Fusion modeling in plasma physics: Vlasov-like systems

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Abstract

The methods developed by authors are applied to some reductions of BBGKY hierarchy, namely, various examples of Vlasov-like systems which are important both for fusion modeling and for particular physical problem s related to plasma/beam physics. We mostly concentrate on phenomena of localization and pattern formation.

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Fusion modeling in plasma physics: Vlasov-like systems

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The methods developed by authors are applied to some reductions of BBGKY hierarchy, namely, various examples of Vlasov-like systems which are important both for fusion modeling and for particular physical problems related to plasma/beam physics. We mostly concentrate on phenomena of localization and pattern formation.

We sketch the applications of our approach based on variational multiresolution technique [1], [2] to the systems with collective type behaviour described by some forms of Vlasov-Poisson/Maxwell equations, some important reduction of general BBGKY hierarchy [3]. Such an approach may be useful in all models in which it is possible and reasonable to reduce all complicated problems related to statistical distributions to the problems described by the systems of nonlinear ordinary/partial differential/integral equations with or without some (functional) constraints. In periodic accelerators and transport systems at the high beam currents and charge densities the effects of the intense self-fields, which are produced by the beam space charge and currents, determinine (possible) equilibrium states, stability and transport properties according to underlying nonlinear dynamics. The dynamics of such space-charge dominated high brightness beam systems can provide the understanding of the instability phenomena such as emittance growth, mismatch, halo formation related to the complicated behaviour of underlying hidden nonlinear modes outside of perturbative KAM regions [3]. Our analysis based on the variational-wavelet approach allows to consider polynomial and rational type of nonlinearities [1], [2]. In some sense in this particular case this approach is direct generalization of traditional nonlinear δF approach [3] in which weighted Klimontovich and related representations are replaced by powerful technique from local nonlinear harmonic analysis, based on underlying symmetries of functional space such as affine or more general. The solution has the multiscale decomposition via nonlinear high-localized eigenmodes, which corresponds to the full multiresolution expansion in all underlying time/phase space scales. Starting from Vlasov-Poisson equations, we consider the approach based on multiscale variational-wavelet formulation. We give the explicit representation for all dynamical variables in the base of compactly supported wavelets/wavelet packets or nonlinear eigenmodes. Our solutions are parametrized by solutions of a number of reduced algebraical problems, one from which is nonlinear with the same degree of nonlinearity as initial problem and the others are the linear problems which correspond to the particular method of calculations inside concrete wavelet scheme. Because our approach started from variational formulation we can control evolution of instability on the pure algebraical level of reduced algebraical system of equations. We consider the following form of equations

$$\Big\{\frac{\partial}{\partial s}+p_x\frac{\partial}{\partial x}+p_y\frac{\partial}{\partial y}-\Big[k_x(s)x+\frac{\partial\psi}{\partial x}\Big]\frac{\partial}{\partial p_x}-\Big[k_y(s)y+\frac{\partial\psi}{\partial y}\Big]\frac{\partial}{\partial p_y}\Big\}f_b(x,y,p_x,p_y,s)=0, \tag{1}$$
$$\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)\psi=-\frac{2\pi K_b}{N_b}\int \mathrm{d}p_x\mathrm{d}p_y f_b, \qquad \int \mathrm{d}x\mathrm{d}y\mathrm{d}p_x\mathrm{d}p_y f_b=N_b.$$

The corresponding Hamiltonian for transverse single-particle motion is given by $H(x,y,p_x,p_y,s)=\frac{1}{2}(p_x^2+p_y^2)+\frac{1}{2}[k_x(s)x^2+k_y(s)y^2]+H_1(x,y,p_x,p_y,s)+\psi(x,y,s)$, where H_1 is nonlinear (polynomial/rational) part of the full Hamiltonian and corresponding characteristic equations are: $\frac{\mathrm{d}^2x}{\mathrm{d}s^2}+k_x(s)x+\frac{\partial}{\partial x}\psi(x,y,s)=0, \quad \frac{\mathrm{d}^2y}{\mathrm{d}s^2}+k_y(s)y+\frac{\partial}{\partial y}\psi(x,y,s)=0$. We obtain our multiscale/multiresolution representations for solutions of these equations via variational-wavelet approach. We decompose the solutions as $f_b(s,x,y,p_x,p_y)=\sum_{i=i_c}^{\infty}\oplus\delta^i f(s,x,y,p_x,p_y), \quad \psi(s,x,y)=\sum_{j=j_c}^{\infty}\oplus\delta^j\psi(s,x,y), \quad x(s)=\sum_{k=k_c}^{\infty}\oplus\delta^k x(s), \quad y(s)=\sum_{\ell=\ell_c}^{\infty}\oplus\delta^\ell y(s)$, where set (i_c,j_c,k_c,ℓ_c) corresponds to the coarsest level of resolution c in the full multiresolution decomposition $V_c\subset V_{c+1}\subset V_{c+2}\subset\dots$ Introducing detail space W_j as the orthonormal complement of V_j with respect to $V_{j+1}: V_{j+1}=V_j\bigoplus W_j$, we have for $f,\psi,x,y\subset L^2(\mathbf{R}):\quad L^2(\mathbf{R})=\overline{V_c\bigoplus_{j=c}^{\infty}W_j}$. In some sense it is some generalization of the old δF approach [3]. Let L be an arbitrary (non) linear differential/integral operator with matrix dimension d, which acts, according to (1), on some set of functions $\Psi\equiv\Psi(s,x)=\Big(\Psi^1(s,x),\dots,\Psi^d(s,x)\Big),\ s,x\in\Omega\subset\mathbf{R}^{n+1}$ from $L^2(\Omega)$: $\quad L\Psi\equiv L(R(s,x),s,x)\Psi(s,x)=0$, where x are the generalized space coordinates or phase space coordinates, and s is "time" coordinate. As a result the solution of equations (1) has the following full multiscale/multiresolution decomposition via nonlinear high-localized eigenmodes:

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$$\Psi(s,\mathbf{x}) = \sum_{(i,j)\in Z^2} a_{ij}\mathbf{U}^i \otimes V^j(s,\mathbf{x}), \quad \mathbf{x}=(x,y,p_x,p_y) \tag{2}$$
$$V^j(s) = V_N^{j,slow}(s) + \sum_{l\geq N} V_l^j(\omega_l s),\ \omega_l \sim 2^l, \quad \mathbf{U}^i(\mathbf{x}) = \mathbf{U}_M^{i,slow}(\mathbf{x}) + \sum_{m\geq M} \mathbf{U}_m^i(k_m\mathbf{x}),\ k_m \sim 2^m,$$

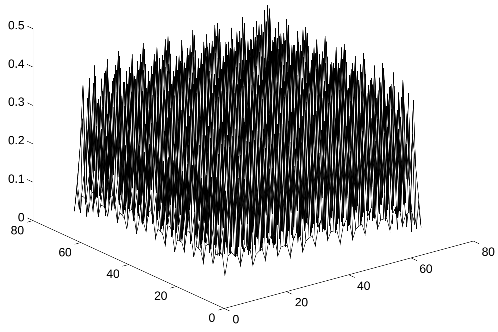

Fig. 1 Chaotic pattern.

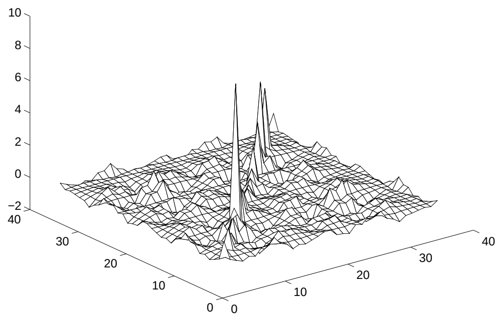

Fig. 2 Localized pattern(waveleton): energy confinement state.

It should be noted that such representations give the best possible localization properties in the corresponding (phase)space/-time coordinates. In contrast with other approaches these formulas do not use perturbation technique or linearization procedures. Modeling demonstrates the appearance of stable patterns formation from high- localized coherent structures or chaotic behaviour (Fig. 1). Such analysis and modeling describes, in principle, a scenario for the generation of controllable localized (meta) stable fusion-like state or waveleton (Fig. 2). Definitely, chaotic-like unstable partitions/states (Fig. 1) dominate during non-equilibrium evolution. It means that (possible) localized (meta) stable partitions have measure equal to zero a.e. on the full space of hierarchy of partitions defined on a domain of the definition in the whole phase space. Nevertheless, our scheme give some chance to build the controllable localized state (Fig. 2) starting from initial chaotic-like partition via process of controllable self-organization. Of course, such confinement states, waveleton, characterized by zero measure and minimum entropy, can be only metastable. But these long-living fluctuations can and must be very important from the practical point of view, because the averaged time of existence of such states may be even more than needed for practical realization, e.g., in controllable fusion processes.

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