ARCHIVES
of
FOUNDRY ENGINEERING

Published quarterly as the organ of the Foundry Commission of the Polish Academy of Sciences

ISSN (1897-3310)
Volume 9
Issue 3/2009

195 – 200

35/3

Determination of Weibull Analysis of the Hypereutectic Silumins Reliability in Failure Time Respect

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Received 20.04.2009; accepted in revised form 24.04.2009

Abstract

The results of dynamic evaluation of the reliability of hypereutectic AlSi17Cu3NiMg silumin under the effect of symmetrical cyclic tensile-compressive stresses were presented. Studies were carried out on a normal-running fatigue testing machine, which was the mechanically driven resonant pulsator. For the needs of quantitative reliability evaluation and the time-to-failure evaluation, the procedures used in survival analysis, adapted to the analysis of failure-free operation with two- and three-parametric Weibull distributions, were applied. The values of the parameters were estimated using the method of maximum reliability and a rank-based non-parametric method. The results of the evaluation of the reliability and damage intensity are an important element in the determination of casting quality and enable a reliable estimation of the operational suitability time.

Keywords: Computer-aided casting production, Weibull analysis, Reliability, Analysis of fault occurrence time

1. Introduction

Components operating under changing loads suffer after some time destruction even if stresses responsible for this destruction are lower than the tensile or yield strength of the examined material. For the safety requirements imposed on equipment used in the automotive and aircraft industries, the fatigue behaviour of materials is of paramount importance.

The effect of changing loads results in the formation of microcracks, invisible until they develop to macroscopic dimensions. Then they start propagating very rapidly, ending in fatigue fracture. The fracture nucleation (initiation) usually occurs on the element surface, in places of the stress concentration or where defects of different types are present.

2. Materials and methods

To ensure safe operation of machines and equipment, the respective materials and structures are examined under the conditions of the changing loads. The conditions of fatigue testing of metallic materials have been determined, among others, by the Polish Standard PN-76/H-04325 „Fatigue testing of metallic materials”. The said standard gives main reference terms and establishes general guidelines for preparation of the specimens and conditions under which the tests should be carried out. The performed tests most often include the tensile-compression test and bending-torsion test, made on both plain and notched specimens, and also on real items.

Each type of the changing load (tensile, compressive, etc.) has a corresponding form of the changing stress. Stresses of the values changing in a repetitive and continuous manner during one loading cycle form a **stress cycle**.

For description of the stress cycle in Figure 1, only changes in the normal stresses σ were taken into consideration, given the fact only these stresses were used in current investigations.

The dynamic fatigue tests were carried out applying the loads of σ_{max}=-σ_{min}=150 [MPa], under the conditions of symmetrical tensile-compressive stresses changing in cycles. Tests were carried out on a normal-running fatigue testing machine, which in this case was the mechanically driven resonant pulsator.

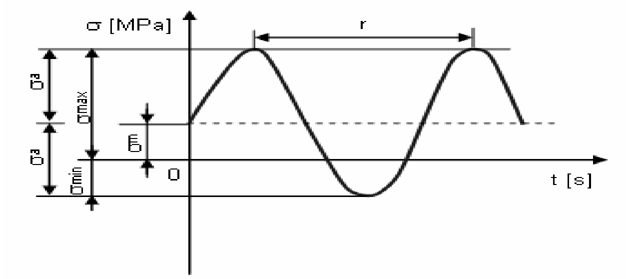


Fig. 1. The sinusoidal fatigue stress cycle

Tests were carried out in the following mode: after setting a load and waiting for a predetermined period of time (an hour and a half minimum), the test was stopped and the load was relieved. If the specimen failed (also before the preset time), the data were considered ***„complete information”***. If the specimen did not fail, the result gave ***„trimmed information”.***

Testing of static fatigue life was made on cast specimens processed according to four different variants: alloy non-modified and non-heat treated, alloy modified, alloy heat-treated, and alloy modified and heat-treated.

To conduct the test properly, it was important to design a test stand in a manner such as to create the testing conditions approaching as much as possible the conditions of the real melting, casting, and solidification. Maintaining the temperature, time and chemical composition constant was of key importance for further statistical analysis, and for computation and correct interpretation of the obtained results.

Alloys were melted from the following charge materials: AR1 aluminium (99,96% Al), technical silicon of 98,5% purity (rest Fe and other elements), electrolytic copper (99, 98% Cu), electrolytic nickel (99, 98% Ni), cast AG10 alloy (about 10 wt.% Mg).

Melting was carried out in a 3 kg capacity magnesite crucible installed in an induction LEYBOLD-HERAEUS IS5/III furnace, using a protective atmosphere of 2NaF and KCl (mixed in a ratio of 20 to 80%, respectively). After preheating the furnace to ~ 820°C, to make preliminary degassing of the examined alloy, the melt was refined with Rafglin-3 added in an amount of 0,3 wt.% respective of the alloy weight. The melt temperature was controlled with an NiCr-NiAl TP-202K-800-1 thermocouple immersed in the melt. Modification was carried out with phosphorus added in an amount of 0,05 wt.% in the form of a Cu-P10 master alloy (~9,95 wt.% P). The samples were next cast into a metal mould. The chemical composition of the alloy was as follows: 16,38% Si, 2,79% Cu, 1,40% Ni, 1,38% Mg, 0,45% Fe, 0,04% Mn, 0,01% Ti, rest Al.

The heat treatment process consisted in precipitation hardening and was basically composed of the two integral operations: solutioning at 500 °C ± 5 °C / 4 h/, cooling in boiling water, and rapid ageing at 175°C ± 5°C / 8 h/ followed by cooling with furnace.

Sampling of the examined cast hypereutectic AlSiCuNiMg alloy, as well as the preparation and processing of specimens were carried out in a way such as to ensure the highest possible homogeneity of samples. The tested sample lot was taken from one alloy melt of the same processing history. Due care was taken to make a very accurate machining, exactly the same for all the specimens included in a lot and providing the roughness values on the specimen surface according to PN-73/M-04251. The techniques of sampling and sample processing were consistent with PN-76/H-04325. The specimen dimensions (Fig. 34) satisfied the requirements of PN-74/H-04327 for an axial tensile-compression test.

Fig. 2. A fatigue specimen

The results of the reliability/fault time test were analysed with two- and three-parametric Weibull distributions, for which the density function of the adopted *parameters of scale* (*b*>0), *shape* (*c*>0) and *location* (γ < x) was plotted in Figure 1 [1]. The variable x is time.

The location parameter γ, determining the minimum fault time, is known and usually of zero value. Sometimes, however, the probability of component failure continues being zero still for some time after the test has been started. If this is the case, then the location parameter of a value larger than zero should be used. [1,2] reports that large values of the shape parameter (i.e. above 6) after fitting the two-parametric Weibull distribution may indicate that, in reality, we have a three-parametric distribution with non-zero location parameter γ.

The evaluation of a cumulative distribution function (irrespective of the distribution type) has been based on j ranks determined for n observations, with the following determination of rank F(x) - median, mean, or White drawing point [1,2]

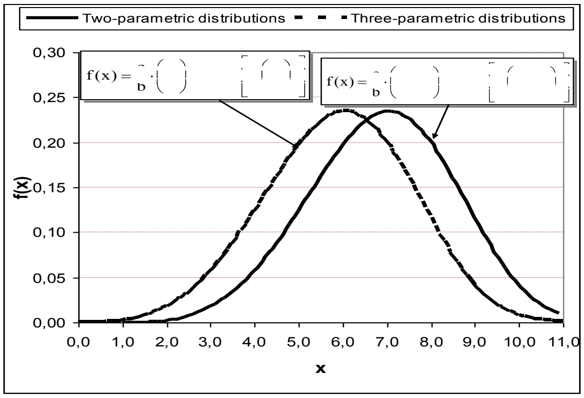
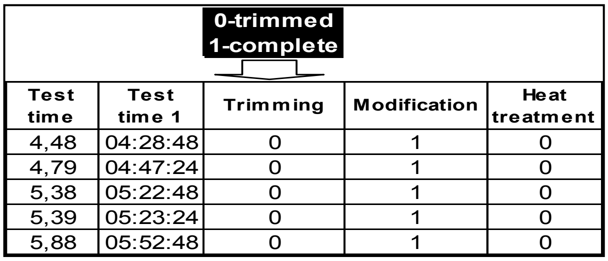


Fig. 3. A probability density function for the two- and three-parametric Weibull distributions

3. Results

The Weibull reliability/fault time analysis was carried out for the results based on time-to-failure data (complete data) and end-of-test data (trimmed data) (Fig.4).



0-trimmed
1-complete

Test time	Test time 1	Trimming	Modification	Heat treatment
4,48	04:28:48	0	1	0
4,79	04:47:24	0	1	0
5,38	05:22:48	0	1	0
5,39	05:23:24	0	1	0
5,88	05:52:48	0	1	0

Fig. 4. The results of a reliability/fault time test

The Weibull probability grids were drawn, first, (Fig. 5) with a non-parametric rank-based estimation of the shape and scale parameters of a two-parametric distribution, thus enabling reading out of the characteristic value (a characteristic operational suitability time), defined as a time limit upon completing of which 63,2% of the population will have failed. This is the value of a proper *parameter in scale b*. From the diagrams we can also estimate the quality of fit of a regression line to the empirical data. If the quality of fit is satisfactory, we are free to proceed with the two-parametric distribution, assuming the location parameter value as equal to zero. For evaluation of the fitting quality on a probability diagram with different values of the *location parameter,* the determination coefficient R^2 was used. Next, the parameters of the two- and three-parametric Weibull distributions were evaluated, applying the method of maximum reliability [5]. The results of this evaluation with Hollander-Proschan and Mann-Scheurer-Ferti goodness-of-fit test are compared in Table 1.

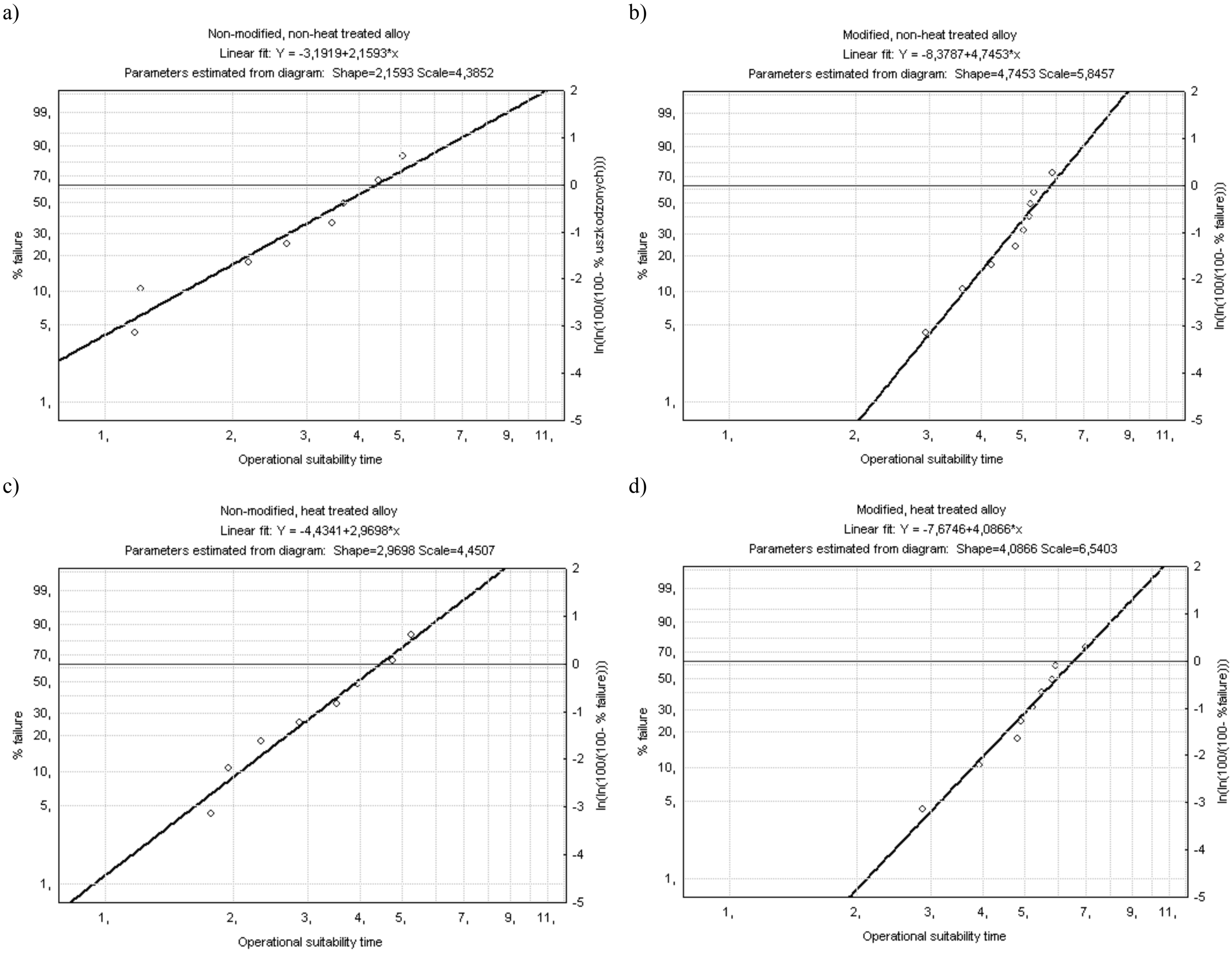
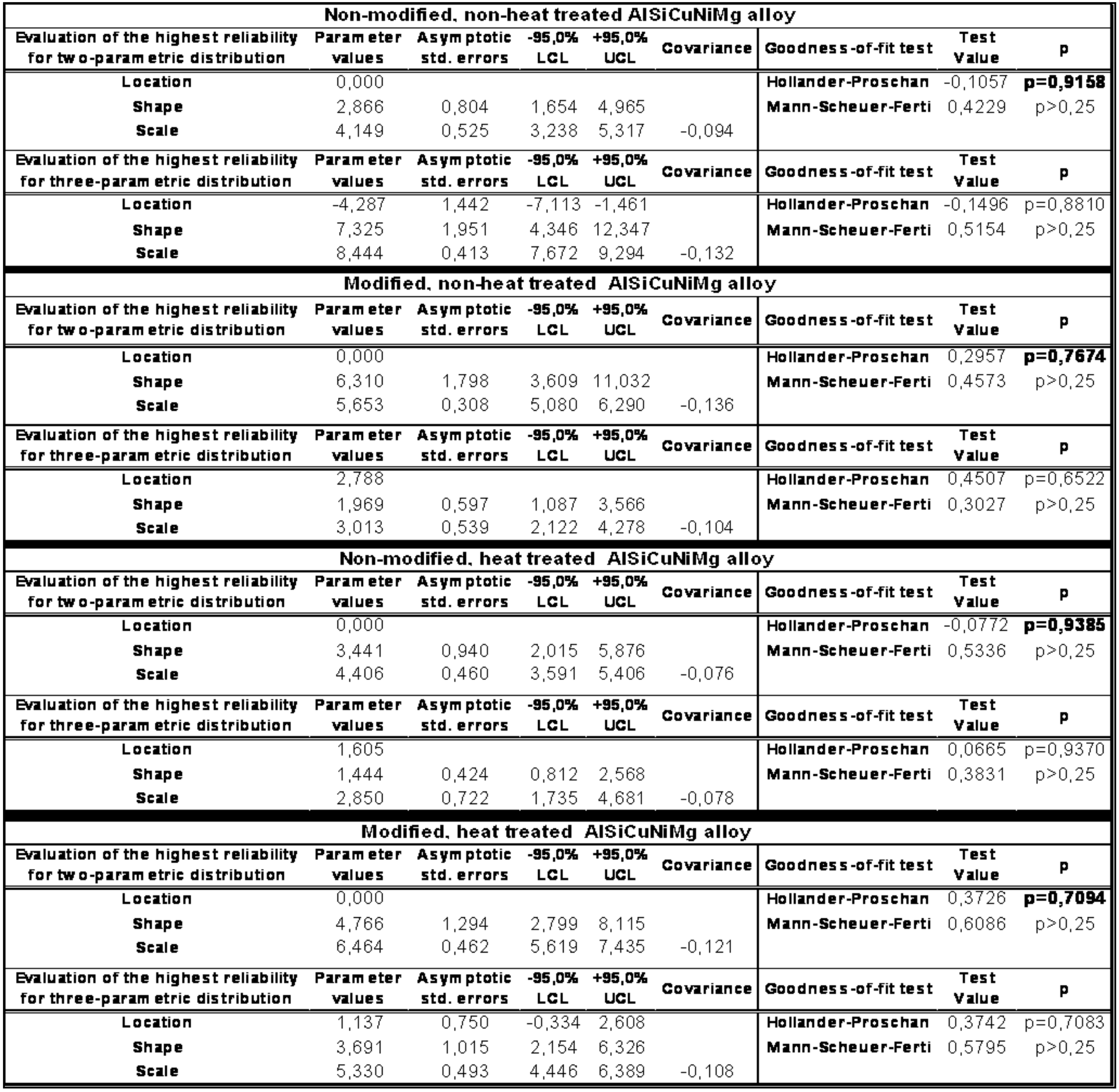


Fig. 5. Weibull probability grids for a two-parametric distribution

Table 1.
Evaluation of parameters for the two- and three-parametric Weibull distributions and the results of Hollander-Proschan and Mann-Scheurer-Ferti goodness-of-fit test



Non-modified, non-heat treated AlSiCuNiMg alloy								
Evaluation of the highest reliability for two-parametric distribution	**Parameter values**	**Asymptotic std. errors**	**-95,0% LCL**	**+95,0% UCL**	**Covariance**	**Goodness-of-fit test**	**Test Value**	**p**
Location	0,000					**Hollander-Proschan**	-0,1057	**p=0,9158**
Shape	2,866	0,804	1,654	4,965		**Mann-Scheuer-Ferti**	0,4229	p>0,25
Scale	4,149	0,525	3,238	5,317	-0,094			
Evaluation of the highest reliability for three-parametric distribution	**Parameter values**	**Asymptotic std. errors**	**-95,0% LCL**	**+95,0% UCL**	**Covariance**	**Goodness-of-fit test**	**Test Value**	**p**
Location	-4,287	1,442	-7,113	-1,461		**Hollander-Proschan**	-0,1496	p=0,8810
Shape	7,325	1,951	4,346	12,347		**Mann-Scheuer-Ferti**	0,5154	p>0,25
Scale	8,444	0,413	7,672	9,294	-0,132			
Modified, non-heat treated AlSiCuNiMg alloy								
Evaluation of the highest reliability for two-parametric distribution	**Parameter values**	**Asymptotic std. errors**	**-95,0% LCL**	**+95,0% UCL**	**Covariance**	**Goodness-of-fit test**	**Test Value**	**p**
Location	0,000					**Hollander-Proschan**	0,2957	**p=0,7674**
Shape	6,310	1,798	3,609	11,032		**Mann-Scheuer-Ferti**	0,4573	p>0,25
Scale	5,653	0,308	5,080	6,290	-0,136			
Evaluation of the highest reliability for three-parametric distribution	**Parameter values**	**Asymptotic std. errors**	**-95,0% LCL**	**+95,0% UCL**	**Covariance**	**Goodness-of-fit test**	**Test Value**	**p**
Location	2,788					**Hollander-Proschan**	0,4507	p=0,6522
Shape	1,969	0,597	1,087	3,566		**Mann-Scheuer-Ferti**	0,3027	p>0,25
Scale	3,013	0,539	2,122	4,278	-0,104			
Non-modified, heat treated AlSiCuNiMg alloy								
Evaluation of the highest reliability for two-parametric distribution	**Parameter values**	**Asymptotic std. errors**	**-95,0% LCL**	**+95,0% UCL**	**Covariance**	**Goodness-of-fit test**	**Test Value**	**p**
Location	0,000					**Hollander-Proschan**	-0,0772	**p=0,9385**
Shape	3,441	0,940	2,015	5,876		**Mann-Scheuer-Ferti**	0,5336	p>0,25
Scale	4,406	0,460	3,591	5,406	-0,076			
Evaluation of the highest reliability for three-parametric distribution	**Parameter values**	**Asymptotic std. errors**	**-95,0% LCL**	**+95,0% UCL**	**Covariance**	**Goodness-of-fit test**	**Test Value**	**p**
Location	1,605					**Hollander-Proschan**	0,0665	p=0,9370
Shape	1,444	0,424	0,812	2,568		**Mann-Scheuer-Ferti**	0,3831	p>0,25
Scale	2,850	0,722	1,735	4,681	-0,078			
Modified, heat treated AlSiCuNiMg alloy								
Evaluation of the highest reliability for two-parametric distribution	**Parameter values**	**Asymptotic std. errors**	**-95,0% LCL**	**+95,0% UCL**	**Covariance**	**Goodness-of-fit test**	**Test Value**	**p**
Location	0,000					**Hollander-Proschan**	0,3726	**p=0,7094**
Shape	4,766	1,294	2,799	8,115		**Mann-Scheuer-Ferti**	0,6086	p>0,25
Scale	6,464	0,462	5,619	7,435	-0,121			
Evaluation of the highest reliability for three-parametric distribution	**Parameter values**	**Asymptotic std. errors**	**-95,0% LCL**	**+95,0% UCL**	**Covariance**	**Goodness-of-fit test**	**Test Value**	**p**
Location	1,137	0,750	-0,334	2,608		**Hollander-Proschan**	0,3742	p=0,7083
Shape	3,691	1,015	2,154	6,326		**Mann-Scheuer-Ferti**	0,5795	p>0,25
Scale	5,330	0,493	4,446	6,389	-0,108			

Basing on the results of the goodness-of-fit test, it has been confirmed that, in each case, the two-parametric Weibull distribution provides a better description of the risk function than the three-parametric distribution.

From evaluations obtained by the method of maximum reliability, a risk function (the damage intensity) was plotted. The lowest damage intensity and the longest operational suitability offered the AlSiCuNiMg alloy after modification and heat treatment (Fig. 6). In this case, the time of the operational suitability, i.e. the condition of full reliability when the component is able to perform its function in a mode consistent with the requirements, amounts to approximately 4 h. In alloy non-modified and non-heat treated, this time is nearly half as long. Also the fragment of the risk curve that illustrates the component aging time is the least steep in the case of alloy modified and heat treated (Fig. 6).

The evaluations obtained by the method of maximum reliability enabled plotting the reliability function in a logarithmic scale, as shown in Fig. 7.

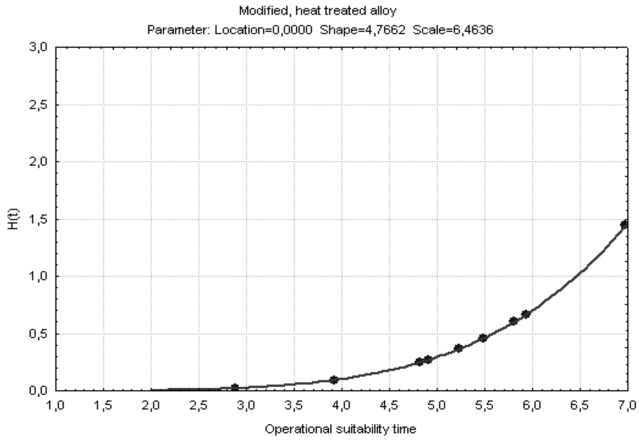


Fig. 6. The risk function (damage intensity) plotted from maximum reliability evaluations for the AlSi17CuNiMg alloy modified and heat treated

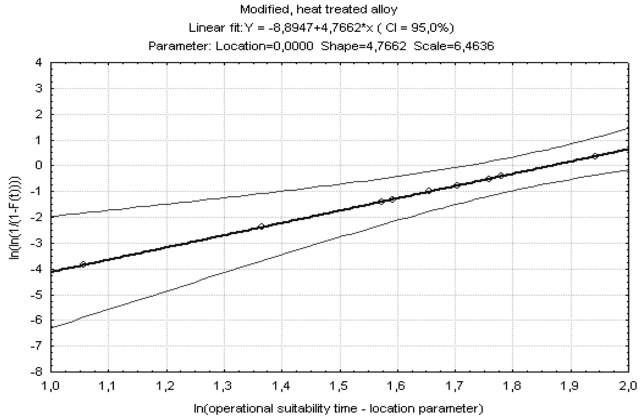


Fig. 7. The reliability function plotted in logarithmic scale with a confidence interval and evaluation of parameters done by the method of maximum reliability

When the shape parameter is smaller than 3, it is recommended to use the diagrams with non-parametric systems, i.e. based on ranks [3]. Figure 7 shows the measuring data, the linear fit, the 95% confidence interval for reliability (i.e. the ln-ln transformation – axis y), and the centre (50th percentile) of a non-parametric confidence interval. From the estimated value of the slope and an absolute term of the fitted straight line, the *shape parameter*, equal to a gradient value, was computed, while the *scale parameter* was computed as an exp(-absolute term/slope).

A very good consistency was obtained between the results of Weibull distribution parameters calculated by a non-parametric method and by the method of maximum reliability. Additionally, in all cases, fitting was characterised by a very high value of the correlation coefficient (R^2 above 0,95).

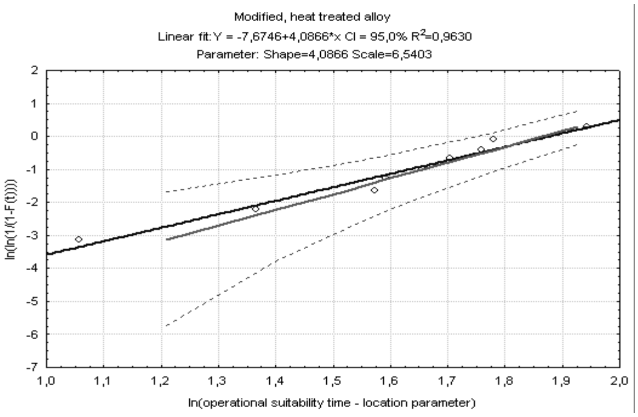


Fig. 8. Plotted reliability function with a confidence interval and parameters evaluated by a non-parametric method

Table 2 shows the results of 50th percentile (median) estimation of the reliability function with a 95% confidence interval.

Table 2.
The values of 50th percentile of the reliability function with a 95% confidence interval for the examined AlSiCuNiMg alloy

AlSiCuNiMg alloy	**Time t [h]**	**-95,0% LCL**	**+95,0% UCL**
Non-modified, non-heat treated	3,651	2,865	4,662
Modified, non-heat treated	5,334	4,808	5,917
Non-modified, heat treated	3,961	3,235	4,851
Modified, heat treated	5,985	5,216	6,868

The time corresponding to 50th percentile for the AlSiCuNiMg alloy non-modified and non-heat treated amounts to about 3,65 hours with a 95% confidence interval extending from 2,86 to 4,65 hours. Hence it can be expected that 50% of all the specimens will suffer damage by the time instant t=3,65 hours.

Figure 9 shows the plotted reliability diagram with a reliability function and 95% confidence intervals. The estimated values of reliability R(t) (the reliability indicator) of the examined component, i.e. the probability of its failure-free operation, are compared in Table 3.

Table 3.
The reliability R(t) values as estimated by the method of maximum reliability and by a non-parametric method

AlSiCuNiMg Alloy							
Non-modified, non-heat treated		**Modified, non-heat treated**		**Non-modified, heat treated**		**Modified, heat treated**	
Time to failure (t)	**R(t)**	**Time to failure (t)**	**R(t)**	**Time to failure (t)**	**R(t)**	**Time to failure (t)**	**R(t)**
1,18	0,973	2,94	0,984	1,78	0,957	2,88	0,979
1,22	0,970	3,61	0,943	1,96	0,940	3,92	0,912
2,19	0,852	4,22	0,854	2,34	0,893	4,82	0,781
2,69	0,749	4,82	0,694	2,88	0,793	4,91	0,764
3,44	0,557	5,03	0,620	3,51	0,633	5,23	0,695
3,67	0,495	5,19	0,558	3,93	0,509	5,49	0,632
4,43	0,299	5,22	0,546	4,75	0,274	5,81	0,548
5,07	0,169	5,33	0,501	5,28	0,155	5,93	0,515
		5,91	0,266			6,98	0,236

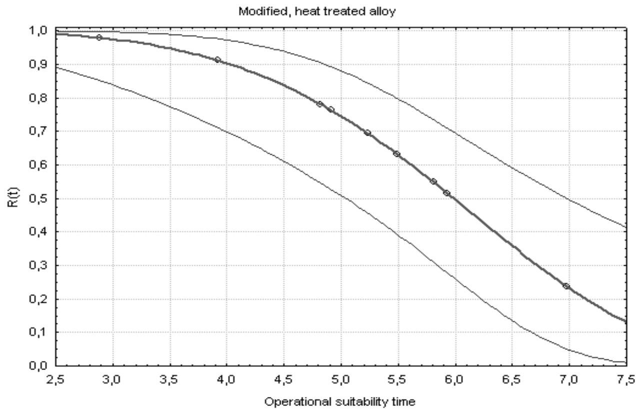


Fig. 9. Reliability - the probability diagram with a confidence interval for parameters evaluated by the method of maximum reliability

The reliability index R(t) is the probability that the component will be able to perform the required function under stated conditions and for a specifed period of time (t_1, t_2):

$$R(t) = P(T \geq t) = 1 - F(t),\ t \geq 0 \qquad (1)$$

where: F(t) – the cumulative distribution function of random variable T of the component operating time until the occurrence of damage, which is called fault (failure) of the component. The runs of function F(t) are shown in Figure 10.

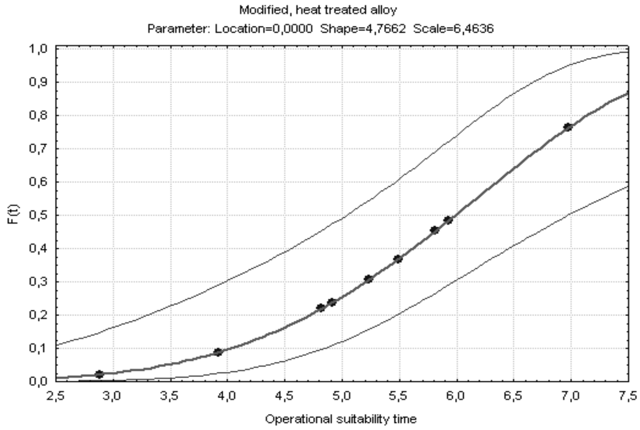


Fig. 10. Plotted cumulative distribution function F(t) (fault) with a confidence interval for parameters evaluated by the method of maximum reliability

Using fitted Weibull distribution, the percentiles of reliability function with 95% confidence intervals (LCL and UCL) were computed by the method of maximum reliability (Table 4).

Table .4.
Percentiles of reliability function with 95% confidence intervals computed by the method of maximum reliability

AlSiCuNiMg alloy	Non-modified, non-heat treated alloy			Modified, non-heat treated alloy		
Percentiles	Time t	-95,0% LCL	+95,0% UCL	Time t	-95,0% LCL	+95,0% UCL
25	2,69	1,98	3,64	4,64	4,06	5,30
50	3,65	2,87	4,65	5,33	4,81	5,92
75	4,65	3,55	6,09	5,95	5,29	6,69
AlSiCuNiMg alloy	Non-modified, heat treated alloy			Modified, heat treated alloy		
Percentiles	Time t	-95,0% LCL	+95,0% UCL	Time t	-95,0% LCL	+95,0% UCL
25	3,07	2,37	3,96	4,98	4,17	5,94
50	3,96	3,23	4,85	5,99	5,22	6,87
75	4,85	3,89	6,03	6,92	5,95	8,06

The information comprised in this table is particularly useful in determination of the expected fraction of components suffering failure after certain period of time. For example, it can be stated that 75% of non-modified and non-heat treated alloy specimens will suffer failure after the period of 4,65 hours, while for the specimens of modified and heat treated alloy this time will be prolonged to approximately 6,92 hours.

4. Summary and conclusions

Only the results of the fatigue tests which allow for the time of loading should be considered a rational and efficient tool in evaluation of the operating reliability of the responsible parts of machines and equipment. The method based on analysis and on the two- and three-parametric Weibull distributions, evaluating parameters by the method of maximum reliability and by a non-parametric method based on ranks, provides the reliable and complex information on, among others, the up time in function of the failed components percent fraction, the cumulative risk in function of up time, the reliability function with estimated percentiles and confidence intervals, and the probability function of reliability with a cumulative distribution function of this probability.

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