLS conventional hedge			
	Mean	Std.Dev.			Mean	Std.Dev.	VR		mean	Std.Dev.	VR	
Corn	0.00005	0.0111			-0.0005	0.0067	0.6396		-0.0003	0.0064	0.6694	
Soybeans	0.0009	0.0097			-0.0001	0.0055	0.6887		0.0001	0.0051	0.7229	
	Corn											
	Normal				Student's t				Skewed t			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
Mean	-0.001	-0.001	-0.001	-0.001	-0.001	0.000	0.000	0.000	-0.001	-0.001	0.000	0.000
VR	0.6618	0.6667	0.6293	0.6562	0.6609	0.6641	0.6674	0.6721	0.6605	0.6628	0.6709	0.6721
Std. dev.	0.0062	0.0060	0.0111	0.0063	0.0065	0.0066	0.0064	0.0064	0.0066	0.0068	0.0063	0.0063
VaR 5%	0.0100	0.0100	0.0107	0.0099	0.0101	0.0105	0.0099	0.0100	0.0102	0.0107	0.0098	0.0099
VaR 1%	0.0260	0.0259	0.0293	0.0262	0.0271	0.0262	0.0272	0.0267	0.0271	0.0264	0.0270	0.0263
ES 5%	0.0111	0.0108	0.0165	0.0104	0.0113	0.0126	0.0105	0.0105	0.0109	0.0128	0.0103	0.0102
ES 1%	0.0113	0.0111	0.0355	0.0112	0.0132	0.0129	0.0145	0.0131	0.0132	0.0130	0.0132	0.0131
	Soybeans											
Mean	0.0003	0.0005	0.0003	0.0003	0.0002	0.0003	0.0003	0.0003	0.0003	0.0002	0.0003	0.0002
VR	0.7098	0.7109	0.6923	0.7057	0.7044	0.7060	0.7065	0.7083	0.7058	0.7039	0.7063	0.7056
Std. dev.	0.0048	0.0047	0.0052	0.0052	0.0054	0.0053	0.0051	0.0052	0.0054	0.0056	0.0053	0.0054
VaR 5%	0.0046	0.0044	0.0087	0.0056	0.0077	0.0073	0.0064	0.0071	0.0079	0.0090	0.0073	0.0085
VaR 1%	0.0184	0.0169	0.0199	0.0170	0.0190	0.0191	0.0188	0.0188	0.0190	0.0191	0.0191	0.0190
ES 5%	0.0105	0.0105	0.0184	0.0113	0.0127	0.0119	0.0105	0.0120	0.0127	0.0128	0.0122	0.0127
ES 1%	0.0117	0.0114	0.0193	0.0115	0.0177	0.0177	0.0172	0.0136	0.0176	0.0177	0.0178	0.0177

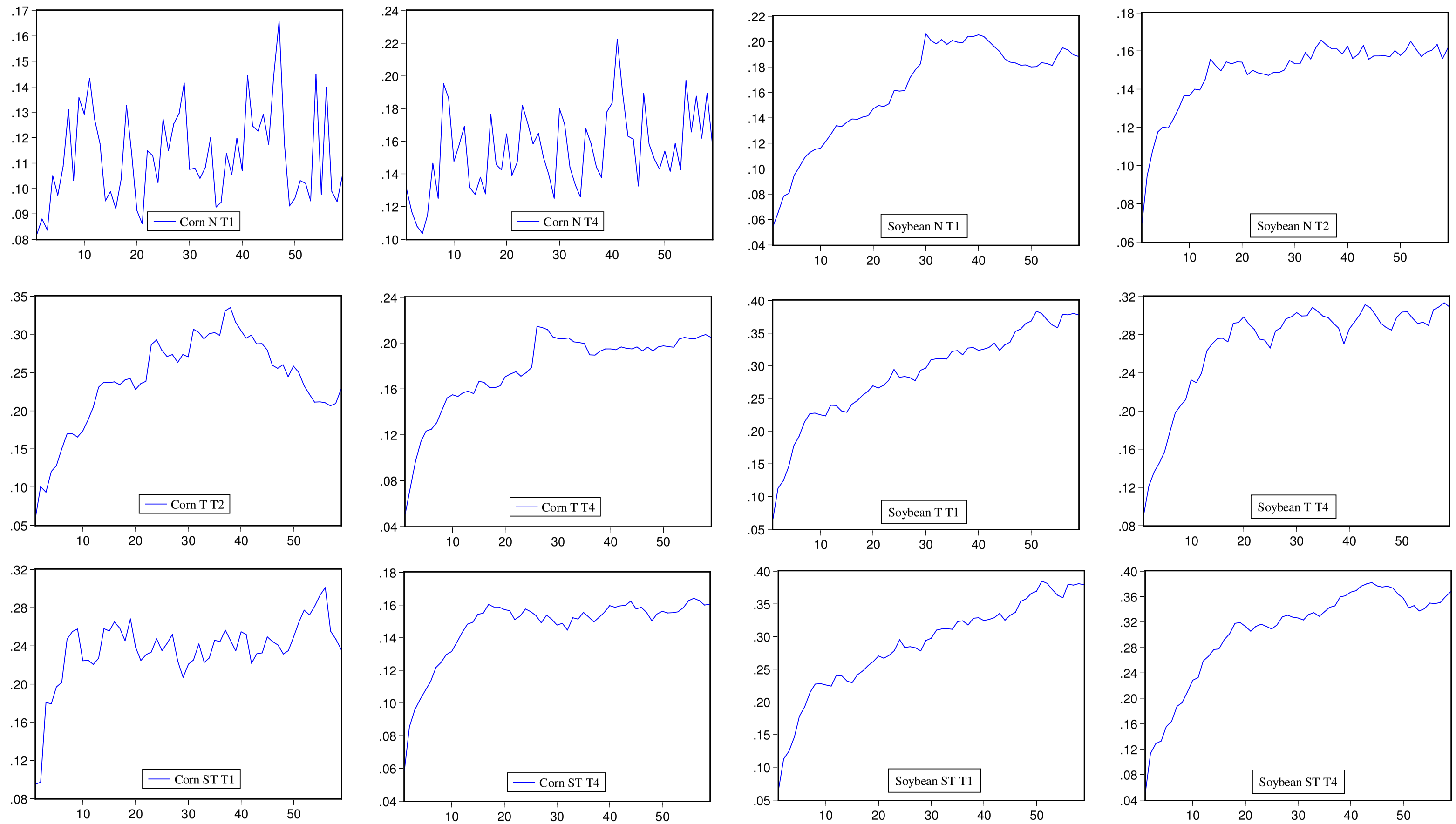
Notes: VR denotes variance reduction calculated by (19). Value at risk (VaR) and expected shortfall (ES) are calculated by (22) and (24), respectively.

Figure. 1. Estimated conditional hedge ratios : Corn and Soybeans



Notes: N and ST denote normal and skewed-t distribution, respectively. Unconditional hedge ratios estimated by OLS method are plotted with horizontal lines.

FIGURE. 2. BOOTSTRAP STANDARD DEVIATION OF CONDITIONAL HEDGE RATIO : CORN AND SOYBEANS



Notes: N and ST denote normal and skewed-t distribution, respectively. Standard deviation is calculated using 500 bootstrapped forecasting conditional hedge ratio at each k for $k = 1, 2, \cdots, 60$.