Emergence of double scaling law in complex system

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Abstract. - We introduce a stochastic model to explain a double power-law distribution which exhibits two different Paretian behaviors in the upper and the lower tail and widely exists in social and economic systems. The model incorporates fitness consideration and noise fluctuation. We find that if the number of variables (e.g. the degree of nodes in complex networks or people's incomes) grows exponentially, normal distributed fitness coupled with exponentially increasing variable is responsible for the emergence of the double power-law distribution. Fluctuations do not change the result qualitatively but contribute to the second-part scaling exponent. The evolution of Chinese airline network is taken as an example to show a nice agreement with our stochastic model.

Introduction. – Power law behaviors are now pervasive in various kinds of studies [1–14], which give an important class of complex networks, namely the scale-free networks. However, in some cases, such a single property is insufficient to describe the distributions in real-world systems in which scaling law is absent in some regions or, even more peculiar, changed at some critical points [15,16]. In contrast to the typical power law, distribution including two different power-law regions is called double power-law whose cumulative distribution, namely the probability that variable K is larger than a specific value k, is given by [16]:

$$P(K > k) = \begin{cases} k^{-\gamma_1}, & k < k_c; \\ k^{-\gamma_2}, & k > k_c \end{cases} \tag{1}$$

where γ_1 and γ_2 are two scaling exponents while k_c is the turning point. This kind of property exists widely in social and economic systems such as the degree distribution of airline network, word network or scientist collaboration network and the distribution of people's incomes [16–20].

Some works related to double power law concentrate on how to fit such distribution by a uniform function rather than treat two power separately. Non-extensive statistical theory and the combination of different power-law functions were applied to the problem [22,23]. However these works cannot tell us how this nontrivial property comes to

its being. To understand its underlying mechanism, Reed proposed a model based on geometric Brownian motion [19]. He proved that such process coupled with exponential distributed evolution time causes, as he called, a double Pareto-lognormal distribution which has a lognormal body but power-law behaviour in both tails. This distribution is shown to provide an excellent fit to observed data on incomes and earnings. Dorogovtsev and Mendes proposed another different model aiming to explain the double power-law degree distribution in word network [24]. The model is constructed by two mechanisms: the preferential attachment and the creation of new links between old nodes which increases with evolution time. By continues approach, they show the degree distribution has two different scaling exponents, -1.5 for upper tail and -3 for lower tail.

Although the above two models can explain incomes and word network respectively, both of them have limitations. In word network model the scaling exponents are fixed. Thus it cannot explain the distributions with distinct scaling exponents. While in Reed's model, the relative increase rate of incomes is assumed to be the same for all people. This is far from our knowledge that persons have heterogeneous ability of making money. Therefore fitness character must be taken into account to generalize the model. Besides, although the previous models reproduced some characters, the evolution of the real-world

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systems have not been investigated to support their model assumptions.

In this Letter, a general stochastic model is developed to explain the double power-law distribution. The model incorporates fitness consideration and noise fluctuation, which is general to describe many real-world system evolution. We find normal distributed fitness coupled with exponentially increasing variables is responsible for the emergence of the double power-law distribution while fluctuation does not change the result qualitatively but contribute to the scaling exponent. We also investigate the evolution of CAN to provide evidence for the proposed model.

Generalized model for double power law. – Let's denote $k_i(t, t')$ the value at time t of the i-th variable which comes into the system at time t' and denote $N(t)$ the number of the total variables in the system at time t. Regardless of the specific meaning of $k_i(t, t')$, its evolution pattern usually shares common features. For an example, the increasing rate of k_i is proportional to k_i itself. This is probably caused by the preferential attachment (also called the rich get richer) that widely exists in self-organized complex systems [4]. Besides, it is natural to assume that the increasing rate is proportional to some of its own attributes which is called fitness, denoted as η_i [25]. In reality fitness can be interpreted as, for examples, capital, social skills, activity levels of individuals and population or Gross Domestic Product (GDP) of cities. The increasing rate can also be influenced by other ingredients which can be normalized to be a time-dependent factors. But as the first step, let us focus on the simplest case where such factors are treated as constants. In this context, the evolution equation of $k_i(t, t')$ is given by:

$$\frac{dk_i}{dt} = \eta_i k_i. \qquad (2)$$

Thus k_i grows exponentially as $k_i(t, t') = e^{\eta_i(t-t')}$ (assuming $k_i(t', t') = 1$). This directly restricts the form of $N(t)$ in some systems such as node degree evolution in a network. Limited by its structure, the exponentially growing degree k_i requires the exponentially growing $N(t)$ (or even faster). Although in some cases such as people's incomes $N(t)$ does not encounter this problem, it has also been assumed to increase exponentially [19]. Therefore we assume

$$N(t) \propto e^{c_n t}. \qquad (3)$$

The distribution of fitness η_i is critical in our model. As we will analyze, normal distributed η_i is essential to produce the double power-law distribution. Note that when we choose the specific parameters for η_i in a network-structured system, we meet with the similar problem to $N(t)$. Since degree of a node can not exceed $n(t) - 1$, to keep a long time evolution η_i should be restricted so that the mean value of η_i, denoted as μ_η, cannot be far larger than c_n while the standard variance, denoted as σ_η, should be bounded properly.

Now let us turn to analyze the distribution under the above-mentioned condition. Using equation $p(k_i(t, t'))dk_i(t, t') = f(\eta_i)d\eta_i$, the distribution of $k_i(t, t')$ is easily derived to follow a lognormal distribution. Since the variables are added exponentially, their lifetime, defined as $T = t - t'$, follows exponential distribution. Therefore the value of k_i we actually examined is lognormal $k_i(T)$ mixed by exponentially distributed T. Thus the distribution reads:

$$p(k) = \int_0^{t_c} \frac{1}{\sqrt{2\pi}\sigma_\eta kT} e^{-\frac{(\ln(k) - \mu_\eta T)^2}{2\sigma_\eta^2 T^2}} e^{-c_n T} dT, \qquad (4)$$

where t_c is the time when we examine $p(k)$. If $\sigma_\eta \to 0$, $\frac{1}{\sqrt{2\pi}\sigma_\eta kT} e^{-\frac{(\ln(k) - \mu_\eta T)^2}{2\sigma_\eta^2 T^2}} \to \delta(T - \frac{\ln(k)}{\mu_\eta})$ and Eq.(4) gives:

$$p(k) \propto k^{-(1+\frac{c_n}{\mu_\eta})} \zeta(k - e^{\mu_\eta t_c}), \qquad (5)$$

where $\zeta(k) = \{^{1,\ k \leq 0}_{0,\ k > 0}$. The variables k_i follow a power-law distribution with a cut-off at $k_c = e^{\mu_\eta t_c}$ which is the maximum value. On the other hand, $\frac{1}{\sqrt{2\pi}\sigma_\eta kT} e^{-\frac{(\ln(k) - \mu_\eta T)^2}{2\sigma_\eta^2 T^2}} \to 0$ if $\sigma_\eta \to \infty$ and the integral becomes k independent, indicating a uniform distribution. However this never happens in a network-structured system since σ_η is limited as we have discussed.

For a finite $\sigma_\eta > 0$, it is difficult to derive analytical result from Eq.(4). Therefore numerical experiments are applied to analyze the problem. The simulation is carried out by the following instructions. At each time step new variables increasing exponentially are added by initial value 1 and are assigned fitness chosen from a normal distribution. Then each variable increases its value according to Eq.(2). We simulate the cumulative distribution for different σ_η as shown in Fig. 1. When $\sigma_\eta = 0$, the distribution follows a power-law form with a cut-off at $e^{\mu_\eta t_c}$, as we have discussed above. With the increase of σ_η, the second part of the distributions decreases more and more slowly while their shapes seem to be a power law. It is noteworthy that the turning point occurs at about $k_c = e^{\mu_\eta t_c}$ which is exact the point at which the cut-off occurs for $\sigma_\eta = 0$. Therefore the turning point is expected to increase with the evolution time t_c as $k_c \sim e^{\mu_\eta t_c}$, as well demonstrated in Fig. 2.

Another interesting discovery is that with the increase of σ_η, the first part of the distributions does not change significantly. From Fig. 1, we see that all the first part of the curves superpose the distribution of $\sigma_\eta = 0$ well. Thus the first part of $p(k)$ must follow the same power-law distribution as the $p(k)$ of $\sigma_\eta = 0$ with exponent $\gamma_1 = \frac{c_n}{\mu_\eta}$ which is independent of σ_η. However $p(k)$ will finally become uniform distribution when $\sigma_\eta \to \infty$. There must have a transition point σ_{tr} at which the first part of the distribution starts to change. By extensive numerical experiments, the transition point is determined to be $\sigma_{tr} \approx \frac{\mu_\eta}{2}$. The distributions of $\sigma_\eta > \sigma_{tr}$ are not studied since we only
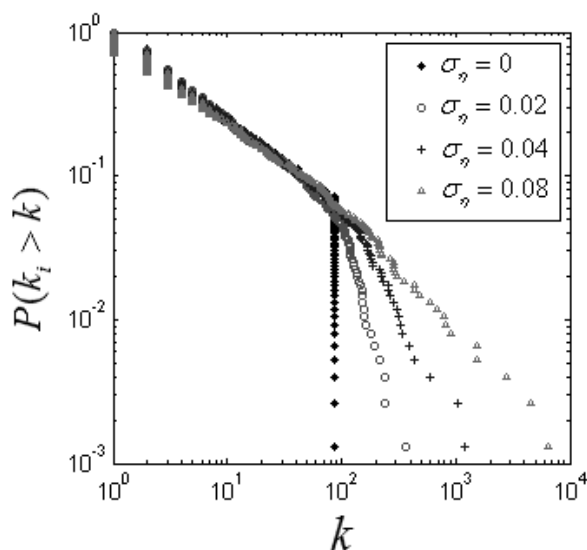
Fig. 1: Simulation of the cumulative distribution of variable k_i. The simulation is carried out with $\mu_\eta = 0.15$, $N(t) = 50e^{0.088t}$ and $t_c = 30$. For other μ_η and exponential increasing $N(t)$, similar results can be obtained. The study is averaged over 50 realizations.

Fig. 2: Simulation of the cumulative distribution of k_i at different t_c. The simulation is carried out with $\mu_\eta = 0.15$, $N(t) = 50e^{0.088t}$ and $\sigma_\eta = 0.02$. Inset: the correlation of k_c and t_c. The fitted line (solid line) represents $y = 0.6e^{0.15x}$, which illustrates that the turning point k_c increases with t_c as $k_c \sim e^{\mu_\eta t_c}$.

concern small σ_η. When $\sigma_\eta = \sigma_{tr}$, the second part of the distribution, as seen in Fig. 1, still decreases faster than $k^{-\frac{c_n}{\mu_\eta}}$. This result indicates that for all the $\sigma_\eta \leq \sigma_{tr}$, the second part of the distributions has an upper bound.

Now let us prove that the second part of $p(k)$ has a lower bound. It is easy to examine that when $\ln(k) > (\sigma_\eta \sqrt{\frac{\ln(t)}{t-1}} + \mu_\eta)t$, the following inequality is valid:

$$\frac{1}{\sqrt{2\pi}\sigma_\eta kT}e^{-\frac{(\ln(k)-\mu_\eta T)^2}{2\sigma_\eta^2 T^2}}e^{-c_n T} >$$
$$\frac{1}{\sqrt{2\pi T}\sigma_\eta k}e^{-\frac{(\ln(k)-\mu_\eta T)^2}{2\sigma_\eta^2 T}}e^{-c_n T}. \qquad (6)$$

Since σ_η is usually very small, the above inequality is approximately considered to be valid when $\ln(k) > \mu_\eta T$. Therefore for any $k > e^{\mu_\eta t_c}$ (namely the second part of $p(k)$. The following discussion is restricted to this condition), the integral of the left term in Eq.(6) from 0 to t_c must be larger than that of the right term. The integral of the left term is exactly the degree distribution $p(k)$ while the integral of the right term, according to Ref. [10], follows asymptotically a power-law function with the exponents related to μ_η, σ_η and c_n. Thus for a specific group of above exponents, $p(k)$ has a lower bound.

If $p(k)$ is not oscillatory, the existence of the upper and the lower bound allows $\frac{\ln(p(k))}{\ln(k)}$ to have a limitation. Assuming it to be $-\gamma$, then $p(k)$ is written as $p(k) \propto l(k)k^{-\gamma}$, where $l(k) \sim o(k^\beta)$ and $k^{-\beta} \sim o(l(k))$ are valid for any $\beta > 0$ when $k \to \infty$. Therefore for large k, given any constant $u > 1$, we have the following inequality:

$$\frac{(uk)^{-\beta}}{k^{-\beta}} < \frac{l(uk)}{l(k)} < \frac{(uk)^\beta}{k^\beta}. \qquad (7)$$
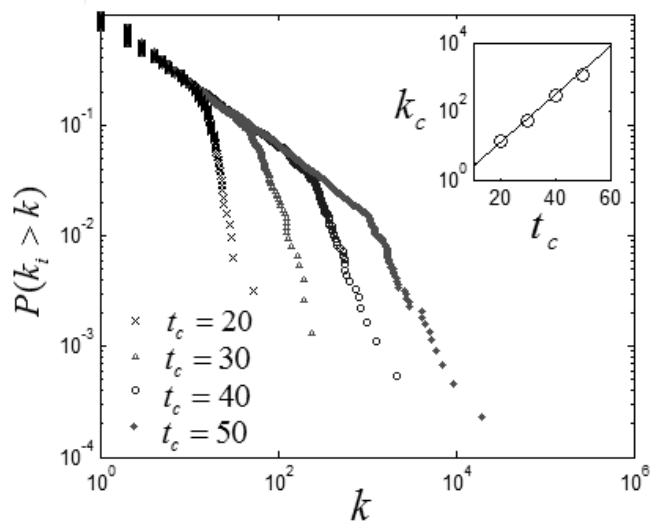
Let $\beta \to 0$, then we have

$$\lim_{k \to \infty} \frac{l(uk)}{l(k)} = 1. \qquad (8)$$

Note that Eq.(8) is also valid for any constant $0 < u \leq 1$. This property of $l(k)$ follows directly from the requirement that $p(k)$ is asymptotically scale invariant. Thus, the form of $l(k)$ only controls the finite extent of the lower tail and will not affect its scaling exponent significantly. So the second part of the $p(k)$ is also power law. For further study, numerical simulation is applied to determine the exponent of the second part of $p(k)$, denoted as γ_2. It is found that

$$\gamma_2 \sim \frac{1}{\sigma_\eta}. \qquad (9)$$

The result is well demonstrated by the simulation as shown in Fig. 3. It is noteworthy that $\sigma_\eta \to 0$ leads to $\gamma_2 \to \infty$. The second part of $p(k)$ degenerates naturally to be a cutoff. The exact formulation of γ_2 may also be related to c_n and μ_η, but extensive simulations indicate that γ_2 is much less sensitive to c_n(or μ_η) than to parameter σ_η.

So far we have provided a possible model to produce double power-law distribution. However real complex systems such as the Internet or WWW usually include fluctuations that may be essential to describe the dynamics of its evolution [10,21]. Therefore, a general model must be able to describe this feature. The generalization can be carried out by modifying Eq.(2):

$$dk_i = \rho\eta_i k_i dt + (1-\rho)(\mu_\eta dt + \sigma dw)k_i, \qquad (10)$$

where dw is white noise of standard normal distribution and σ is the standard variance of fluctuations. $\rho \in [0,1]$ is
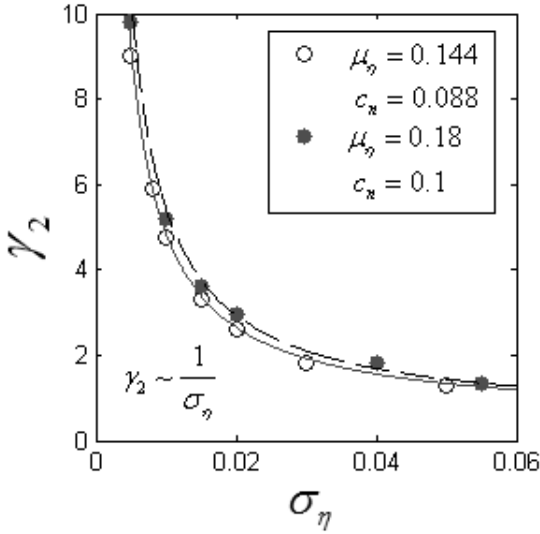
Fig. 3: Numerical studies of γ_2. The evolution time t_c is selected properly to keep the number of variables at about 1000 while the initial number of variables equals to 50. The fitted lines (solid line for $\mu_\eta = 0.15$, $c_n = 0.088$ and dashed line for $\mu_\eta = 0.18$, $c_n = 0.1$) is given by $\gamma_2 = 0.46 + \frac{0.044}{\sigma_\eta}$ and $\gamma_2 = 0.44 + \frac{0.05}{\sigma_\eta}$, respectively. The study is averaged over 50 realizations. For other μ_η and c_n, similar results can be obtained.

Fig. 4: Numerical studies of the cumulative distribution for different ρ. The simulation is carried out with $\mu_\eta = 0.17$, $N(t) = 50e^{0.088t}$ and $t_c = 30$. '$\times$' represents $\sigma_\eta = \sigma = 0$ and the others represent $\sigma_\eta = \sigma = 0.02$. When $\sigma_\eta = \sigma = 0$ the distribution is a power law independent of ρ. When $\sigma_\eta = \sigma > 0$, the first-part scaling exponent does not change while the second-part exponent decreases with ρ. The study is averaged over 50 realizations.

a parameter representing the relative contribution of the noise and the fitness, which can be considered as a measure of the degree of the disorder in real-world systems. Note that if $\rho = 1$, Eq.(10) becomes Eq.(2) while if $\rho = 0$, it leads to the geometric Brownian motion. Fluctuation described by the term $\sigma k_i dw$ is based on a general fact that in real systems such as WWW, site with large number of connections are likely to lose or gain more links than the site with small one. The solution of Eq.(10) is solved to be $e^{\rho \eta_i t} e^{(1-\rho)[(\mu_\eta - 0.5\sigma^2)t + \sigma w]}$ which follows lognormal distribution with logarithmic mean $\rho \mu_\eta t + (1 - \rho)(\mu_\eta - 0.5\sigma^2)t$ and logarithmic variance $(\rho \sigma_\eta t)^2 + (1-\rho)^2 \sigma^2 t$. By the similar methods used above, one can verify the distribution is still a double power law but the second scaling exponent is controlled by parameter ρ. In Fig. 4 we show the cumulative distribution for different ρ. It is found that the first part power-law behavior is still independent of ρ, leading to $\gamma_1 = \frac{c_n}{\mu_\eta}$, but the second power-law exponent decreases with ρ. Therefore noise fluctuations do not change the distribution qualitatively but contribute to the second scaling exponent.

The present model (Eq.(10)) indicates that evolution of a complex system may be characterized by two parts: a leading ingredient influencing the evolution and noise fluctuations. This will be interpreted as follow. Since there are usually various ingredients related to the evolution of complex system, practically we cannot take all of them into account. A feasible method is to consider the most important ingredient as the fitness while all other minor ones as contributions to fluctuations. Then parameter ρ represents how much the leading ingredient contributes to
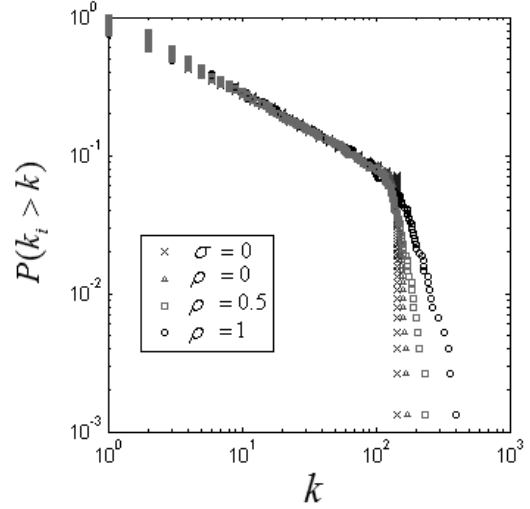
the evolution. If the evolution is totally governed by the leading ingredient, then $\rho \to 1$, indicating a deterministic pattern. On the other hand, if there is no apparent leading ingredient, it leads to $\rho \to 0$, indicating a random picture. Therefore our model is general to describe various real-world systems which evolve between order and disorder, and provide a better understanding on their evolution. As we will see in the following section, CAN is a typical example that follows such an evolution mechanism.

An example: Evolution of CAN. – In this section, we will analyze the evolution of CAN to provide evidence for our proposed model.

Chinese airline system can be modeled as a complex network with cities representing nodes and flights representing edges. The degree of a node is defined as the sum of the airlines connecting to it. The degree distribution of CAN has been investigated by several studies which all indicate a double power-law behavior [16,17,22]. Here we will report some useful information. First we have analyzed the total number of nodes $N(t)$ existing at time t. It grows as $N(t) \propto e^{c_n t}$ with $c_n \approx 0.088$, which is consistent with our assumption that nodes increases exponentially. The number of edges $M(t)$ also increases exponentially with time as $M(t) \propto e^{c_m t}$ with $c_m \approx 0.154$. We have also measured the parameters of the degree distribution of CAN from 1999 to 2003 and a single year 2008, summarized in Table. 1. The exponent γ_1 stabilizes at about 0.51 while γ_2 fluctuates from 2.1 to 2.7. Note that the stabilization of γ_1 is indicated by our model where γ_1 is independent of fluctuations. The turning point k_c shows an increase from 18 to 30, which is also consistent with

Table 1: Two scaling exponents (γ_1 and γ_2) and the turning point (k_c) of cumulative degree distribution in CAN

	1999	2000	2001	2002	2003	2008
γ_1	0.46	0.51	0.52	0.51	0.51	0.51
γ_2	2.2	2.1	2.5	2.3	2.7	2.7
k_c	18	18	20	21	22	30

our result that the turning point increases with evolution time.

In the present paper the evolution of CAN is studied by investigating the correlation between GDP and degree. The economic growth such as the size of tertiary industry has been recently demonstrated to be a leading ingredient in shaping the topology of CAN [26]. According to the discussion in the last section, we consider GDP relates to the fitness of the corresponding nodes. Note that the evolution of degree can also be studied by directly measuring the logarithmic ratio of the degree of successive two years. But this method can neither help to distinguish the identity of the fitness nor provide useful information of the corresponding parameters which is important in our analysis. As shown in Fig. 5, we found that the degree forms a linear function with its corresponding GDP ($R^2 > 0.62$) while the fluctuations are obvious. Despite the continuing evolution of both GDP and degree, this correlation has maintained for at least six years ($1998 - 2003$) since it has been first observed in 1998. Therefore it provides some key information about the evolution of degree in CAN and cannot be viewed as just a coincidence.

Considering the time evolution, the correlation can be described as:

$$k_i(t) = D(t)G_i(t), \qquad (11)$$

where $G_i(t)$ is the GDP of city i at year t. It grows exponentially as $G_i(t) \propto e^{\lambda_i t}$ where λ_i follows normal distribution with the mean of 0.18 and the standard variance of 0.02. The strong positive correlation confirms that economy may govern the evolution of the degree in CAN while the fluctuations, as we mentioned previously, are considered to result from some minor ingredients (such as population density, public administration, geographical constraints, etc). Both the two aspects contribute to $D(t)$, leading to an expression given by $D(t) = e^{a(t)}e^{\varepsilon(t)}$, where term $e^{a(t)}$ is the time-dependent slope and the term $e^{\varepsilon(t)}$ represents the fluctuations.

The specific form of $a(t)$ is easy to be evaluated. Summing Eq.(11) for all nodes we get $\langle D(t) \rangle = e^{a(t)} = \frac{2M(t)}{\sum_i G_i(t)} \sim e^{-0.036t}$, where $\sum_i G_i(t)$ is measured to be proportional to $e^{0.19t}$. Thus $a(t) = -0.036t$. Then $\varepsilon(t)$ can be investigated from data by calculating $\varepsilon(t) = \ln(\frac{k_i(t)}{G_i(t)}) - a(t)$. However what we concern here is the increment of $\varepsilon(t)$, defined as $d\varepsilon(t) = \varepsilon(t) - \varepsilon(t-1)$. In Fig. 6 we plot the distribution of $d\varepsilon(t)$ for all the five years. It follows a normal distribution with the mean of 0 and the standard variance of 0.09. Furthermore we
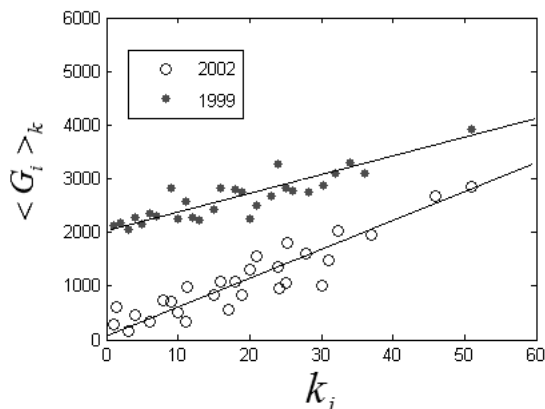

Fig. 5: The correlation of degree and its corresponding GDP for year 1999 and year 2002. Both of them exhibit linear correlation. $\langle G_i(t) \rangle_k$ is an average over all the nodes with the same degree k. Note that the data of the two years are not drawn for better visualization. ($G_i(1999)$ here increases 2000 from the origin data.) The correlation does not change from 1998 to 2003.

calculate the self-correlation function of $d\varepsilon(t)$, defined as $\langle d\varepsilon(t) d\varepsilon(t+\tau) \rangle = \frac{1}{N(t)} \sum_i d\varepsilon_i(t) d\varepsilon_i(t+\tau)$ (τ is the time interval). As listed in Table. 2, it exhibits very weak correlation (correlation coefficient < 0.1) when $\tau \neq 0$. Therefore $d\varepsilon(t)$ can be regarded as white noise and expressed as $d\varepsilon(t) = 0.09dw$. Then $D(t)$ is written as $D(t) \sim e^{0.09w - 0.036t}$. Substituting it into Eq.(11) and applying differentiation we have [27]

$$dk_i = (\lambda_i - 0.036)k_i dt + 0.09 k_i dw. \qquad (12)$$

Eq.(12) is exact the form of our model which can give rise to double power-law distribution. To further demonstrate its agreement with the real evolution of CAN, we simulate the degree distribution according to Eq.(12). We obtained $\gamma_1 = 0.61$ (it can also be calculated from $\gamma_1 = \frac{c_n}{\mu_\eta} = \frac{c_n}{\langle \lambda_i \rangle - 0.036} = 0.61$), comparable to the first exponent 0.51 while $\gamma_2 = 2.84$, good agreement with the second exponent 2.7.

Conclusion. – We have proposed a general model to explain the emergence of the double power-law distribution. The model incorporates fitness consideration and noise fluctuation which indicates that evolution of a complex system may be characterized by two parts: a leading ingredient and noise fluctuations. We find that normal distributed fitness coupled with exponentially increasing variables is responsible for the emergence of the double power-law distribution. Fluctuations do not change the result qualitatively but contribute to the value of scaling exponent. We have also studied empirically the CAN which turns out to follow the same evolution pattern as our proposed model.

We have only discussed the behavior of our model when $\sigma_\eta < \frac{\mu_\eta}{2}$. If σ_η is not much larger than $\frac{\mu_\eta}{2}$, the distribution still decays like a double power-law but both the
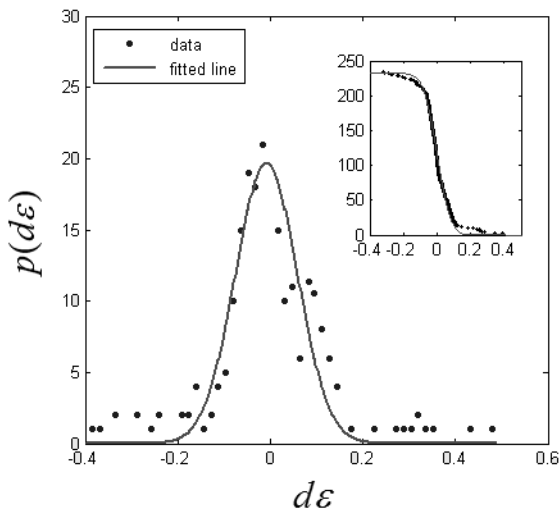
Fig. 6: The distribution of the increment of $\varepsilon(t)$ for all the five years. The data is binned into classes. The fitted line is $p(d\varepsilon) = 19.4e^{\left(-\frac{(d\varepsilon)^2}{2*0.09^2}\right)}$. Therefore it follows normal distribution with mean 0 and standard variance 0.09. Inset: The cumulative distribution of $d\varepsilon(t)$. It is well fitted by $P(d\varepsilon(t) > d\varepsilon) = 116 erfc(\frac{d\varepsilon}{0.127})$ where $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-x^2} dx$.

exponents are different from previous ones. With the continuing increasing of σ_η, the double power-law behavior turns out to be unconspicuous. It results from that the second scaling exponent is gradually close to the first one and finally becomes indistinguishable.

Finally, we would like to mention that we have done tests for six usual distributions, namely exponential distribution, uniform distribution, power-law distribution, Poisson distribution, Rayleigh distribution and Weibull distribution for the fitness instead of normal distribution, the results show that none of them is able to achieve double power-law distribution. Therefore we believe that normal distribution of fitness is a key ingredient responsible for the double power-law distribution.

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Table 2: The self-correlation function $\langle d\varepsilon(t)d\varepsilon(t+\tau)\rangle$

	1999	2000	2001	2002	2003
1999	1	0.041	-0.019	-0.02	-0.01
2000	0.041	1	-0.06	-0.023	0.06
2001	-0.019	-0.06	1	-0.1	0.01
2002	-0.02	-0.023	-0.1	1	-0.035
2003	-0.01	0.06	0.01	-0.035	1

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[27] Actually dw as well as k_i defined here is time-discrete, therefore Eq.(12) is better to be expressed as a difference equation. However we omit rigor, for simplicity.