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Identification of boundary condition on the contact surface of continuous casting mould

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Abstract

In the paper the methods of inverse problems solution are applied for the identification of substitute heat transfer coefficient in the primary cooling zone of continuous casting plant. The heat exchange between the cast strand and the continuous casting mould proceeds in the very complex conditions (air gap, layer of mould dressing, fouling etc.). For the needs of casting solidification modelling the real boundary condition on the contact surface should be simplified and one can introduce the mean boundary heat flux (the Neumann condition) or the substitute heat transfer coefficient (the Robin condition). These values can be found using the inverse approach. In order to solve the problem of identification, the gradient methods are here applied (the least square criterion with regularization parameter).The algorithm presented bases on the sensitivity analysis methods, in other words the values of temperature derivatives with respect to the unknown nodal heat transfer coefficients must be known. The basic and additional boundary problems connected with the sensitivity coefficients computations are solved using the generalized variant of finite differences method. In the final part of the paper the example of computations is shown.

Keywords: Application of information technology to the foundry industry, Solidification process, Continuous casting process, Inverse problems, Numerical techniques

1. Introduction

The very important problem connected with design of continuous casting technology consists in the computations of temperature field, heat fluxes and temperature gradients in domain of cast strand and continuous casting mould (CCM). The thermal characteristic of continuous casting mould has the basic influence on the course of cast strand solidification in the region of primary cooling zone. In particular, the very essential thing is the proper choice of the cooling system in domain of the plant considered [1, 2]. The boundary conditions determining the thermal processes proceeding in the domain of CCM are rather simple and the coefficients appearing in these conditions can be found using the typical presented in literature procedures. The exception to the rule takes place on a contact surface between cast strand and CCM. Here the mutual thermal interactions are very complex and a lot of thermal phenomena should be considered. In a practice, the simplified approach to the formulation of boundary condition discussed is, as a rule, used. The real boundary condition is substituted by the Neumann or Robin ones and the parameters appearing in these conditions are determined using the experimental methods.

In this paper the following inverse problem is analyzed. On the basis of the knowledge of temperature values at the internal points of continuous casting mould the substitute heat transfer coefficient between crystallizer and slab is identified.

The heat transfer processes in continuous casting mould are described by the partial differential equation (energy equation) concerning either the pseudo-steady state or the transient one. In the undisturbed technological conditions in domain of CCM the pseudo-steady state is generated and a such case is considered. The energy equation is supplemented by the boundary conditions given on the outer surface of the system, on the contact surface between cast slab and mould and also on the cooling pipes surfaces but we assume that the boundary condition between CCM and cast slab is unknown (in particular, the substitute heat transfer coefficient in the Robin condition).

The problem above formulated has been solved using the least squares criterion with regularization parameter. This criterion contains the sensitivity coefficients, this means the values of temperature derivatives with respect to the unknown nodal heat transfer coefficients. In order to solve the task discussed, the additional boundary problems connected with the sensitivity coefficients determination must be solved. On the stage of numerical computations the generalize finite differences method has been used. In the final part of the paper the example of the heat transfer coefficient identification is presented.

2. Governing equations

The following inverse problem is considered - Figure 1:

$$\begin{cases} x \in \Omega : & \nabla^2 T = 0 \\ x \in \Gamma_1 : & q(x) = -\lambda\, \partial T/\partial n = \alpha\left[T(x) - T^\infty\right],\ \alpha = ? \\ x \in \Gamma_2 : & T(x) = T_b \\ x \in \Gamma_3 : & q(x) = -\lambda\, \partial T/\partial n = q_b \\ x \in \Gamma_4 : & q(x) = -\lambda\, \partial T/\partial n = \alpha_0\left[T(x) - T_0^\infty\right] \\ x_e \in \Omega : & T_{de} = T_d(x_e) - \text{given},\ e = 1, 2, ..., M \end{cases} \tag{1}$$

where: λ is a thermal conductivity, $\partial T/\partial n$ is a normal derivative at boundary point x, T_b , q_b are the given boundary temperature and heat flux, α_0 is the known heat transfer coefficient, T^∞ , T_0^∞ are the ambient temperatures corresponding to Γ_1 and Γ_4 , α is the unknown substitute heat transfer coefficient. At the internal points x_e the values of temperature T_d^i are given.

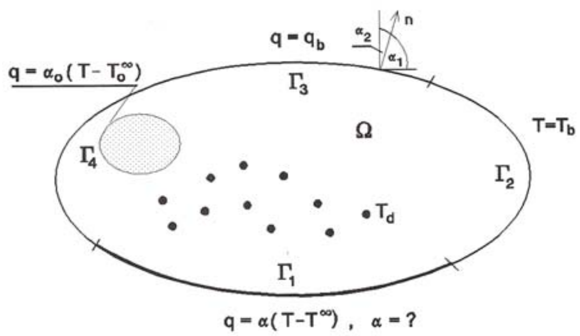

Fig. 1. Domain considered

3. Solution of inverse problem

In order to create the sensitivity model we differentiate the equations (1) with respect to the unknown boundary value α_k at the point $x^k \in \Gamma_1$ [3, 4, 5]

$$\begin{cases} x \in \Omega : & \nabla^2 U_k(x) = 0 \\ x \in \Gamma_1 : & P_k(x) = \begin{cases} T_k - T^\infty + \alpha_k U_k(x), & x = x^k \\ 0 & , \ x \neq x^k \end{cases} \\ x \in \Gamma_2 : & U_k(x) = 0 \\ x \in \Gamma_3 : & P_k(x) = 0 \\ x \in \Gamma_4 : & P_k(x) = \alpha_0 U_k(x) \end{cases} \tag{2}$$

where: $k = 1,2,...,N$,

$$U_k = \frac{\partial T}{\partial \alpha_k},\ P_k = -\lambda \frac{\partial U_k}{\partial n} \tag{3}$$

At first, we solve the problem (1) arbitrary assuming the value of heat transfer coefficients $\alpha = \alpha^*$ at the boundary points $x^k \in \Gamma_1$. The solution obtained we denote as T^* . In this way we can solve the additional boundary problems (2), which are of the form

$$\begin{cases} x \in \Omega : & \nabla^2 U_k(x) = 0 \\ x \in \Gamma_1 : & P_k(x) = \begin{cases} T_k^* - T^\infty + \alpha_k^* U_k(x), & x = x^k \\ 0 & , \ x \neq x^k \end{cases} \\ x \in \Gamma_2 : & U_k(x) = 0 \\ x \in \Gamma_3 : & P_k(x) = 0 \\ x \in \Gamma_4 : & P_k(x) = \alpha_0 U_k(x) \end{cases} \tag{4}$$

Determined in this way the sensitivity coefficients can be collected in the following matrix

$$\mathbf{U} = \begin{bmatrix} U_{11} & U_{21} & ... & U_{k1} & ... & U_{N1} \\ U_{12} & U_{22} & ... & U_{k2} & ... & U_{N2} \\ ... & ... & ... & ... & ... & ... \\ U_{1M} & U_{2M} & ... & U_{kM} & ... & U_{NM} \end{bmatrix} \tag{5}$$

In this matrix the first index corresponds to the boundary node where the heat transfer coefficient is unknown, the second one corresponds to internal node where the temperature is measured.

Now, we expand the temperature T_e into the Taylor series in the vicinity of point T^{*i} taking into account the first and second component

$$T_e = T_e^* + \sum_{k=1}^{N} \left(\frac{\partial T}{\partial \alpha_k} \right)_e \left(\alpha_k - \alpha_k^* \right) \tag{6}$$

or

$$T_e = T_e^* + \sum_{k=1}^{N} U_{k\,e} \left(\alpha_k - \alpha_k^* \right) \tag{7}$$

In above formulas the values α_k are unknown. We apply the least squares criterion in the form [6, 7]

$$\begin{gathered} S(\alpha_1, \alpha_2, ..., \alpha_N) = \sum_{e=1}^{M} \left[T_e^* + \sum_{k=1}^{N} U_{ke} \left(\alpha_k - \alpha_k^* \right) - T_{d\,e} \right]^2 + \\ + \gamma \sum_{k=1}^{N} \alpha_k^2 \end{gathered} \tag{8}$$

where γ is the regularization parameter.
Differentiating the criterion (8) with respect to the unknown coefficients α_k and using the necessary condition of minimum one obtains

$$\begin{gathered} \sum_{e=1}^{M} \left[T_e^* + \sum_{k=1}^{N} U_{k\,e} \left(\alpha_k - \hat{\alpha}_k \right) - T_{d\,e} \right] U_{l\,e} + \gamma \alpha_l = 0 \,, \\ l = 1, 2, ..., N \end{gathered} \tag{9}$$

or

$$\begin{gathered} \sum_{e=1}^{M} \sum_{k=1}^{N} U_{k\,e} U_{l\,e} \alpha_k + \gamma \alpha_l = \\ = \sum_{e=1}^{M} \left[U_{l\,e} \left(T_{d\,e} - T_e^* \right) + \sum_{k=1}^{N} U_{k\,e} U_{l\,e} \alpha_k^* \right], \; l = 1, 2, ..., N \end{gathered} \tag{10}$$

The last system of equations can be written in the matrix form

$$\left(\mathbf{U}^{\mathrm{T}} \mathbf{U} + \gamma \mathbf{I} \right) \boldsymbol{\alpha} = \mathbf{U}^{\mathrm{T}} \left(\mathbf{T}_{\mathrm{d}} - \mathbf{T}^* \right) + \mathbf{U}^{\mathrm{T}} \mathbf{U} \boldsymbol{\alpha}^* \tag{11}$$

where **I** is the identity matrix, **U** is the matrix (5), while

$$\mathbf{T}^* = \begin{bmatrix} T_1^* \\ T_2^* \\ \dots \\ T_M^* \end{bmatrix}, \quad \mathbf{T}_{\mathrm{d}} = \begin{bmatrix} T_{d1} \\ T_{d2} \\ \dots \\ T_{dM} \end{bmatrix} \tag{12}$$

Solving the system (11), we determine the values of α_k. The solution obtained we denote as α_k^*. The problem (1) and additional problems (2) we solve again for the new set of obtained heat transfer coefficients and next the system of equations (11) can be solved. In this way we start with the iterative process which allows to identify the values of heat transfer coefficients at the boundary nodes $x^k \in \Gamma_1$.

4. Generalized FDM

Below, a short information concerning the numerical solution of problems (1) or (2) will be presented. The domain Ω and its boundary is covered by the practically optional set of points and next a certain way of internal and boundary stars definitions is introduced (c.f. Fig. 2).

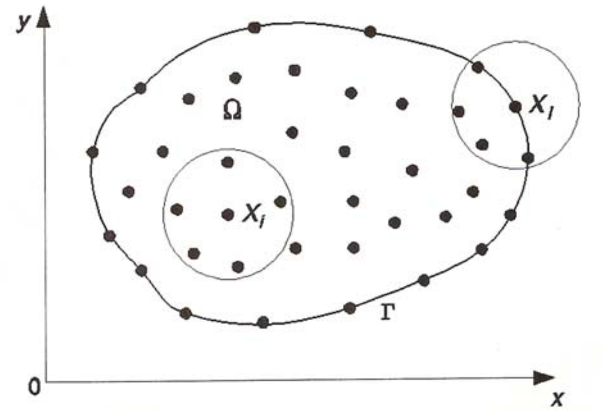

Fig. 2. Internal and boundary stars

A single internal star oriented in a local co-ordinate system is shown in Figure 3.

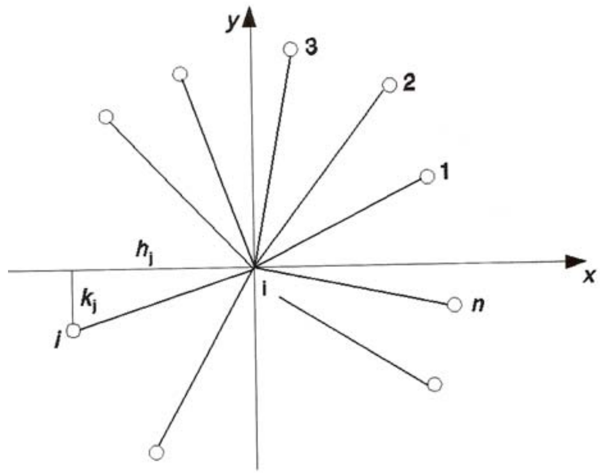

Fig. 3. $n+1$ points star

Function $T = T(x, y)$ is expanded into the Taylor series (taking into account the second derivatives). We denote $x_j - x_i = h_j$, $y_j - y_i = k_j$, while derivatives of function T at a central star point as $(T_x)_i$, $(T_y)_i$, … , $(T_{xy})_i$ and then

$$\begin{gathered} T_j = T_i + (T_x)_i h_j + (T_y)_i k_j + 0.5 (T_{xx})_i h_j^2 + \\ + 0.5 (T_{yy})_i k_j^2 + (T_{xy})_i h_j k_j \end{gathered} \tag{13}$$

Now, the least squares criterion is formulated

$$J=\sum_{j=1}^{N}\left\{\left[T_i-T_j+\left(T_x\right)_i h_j+\left(T_y\right)_i k_j+0.5\left(T_{xx}\right)_i h_j^2+\right.\right.$$
$$\left.\left.+\,0.5\left(T_{yy}\right)_i k_j^2+\left(T_{xy}\right)_i h_j k_j\right]\frac{1}{\rho_j^m}\right\}^2=\min \tag{14}$$

where:

$$\rho_j=\sqrt{\left(x_j-x_i\right)^2+\left(y_j-y_i\right)^2} \tag{15}$$

and m is a natural number (e.g. $m=3$).
Using the necessary criterion of functional (14) minimum one obtains a system of equations in the form

$$\sum_{j=1}^{n}\left[T_i-T_j+\left(T_x\right)_i h_j+\left(T_y\right)_i k_j+0.5\left(T_{xx}\right)_i h_j^2+\right.$$
$$\left.+0.5\left(T_{yy}\right)_i k_j^2+\left(T_{xy}\right)_i h_j k_j\right]\frac{h_j}{\rho_j^{2m}}=0$$
$$\sum_{j=1}^{n}\left[T_i-T_j+\left(T_x\right)_i h_j+\left(T_y\right)_i k_j+0.5\left(T_{xx}\right)_i h_j^2+\right.$$
$$\left.+0.5\left(T_{yy}\right)_i k_j^2+\left(T_{xy}\right)_i h_j k_j\right]\frac{k_j}{\rho_j^{2m}}=0$$
$$\sum_{j=1}^{n}\left[T_i-T_j+\left(T_x\right)_i h_j+\left(T_y\right)_i k_j+0.5\left(T_{xx}\right)_i h_j^2+\right.$$
$$\left.+0.5\left(T_{yy}\right)_i k_j^2+\left(T_{xy}\right)_i h_j k_j\right]\frac{h_j^2}{2\rho_j^{2m}}=0$$
$$\sum_{j=1}^{n}\left[T_i-T_j+\left(T_x\right)_i h_j+\left(T_y\right)_i k_j+0.5\left(T_{xx}\right)_i h_j^2+\right.$$
$$\left.+0.5\left(T_{yy}\right)_i k_j^2+\left(T_{xy}\right)_i h_j k_j\right]\frac{k_j^2}{2\rho_j^{2m}}=0$$
$$\sum_{j=1}^{n}\left[T_i-T_j+\left(T_x\right)_i h_j+\left(T_y\right)_i k_j+0.5\left(T_{xx}\right)_i h_j^2+\right.$$
$$\left.+0.5\left(T_{yy}\right)_i k_j^2+\left(T_{xy}\right)_i h_j k_j\right]\frac{h_i\,k_i}{\rho_j^{2m}}=0 \tag{16}$$

from which the local optimal values of the second (equation) and the first (boundary conditions) derivatives approximation can be found. Finally the system of linear algebraic equations must be solved and the temperature (sensitivity) field is found. The details concerning the GFDM are discussed in [8].

5. Example of computations

The lateral section of the copper continuous casting mould is shown in Figure 4. The symmetrical fragment of this domain (Fig. 5) is considered. We assume that the values of temperature in the set of internal points are known.

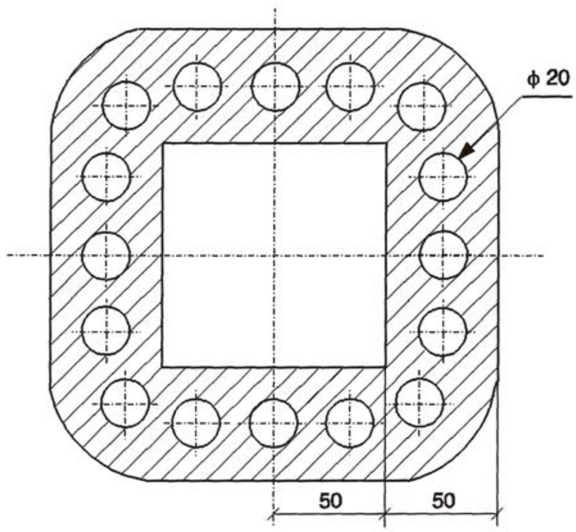

Fig. 4. The section of CCM

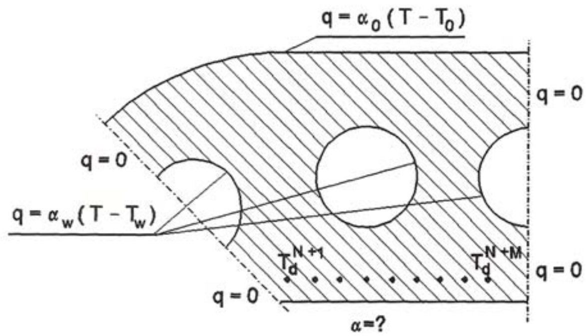

Fig. 5. The fragment of CCM

Thermal conductivity of CCM $\lambda=330$ [W/mK], heat transfer coefficient on the outer surface of the CCM $\alpha_0=15$ [W/m^{2}K], heat transfer coefficient between CCM and cooling pipes $\alpha_w=3675$ [W/m^{2}K], ambient temperature $T_0=30$ [°C], cooling water temperature $T_w=50$ [°C]. The substitute heat transfer coefficient α on the contact surface Γ_1 has been found assuming that the conventionally assumed 'ambient temperature' equals $T^\infty=50$ [°C]. The following temperatures at internal points have been introduced (node number - temperature) - c.f. Figure 6:

$$\begin{array}{lllll} 1-199, & 2-207, & 3-214, & 4-219, & 5-223, \\ 6-226, & 7-228, & 8-230, & 9-231, & 10-232, \\ 11-186, & 12-200, & 13-209, & 14-214, & 15-217 \end{array} \tag{17}$$

These values result from the direct solution ($\alpha = 1330$ [W/m^{2}K] disturbed in a random way. The reconstructed after 5 iterations values of substitute heat transfer coefficients α_k, $k = 1, 2, ..., 10$ are equal to

$$\alpha_1 = 1220, \quad \alpha_2 = 1280, \quad \alpha_3 = 1310, \quad \alpha_4 = 1310, \quad \alpha_5 = 1320,$$
$$\alpha_6 = 1330, \quad \alpha_7 = 1330, \quad \alpha_8 = 1315, \quad \alpha_9 = 1315, \quad \alpha_{10} = 1305,$$

while the mean value of α equals about 1305 [W/m^{2}K].
In Figure 6 the temperature distribution in the lateral section of CCM is shown.

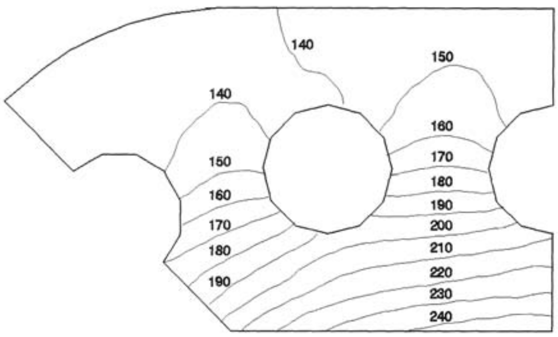

Fig. 6. Temperature field

Summing up, the algorithm proposed constitutes the quite effective tool to the identification of boundary heat transfer coefficient.

6. Conclusions

The problem discussed is rather complex. It results both from the domain geometry and also from the big number of unknown parameters (local values of substitute heat transfer coefficients). The complexity of object geometry was a reason of generalized FDM application. The irregular mesh allows to locate the nodes directly at the internal and external boundaries. For the boundary nodes 5-points stars, while for the interior 9-points stars have been taken into account. The sensitivity models are similar to the basic one and on the stage of computations the same numerical procedures have been applied. The quality criterion (8) was sufficiently effective to identify the unknown coefficients. The mean value of substitute heat transfer coefficient is close to the assumed 'real' one.

The solutions of the others types of inverse problems connected with the thermal theory of foundry processes ca be found in [9, 10, 11]

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