Induced gauge interactions revisited

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Abstract

It has been shown that the old-fashioned idea of Sakharov's induced gravity and gauge interactions in the "one-loop dominance" version works astonishingly well yielding reasonable parameters. It appears that induced coupling constants of gauge interactions of the standard model assume qualitatively realistic values. Moreover, it is possible to induce the Barbero–Immirzi parameter of canonical gravity from the fields entering the standard model.

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1 Introduction

The idea that fundamental interactions might be not so fundamental as they appear, but induced by quantum fluctuations of the vacuum emerges from the fifties of the 20th century when in 1967 Sakharov and Zel'dovich published their papers about induced gravity [1] and induced electrodynamics [2]. After a while it was followed by many people (see, e.g. [3]) and firstly applied to the context of the standard model by Terazawa [4], who also derived logarithmic relation between the gauge coupling constants and the Newton gravitational constant [5] (see also [6]). Some further, explorations have been presented in [7],[8].

It seems that successful application of the idea of quantum vacuum induced interactions to the two fundamental interactions subsequently renewed interest in this subject. At present, the very idea lacks a clear theoretical interpretation. It is supposed to be an interesting curiosity as well as an unexplained deeper phenomenon. Despite of an interpretation, striking coincidences have forced us to claim that the idea of quantum induced interactions does work and can be used practically.

The aim of our paper is to show that the idea of induced gauge interactions in its primary, old-fashioned, "one-loop dominance" Sakharov's version [9] yields phenomenologically very realistic results. This standpoint assumes that in the classical action at the beginning there are only (fundamental) matter fields coupled (minimally) to external gauge fields. Classical terms for gauge fields and gravity do not exist autonomously, but they appear by virtue of the low-order one-loop calculations. (The superior role of the matter fields awaits an explanation in this framework.) Interestingly, classical terms produced this way have not only appropriate functional term but also realistic numeric coefficients. The former fact is akin to renormalizability.

In our paper, gauge interactions and gravity in the modern Ashtekar's canonical formulation are treated and analyzed in the framework of the very convenient method: Schwinger's proper time and the Seeley–DeWitt heat-kernel expansion. Our approach to the gauge interactions uses flat Lorentzian manifold whereas for the Ashtekar's gravity our calculations are carried out on the Riemannian one with torsion (extension including torsion has been given in [10]), so that the one-loop effective actions for each case are different and need individual treatment.

2 Induced gauge interactions

2.1 Heat-kernel method

According to the idea of quantum vacuum induced interactions, its dynamics emerges from dominant contributions to the one-loop effective action of non-self-interacting matter fields coupled to these interactions. In the framework of the Schwinger proper-time method, the expected terms for gauge interactions can be extracted from the 2nd coefficient of the Seeley–DeWitt heat-kernel expansion. ("Cosmological constant" and gravity are obtainable from the 0th and 1st coefficient respectively, see our work [11].) In Minkowskian signature [12],[13]

$$S_{\text{eff}} = i\kappa \log\det\mathcal{D} = i\kappa \mathrm{Tr}\log\mathcal{D} = -i\kappa\int\frac{\mathrm{d}s}{s}\,\mathrm{Tr}\,e^{-is\mathcal{D}}, \tag{2.1}$$

where $\mathcal{D}$ is an appropriate second-order differential operator, and κ depends on the kind of the "matter" field (its statistics, in principle). E.g. for a bosonic single mode, $\kappa = \frac{1}{2}$. Seeley–DeWitt heat-kernel expansion (for detailed introduction, see Appendix) in four dimensions, reads

$$\mathrm{Tr}\,e^{-is\mathcal{D}} = \frac{1}{16\pi^2\,(is)^2}\left[A_0 + A_1\,(is) + A_2\,(is)^2 + \cdots\right], \tag{2.2}$$

where A_n is the nth Seeley–DeWitt coefficient. Imposing appropriate cutoffs, i.e. an UV cutoff ε for A_0, A_1 and A_2, and an IR cutoff Λ for A_2, we obtain

$$S_{\text{eff}} = \frac{\kappa}{16\pi^2}\left(\frac{1}{2}A_0\varepsilon^{-2} + A_1\varepsilon^{-1} + A_2\log\frac{\Lambda}{\varepsilon} + \cdots\right). \tag{2.3}$$

As we stated earlier, we are especially interested in the gauge part connected with the A_2. Collecting contributions from various modes, we get the following gauge Lagrangian density:

$$\mathcal{L}_2 = \frac{1}{384\pi^2}\log\frac{\Lambda}{\varepsilon}\left(N_0 + 4N_{\frac{1}{2}}\right)\mathrm{tr}F^2 \tag{2.4}$$

(see, Table 2 in Appendix for the origin of the numeric coefficient), where:

$$\begin{aligned} &N_0 = \text{number of minimal scalar degrees of freedom (dof)},\\ &N_{\frac{1}{2}} = \text{number of two-component fermion fields} = \text{half the number of fermion dof}, \end{aligned} \tag{2.5}$$

and F is the strength of a gauge field (see, the definition (B.1).) Higher-order terms are in principle present (even in classical case), but they are harmless in typical situations because of small values of the coefficients following from the cutoffs.

gauge group	U(1)	SU(2)	SU(3)
coupling constant	g'	g	f
"coupling matrix"	$\frac{i}{2}Yg'$	$\frac{i}{2}\vec{\tau}g$	$\frac{i}{2}\vec{\lambda}f$
conventions	Y—hypercharge	$\mathrm{tr}(\tau^a\tau^b)=2\delta^{ab},\ a,b=1,2$	$\mathrm{tr}(\lambda^i\lambda^j)=2\delta^{ij},\ i,j=1,2,3$
FERMIONS (one family) $a_2=-\frac{1}{3}\mathrm{tr}F^2$			
numeric formula	$-\frac{1}{3}\left(-\frac{1}{4}\right)Y^2{g'}^2=\frac{1}{12}Y^2{g'}^2$	$-\frac{1}{3}\cdot 2\left(-\frac{1}{4}\right)g^2=\frac{1}{6}g^2$	$-\frac{1}{3}\cdot 2\left(-\frac{1}{4}\right)f^2=\frac{1}{6}f^2$
LEPTONS			
left, $Y=-1$	$\frac{1}{12}\cdot 2{g'}^2=\frac{1}{6}{g'}^2$	$\frac{1}{6}g^2$	0
right, $Y=-2$	$\frac{1}{12}\cdot(-2)^2{g'}^2=\frac{1}{3}{g'}^2$	0	0
QUARKS			
left, $Y=\frac{1}{3}$	$\frac{1}{12}\cdot 2\cdot 3\left(\frac{1}{3}\right)^2{g'}^2=\frac{1}{18}{g'}^2$	$\frac{1}{6}\cdot 3g^2=\frac{1}{2}g^2$	$\frac{1}{6}\cdot 2f^2=\frac{1}{3}f^2$
right, $Y=\frac{4}{3}$	$\frac{1}{12}\cdot 3\left(\frac{4}{3}\right)^2{g'}^2=\frac{4}{9}{g'}^2$	0	$\frac{1}{6}f^2$
right, $Y=-\frac{2}{3}$	$\frac{1}{12}\cdot 3\left(-\frac{2}{3}\right)^2{g'}^2=\frac{1}{9}{g'}^2$	0	$\frac{1}{6}f^2$
BOSONS $a_2=-\frac{1}{12}\mathrm{tr}F^2$			
Higgs, $Y=1$	$-\frac{1}{12}\cdot 2\left(-\frac{1}{4}\right){g'}^2=\frac{1}{24}{g'}^2$	$-\frac{1}{12}\cdot 2\left(-\frac{1}{4}\right)g^2=\frac{1}{24}g^2$	0

***Table 1**: Various "matter field" species contributions to induced gauge coupling constants in the framework of the standard model.*

2.2 Standard model contributions

Now, we will concentrate on the possibility of quantum generation of gauge interactions in the context of the standard model. We are interested in the contributions coming from appropriate matter fields taken into account in (2.4). To this end we should adapt (2.4) to the context of the standard model. Adopting the matter contents of the Lagrangian of the standard model we display all contributions to the respective gauge parts in Table 1. The assumed implicit convention for the operator of covariant derivative is

$$D_\mu = \partial_\mu + \vec{X}\cdot\vec{A}_\mu, \tag{2.6}$$

where $\vec{X}$ is the "coupling matrix" given in the third row of Table 1. Strictly speaking, $\vec{X}$ is a tensor product with two matrix units corresponding to the other two gauge groups, yielding additional coefficients, 2 or 3. In principle, the coefficients given in each column and multiplied by

$$\frac{1}{16\pi^2}\log\frac{M}{m}, \tag{2.7}$$

where M and m is an UV and an IR cutoff, respectively, in mass units, should sum up to $\frac{1}{4}$, a standard normalization term in front of F^2, so that, we have the following theoretical constraint:

$$\frac{{g_i}^2}{16\pi^2}\sum_n \alpha_{(i)n}\log\frac{M_n}{m_n} = \frac{1}{4}, \tag{2.8}$$

where g_i $(i = 1, 2, 3)$ is one of the three coupling constants, $\alpha_{(i)n}$ are corresponding numeric coefficients from Table 1, and the sum concerns all matter fields. We can confidently set $M_n = M_{\rm P}$ (Planck mass) and m_n are masses of lightest particles.

Now, we can utilize the data given in Table 1 in order to reproduce a number of phenomenologically realistic results. Assuming for simplicity (or as an approximation) fixed values of M_n and m_n for all species of matter particles, we can uniquely rederive following [14] the Weinberg angle $\theta_{\rm w}$,

$$\sin^2\theta_{\rm w} = \frac{g'^2}{g^2 + g'^2} \approx 0.38\ . \tag{2.9}$$

Estimation of the coupling constants requires definite values of infrared cutoffs m_n. For m_n of the order of the mass of lighter particles of the standard model we obtain

$$\alpha = \frac{e^2}{4\pi} = \frac{g^2\sin^2\theta_{\rm w}}{4\pi} = \mathcal{O}(0.01), \tag{2.10}$$

and

$$g = f = \mathcal{O}(1), \tag{2.11}$$

which is phenomenologically a very realistic estimate.

Alternatively, the constraint (2.8) can give some limitations on the ratio of the two scales M and m, provided the scale of interactions $g = f = \mathcal{O}(1)$ is assumed.

3 Ashtekar gravity: the Barbero–Immirzi parameter

Introduction

The Barbero–Immirzi (BI) parameter γ is an a priori free parameter in the framework of the modern approach to canonical gravity (Ashtekar's formalism) [15]. In the Holst extended action for gravity [16] the BI parameter γ resides in the additional term of the full (Holst) action. One can easily further extend the Holst contribution [17] yielding, in particular, the Nieh–Yan (NY) term [18]. (The role that NY invariant plays in gravity has been studied in [19], while an extension to a possible new scenario where BI parameter is promoted to a field, has been studied in [20]). Topological nature of the NY term, means that it influences quantum theory. Interestingly, it appears, and we will show it, that, the NY term can be quantumly induced by dominant part of one-loop contributions coming from chiral matter fields.

3.1 Formalism

We will work in the framework of euclidean formalism applying the Sakharov idea of induced gravity (one-loop dominance) to the standard model of particle physics. We are especially interested in showing that chiral fields entering the standard model (left-handed leptons, i.e. neutrinos, in our case) will yield an additional term [21], the NY term. Consequently, the induced BI parameter depends only on the number and kind of particle species entering the standard model. From purely technical point of view the calculus is partially akin to the derivation of the Adler–Bell–Jackiw (ABJ) chiral anomaly in space-time with torsion [22] (see [23] for an extended discussion on the anomaly issue).

According to our realization of the Sakharov idea, we are interested in a dominant part of one-loop contributions coming from left-handed leptons. We will work in the (euclidean) Schwinger proper-time formalism and in the framework of the Seeley–DeWitt heat-kernel expansion on manifolds with torsion [24]. Our starting object is the Dirac differential operator

$$D \equiv i \not{\nabla} \equiv i\gamma^a e_a^\mu \nabla_\mu, \tag{3.1}$$

where e_a^μ is a vierbein field, ∇_μ is a covariant derivative in space with torsion, and γ^a are euclidean Dirac matrices. Now

$$D^2 = -\Box + \frac{1}{2} e_a^\mu e_b^\nu \sigma^{ab} T_{\mu\nu}^\lambda \nabla_\lambda - \frac{1}{8} e_a^\mu e_b^\nu \sigma^{ab} \sigma^{cd} R_{cd\mu\nu}, \tag{3.2}$$

where

$$\Box \equiv \nabla_\mu \nabla^\mu, \;\; \sigma^{ab} \equiv \frac{1}{2}\left[\gamma^a, \gamma^b\right], \;\; \left[\nabla_\mu\, , \nabla_\nu\right] V^a = R^a_{\ b\mu\nu} V^b - T_{\mu\nu}^\lambda \nabla_\lambda V^a. \tag{3.3}$$

Introducing the two chiral projectors

$$P_{\rm L} \equiv \frac{1-\gamma^5}{2}, \;\; P_{\rm R} \equiv \frac{1+\gamma^5}{2}, \tag{3.4}$$

with $\gamma^5 \equiv \gamma^1\gamma^2\gamma^3\gamma^4$, we can write in the chiral representation

$$D^2 = D^2 P_{\rm L} \oplus D^2 P_{\rm R}, \tag{3.5}$$

and consequently

$$\det D = \sqrt{\det D^2} = \sqrt{\det_{\rm L} D^2 \det_{\rm R} D^2}, \tag{3.6}$$

because D^2 is diagonal-blocked in the subspaces L and R. From now on we will confine ourselves to $\sqrt{\det_{\rm L} D^2}$ corresponding to the left-handed lepton.

3.2 Effective action

The effective action for the chiral (left-handed) lepton is of the following form

$$S = -\frac{1}{2}\log \det{}_{\text{L}} D^2 = \frac{1}{2}\int \frac{\mathrm{d}s}{s}\mathrm{Tr}\left(e^{-sD^2}P_{\text{L}}\right). \tag{3.7}$$

Then, corresponding chiral part of the M^2-regularized effective Lagrangian density reads (see, [22])

$$\mathcal{L} = \frac{1}{2}\int\limits_{M^{-2}}^{\infty} \frac{\mathrm{d}s}{s}\frac{s}{(4\pi s)^2}\left(-\frac{1}{2}\right)\mathrm{tr}\left(a_1\gamma^5\right) \approx -\frac{1}{4}\left(\frac{M}{4\pi}\right)^2 \mathcal{NY}, \tag{3.8}$$

where a_1 is the 1st Seeley–DeWitt coefficient [24], and the NY term $\mathcal{NY}$ is defined by

$$\mathcal{NY} \equiv \mathrm{d}_\omega e^a \wedge \mathrm{d}_\omega e_a - e^a \wedge e^b \wedge R_{ab} \equiv T^a \wedge T_a - e^a \wedge e^b \wedge R_{ab}. \tag{3.9}$$

The extended Lagrangian density of general relativity assumes the form

$$\mathcal{L} = \alpha \star \left(e^a \wedge e^b\right) \wedge R_{ab} - \beta\left(T^a \wedge T_b - e^a \wedge e^b \wedge R_{ab}\right), \tag{3.10}$$

where the first term is the standard Einstein–Hilbert (EH) one, and the second term is the extended Holst or the NY one. The Barbero–Immirzi parameter γ is now given by

$$\gamma \equiv \frac{\alpha}{\beta}. \tag{3.11}$$

Using the result of [11] we have

$$\mathcal{L}_{\text{EH}} = -\frac{1}{12}\left(\frac{M}{4\pi}\right)^2 \left(N_0 + N_{\frac{1}{2}} - 4N_1\right) \star \left(e^a \wedge e^b\right) \wedge R_{ab}, \tag{3.12}$$

where N_0, $N_{\frac{1}{2}}$ are defined by (2.5), and N_1 is the number of gauge fields. Therefore, by virtue of (3.8), (3.9) and (3.10)–(3.12)

$$\gamma = \frac{-\frac{1}{12}\left(N_0 + N_{\frac{1}{2}} - 4N_1\right)}{-\frac{1}{4}N_{\text{L}}}, \tag{3.13}$$

where N_{L} is the number of chiral left-handed modes, and the UV cutoffs $(M/4\pi)^2$ canceled out in (3.13).

For example, exactly in the framework of the standard model, we insert the following numbers of fundamental modes: $N_0 = 4$ (Higgs), $N_{\frac{1}{2}} = 45$, $N_1 = 12$, $N_{\text{L}} = 3$ (neutrinos), yielding $\gamma = \frac{1}{9} \approx 0.11$, which is quite close to the (a bit obsolete) Ashtekar–Baez–Corichi–Krasnov value, $\gamma_{\text{ABCK}} = \frac{\ln 2}{\pi\sqrt{3}} \approx 0.13$ [25],[26] (see [27], for a better estimation). Nevertheless

we should remember that γ induced that way depends on the number and kinds of fundamental modes, and moreover the whole calculus is valid in the framework of euclidean formalism.
One should stress that the result $\gamma_{\text{ABCK}} \approx 0.13$ is obtained in the framework of an approach using the black-hole entropy. We have shown, and this is the main objective of our considerations, that our method of the Sakharov's induced NY term also fix the BI parameter γ, and moreover it does it in an independent way.

4 Final remarks

In this paper, we have presented a number of arguments supporting the idea of the old-fashioned "one-loop dominance" version of gauge interactions in the spirit of Sakharov. All coupling constants of fundamental gauge interactions have been shown to assume phenomenologically realistic values, provided the Planckian value of the UV cutoff is given. As an independent check of the idea, we have proposed Sakharov's approach to the modern canonical Ashtekar's gravity. The a priori free Barbero–Immirzi parameter, by virtue of the chiral fields contributions coming from the standard model, also assume an acceptable value in the euclidean framework.

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Appendix

A Heat-kernel expansion

The functional trace in Eq. (2.2), by definition reads:

$$\mathrm{Tr}\, e^{-is\mathcal{D}} \equiv \int\limits_{\mathcal{M}} \mathrm{d}x \, \mathrm{tr} \left\langle x \right| e^{-is\mathcal{D}} \left| x \right\rangle , \tag{A.1}$$

where "tr" is an ordinary algebraic trace. In order to calculate this integrand, let us consider the, so called, heat-kernel, defined by:

$$\mathcal{G}(x, y; s) \equiv \left\langle x \right| e^{-s\mathcal{D}} \left| y \right\rangle . \tag{A.2}$$

It is easy to check, that non-interacting version of the heat-kernel equation assumes the form:

$$\frac{\partial \mathcal{G}_0(x, y; s)}{\partial s} = \partial^2 \mathcal{G}_0(x, y; s), \tag{A.3}$$

where

$$\mathcal{G}_0(x, y; s) = \left\langle x \right| e^{-s\mathcal{D}_0} \left| y \right\rangle \tag{A.4}$$

is defined by a free operator $\mathcal{D}_0$. Consequently, solving out Eq. (A.3), in d dimensions, we have:

$$\mathcal{G}_0(x, y; s) = \frac{1}{(4\pi s)^{d/2}} e^{-|x-y|^2/4s}. \tag{A.5}$$

Making use of a perturbation techniques with respect to s, we can write the final solution for a general operator $\mathcal{D}$:

$$\mathcal{G}(x, y; s) = \frac{1}{(4\pi s)^{d/2}} e^{-|x-y|^2/4s} F(x, y; s), \tag{A.6}$$

where $F(x, y; s)$ is a matrix valued function represented by the perturbative expansion:

$$F(x, y; s) \equiv \sum_{j=0}^{\infty} A_j(x, y) s^j. \tag{A.7}$$

The $A_j(x, y)$ coefficients are Seeley–DeWitt (Hamidew) coefficients widely explored in numerous papers, e.g. [28],[29]. Recapitulating, going back to Eq. (A.2) with $x = y$ condition, for $d = 4$ in the relativistic version (Wick's rotation), we finally obtain Eq. (2.2).

B Seeley–DeWitt coefficients

The Seeley–DeWitt ("Hamidew") coefficients assume the values presented below (in Table 2). Our sign convention corresponds to the Landau–Lifshitz timelike one, i.e. the metric signature is $(+ - --)$. Our conventions concerning gauge fields are as follows:

$$\begin{aligned} D_\mu &= \partial_\mu + A_\mu, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]\,. \end{aligned} \tag{B.1}$$

particle	Seeley–DeWitt coefficient k_2
minimal scalar	$\frac{1}{12}$ [13]
Weyl spinor	$-\frac{1}{3}$ [13]
massless vector	$-\frac{11}{24}$ [29]

Table 2: *Seeley–DeWitt coefficients. In brackets, we have given the references where the coefficients can be found explicitly or almost explicitly (i.e. after few-minute calculations).*

We have assumed the following notation:

$$a_2(x) = k_2 \,\mathrm{tr} F^2 + k_2' \text{“curvature terms”}. \tag{B.2}$$

Interested reader can find k_2' in [9], [12] or [29].

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