ARCHIVES of FOUNDRY ENGINEERING

ISSN (1897-3310)
Volume 7
Issue 3/2007

183 – 186

35/3

Published quarterly as the organ of the Foundry Commission of the Polish Academy of Sciences

Using Fuzzy Optimisation Method in Calculation of Charge Burden to Correct the Chemical Composition of Metal Melt

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Received on 19.04.2007; Approved for printing on: 27.04.2007

Abstract

The article describes a mathematical model of an algorithm used in calculation of the charge burden to correct the misadjusted chemical composition of metal melt. The model assumes that the charge materials covered by calculations are characterised by a fuzzy (uncertain) chemical composition. The model also assumes different yield of chemical elements from the metal melt and charge materials. The discussion is completed with an example of calculations illustrating practical application of the said algorithm.

Keywords: Computer-aided foundry process; Correction of chemical composition in metal melt

1. Introduction

When metal is melted in a foundry furnace, quite often we have to face the situation when the chemical composition of the metal melt is not consistent with the target composition assumed at the beginning of the process. If this happens, it is necessary to correct the chemical composition and bring it to a required level. The corrections can be done in a foundry furnace or in a ladle. The calculation of a correcting charge burden can be done with the help of optimisation algorithms which will enable determination of the charge components in terms of both quantity and quality. When various charge materials are used, often including numerous chemical elements whose content is either unknown or uncertain, the calculation of a correcting charge burden requires the development of special mathematical models. Among many issues related with the calculation of charge burden used to correct the misadjusted chemical composition of metal melt, special attention require the following ones:

- it is assumed that the chemical composition of metal melt will be corrected in an electric foundry melting unit, e.g. in an induction crucible furnace or resistance furnace, or in a foundry ladle,
- the chemical composition of the metal melt held in the furnace is known because it has been examined previously in a laboratory,
- also the weight of the metal melt whose chemical composition is to be corrected is known,
- any possible changes in the content of the individual chemical elements resulting from mixing of molten metal with the solid correcting charge will be taken into consideration. The chemical elements present in molten metal and in the solid charge will change their content due to melting loss but the values of this loss will vary quite considerably in each specific case,
- when the weight of the correcting charge has been determined, the melt level in the furnace or ladle should be checked to avoid overfilling after the correcting charge has been added. If the correcting charge is too voluminous, it may be advisable to carry out the corrections rather as a step-

adjustment process on the successive and therefore smaller batches of molten metal.

2. Mathematical model of correction of the chemical composition of molten metal

Modelling of corrections introduced to rectify the misadjusted chemical composition of molten metal requires that, first, an objective function and boundary conditions are determined in the optimisation tasks. These conditions reflect the technological and technical potential of a foundry plant, allowing also for its production program.

2.1. Assumptions made for a model description of the fuzzy chemical composition of charge materials

In most cases, the deterministic (precisely determined) values of parameters, such as the chemical composition or the unit price of charge materials, are adopted because:

- the values of the examined parameters are usually averaged, since the results of laboratory measurements of the chemical composition done on large batches of the charge materials are not fully reliable, and the data in material certificates are often oversimplified,
- some typical characteristics (chemical composition, unit price) present in different batches of the same charge component stored in the same container are often neglected,
- the mathematical models used so far in the description of optimisation tasks impose certain limitations, and access to computer programs which could solve these tasks is often very difficult.

In the situation when the precisely determined values of the parameters are the source of simplifications considerably reducing the range of their practical application, one can think of the possibility of using the theory of fuzziness in describing the examined domain of reality. The uncertainty of the values of the numerical variables can be described with the, so called, fuzzy sets using various membership functions [1]. In description of the non-precise (uncertain) or, in other words, fuzzy values of the parameters characteristic of the chemical composition and, possibly also, of the unit price of charge materials, the membership functions of different types can be used, for example:

- polygonal membership functions,
- symmetric and asymmetric Gaussian functions as well as symmetric Gaussian functions with finite support,
- sigmoidal membership functions (left and right),
- harmonic symmetric and asymmetric membership functions (internal, right external and left external)
- polynomial membership functions (of the second and higher orders).

For further considerations we shall adopt a rectangular function of membership, which is one of the variants of polygonal functions. This assumption will not complicate the optimisation task, and at the same time it will describe relatively well the uncertain chemical composition of charge materials. The rectangular function of membership can be modelled according to a relationship:

$$\mu(x)=\begin{cases}0 & for\ x<a\ or\ x>b\\ 1 & for\ a\le x\le b\end{cases} \tag{1}$$

Figure 1 shows graphic interpretation of this membership function.

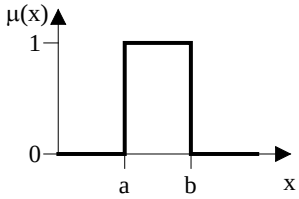

Fig. 1. Graphic interpretation of a rectangular membership function

The adopted membership function will serve as a tool in defining the chemical composition of charge materials as a range of contents of various chemical elements.

2.2. Objective functions in the task of optimising a correcting charge burden

In the task of optimising a misadjusted chemical composition of molten metal it is recommended, first of all, to try to make the weight of the correcting charge as small as possible. This is dictated by the fact that the correcting charge will be put into a foundry furnace already filled with molten metal. The total weight of the metal melt and of the correcting charge must not exceed the rated capacity of a furnace or ladle. This means that the objective function used in an optimisation task should assume the following form:

$$m_k=\sum_{j=1}^{N}x_j \tag{2}$$

where:

m_k - the total weight of the charge used to correct the chemical composition of molten metal, kg,

x_j - the content of the j-th charge material in the correcting charge (j=1,2,…,N; N – the number of charge components included in calculations).

2.3. The system of boundary conditions

When modelling the system of constraints it is necessary to introduce the parameters of the melting loss suffered by individual chemical elements in both molten metal and correcting charge.

In a rectangular fuzzy model of the chemical composition, the system of boundary conditions assumes the form of:

$$\begin{cases}\sum_{j=1}^{N}U_{ij}\underline{a}_{ij}x_j+U_{ai}A_i m_a\ge \underline{b}_i\left(m_a+\sum_{j=1}^{N}x_j\right)\\ \sum_{j=1}^{N}U_{ij}\overline{a}_{ij}x_j+U_{ai}A_i m_a\le \overline{b}_i\left(m_a+\sum_{j=1}^{N}x_j\right)\end{cases} \tag{3}$$

where:

U_{ij} - the yield of the i-th chemical element in the j-th charge component, %,

$\underline{a}_{ij}$, $\overline{a}_{ij}$ - the lower and upper content level, respectively, of the i-th chemical element in the j-th charge component, %,

U_{ai} - the yield of the *i*-th chemical element in molten metal held in the furnace, %,

A_i - the content of the *i*-th chemical element in molten metal, %,

m_a - the volume (weight) of molten metal subjected to correction of the chemical composition, %,

$\underline{b}_i, \overline{b}_i$ - the assumed lower and upper content level, respectively, of the *i*-th chemical element in molten metal after correction, %.

The system of constraints (3) expresses a balance in the chemical composition of molten metal after correction. To preserve this system in optimisation tasks it should be transformed to the following form:

$$\begin{cases} \sum_{j=1}^{N} x_j \left(U_{ij}\,\underline{a}_{ij} - \underline{b}_i\right) \geq m_a \left(\underline{b}_i - U_{ai} A_i\right) \\ \sum_{j=1}^{N} x_j \left(U_{ij}\,\overline{a}_{ij} - \overline{b}_i\right) \leq m_a \left(\overline{b}_i - U_{ai} A_i\right) \end{cases} \tag{4}$$

The task of determining an optimum charge burden to correct the misadjusted chemical composition of molten metal consists in finding such values of x_j for which the objective function (2) will have minimum values under constraints (4). A very important implicit assumption in this optimisation task is the non-negative character of the value of x_j. This means that the content of each charge material can be only higher than or equal to zero.

3. Algorithm for calculation of the correcting charge burden

In the case of function (2), the optimisation task for calculation of the correcting charge burden is of a linear nature, since the system of boundary conditions (4) has functions in the linear form.

To solve this optimisation task one can use a selected method of linear programming, e.g. a version of simplex algorithm. It is also possible to use a properly modified version of the algorithm of square programming in which it is allowed to apply the objective function in a square or linear form. The algorithm of square programming implemented in the computer program aids the decision-taking process regarding burdening of foundry furnaces and can serve equally well as a tool in other similar calculations, described in a more comprehensive way in [2].

The method of square programming searches for a set of values of the vector of decision variables **x** such that will bring the value of an objective function to its minimum in the form of:

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{t}^{\mathrm{T}}\mathbf{x} \tag{5}$$

when the following constraints hold good:

$$\mathbf{r}_i^{\mathrm{T}}\mathbf{x} \leq \mathbf{s}_i \quad \text{for} \quad i=1,\ldots,l_g, \tag{6}$$

$$\mathbf{r}_i^{\mathrm{T}}\mathbf{x} = \mathbf{s}_i \quad \text{for} \quad i=l_g+1,\ldots,l_g+l_h, \tag{7}$$

$$x_j \geq 0 \quad \text{for} \quad j=n_u+1,\ldots,n. \tag{8}$$

where:

x - the vector of decision variables of the size *n*, where the first n_u of the decision variables may assume any arbitrary sign, while the remaining variables must be non-negative,

Q - the positively semi-defined symmetrical matrix of the size $n \times n$, comprising values of the coefficients of a non-linear fragment of the objective function f(x),

t - the vector of the coefficients of a linear fragment of the objective function f(x),

$\mathbf{r}_i$ - the vector of the coefficients of the linear functions defined on the left side of the inequality constraints of the size l_g,

$\mathbf{s}_i$ - the vector of the coefficients present on the right side of the inequality and equality constraints. The vector has the size l_g+l_h, where the first l_g of the coefficients refer to the inequality constraints, while the next l_h of the coefficients refer to the equality constraints,

$\mathbf{r}_i$ - the vector of the size l_h, comprising the coefficients of the linear functions defined on the left side of the equality constraints.

Detailed description of the algorithm and an original computer program with the algorithm of square programming written in FORTRAN language are given in [3]. Basing on this algorithm, a version of the program has been written in PASCAL language using a DELPHI 2005 compilator. This version of the program, assigned for operation in a MS Windows graphical environment, uses the dynamic tables and has a graphic interface for data input and display of the results of calculations.

4. Example of calculations

To better illustrate the structure of a mathematical model used for correction of the chemical composition of molten metal, it has been assumed that a foundry furnace holds 1000 kg of molten metal. The chemical composition of this metal, both preset and given by laboratory analysis, is compiled in Table 1.

Table 1.
Chemical composition of molten metal in foundry furnace as preset and given by laboratory analysis

Chemical element	Chemical composition, %	
	Preset	Given by analysis
C	3.2÷3.5	3.14
Si	0.8÷1.1	0.72
Mn	0.3÷0.7	0.65

As follows from Table 1, the content of two chemical elements, i.e. of C and Si, is lower than the preset level, while the content of Mn is comprised in the preset range of values. Table 2 shows the chemical composition of the charge materials which will be used to correct the misadjusted chemical composition of molten metal. In calculations, a rectangular model has been adopted to describe the fuzziness of the chemical composition of the charge materials.

Table 3 completes the set of input data for a model calculation of the corrected chemical composition of molten metal. This table also states the yield coefficients of the individual chemical elements from charge materials. To make this task simpler it has been assumed that the yield of these elements from molten metal is of a value 1, which means that it equals 100%. No constraints have been imposed on the content of the individual charge materials.

Table 2.
Chemical composition of charge materials

Charge material	Chemical composition, %		
	C	Si	Mn
Comp. 1	0.20÷0.40	0.2÷0.3	0.1÷0.2
Comp. 2	0.85÷1.00	1.5÷1.8	0.6÷0.9
Comp. 3	3.80÷4.20	0.6÷0.9	0.4÷0.8
Comp. 4	3.90÷4.10	1.3÷1.4	0.3÷0.6

Table 3.
Chemical composition of charge materials

Charge material	Yield, [-]		
	C	Si	Mn
Comp. 1	0.90	0.84	0.83
Comp. 2	0.85	0.86	0.81
Comp. 3	0.94	0.82	0.80
Comp. 4	0.92	0.81	0.78

The solution of optimisation task consists in determining the content of the individual charge materials in a way such as to make the objective function (2) reach for N=4 its minimum value under the constraints in the following form:

$$\begin{cases} -3.02x_1 - 2.775x_2 + 0.372x_3 + 0.388x_4 \geq 60 \\ -3.14x_1 - 2.65x_2 + 0.448x_3 + 0.272x_4 \leq 360 \\ -0.632x_1 + 0.49x_2 - 0.308x_3 + 0.253x_4 \geq 80 \\ -0.848x_1 + 0.448x_2 - 0.362x_3 + 0.034x_4 \leq 380 \\ -0.217x_1 + 0.186x_2 + 0.02x_3 - 0.066x_4 \geq -350 \\ -0.534x_1 + 0.029x_2 - 0.06x_3 - 0.232x_4 \leq 50 \end{cases} \quad (9)$$

For a set of constraints exposed in (9), the same implicit assumption derived from the algorithm of mathematical programming imposing the non-negative values on vector **x** holds good.

Having introduced the values of the coefficients of objective function and a system of constraints to the respective vectors and a matrix, in the computer program comprising procedures for an algorithm of the square programming the results depicted in Figure 2 were obtained.

As follows from Figure 2, to correct the misadjusted chemical composition of molten metal it is enough to use only the second and fourth component. Then the smallest total weight of the correcting burden will be 299.6 kg, that is, almost 30% of the molten metal weight before correction. So large volume of the correcting burden respective of the weight of molten metal is an effect of the use of charge materials which introduce each of the considered chemical elements. If the charge materials were used which introduce only one specific chemical element, e.g. a selected type of carburiser or ferroally (ferrosilicon or ferromanganese), then the correcting burden could contain much lower content of the element which needs to be adjusted.

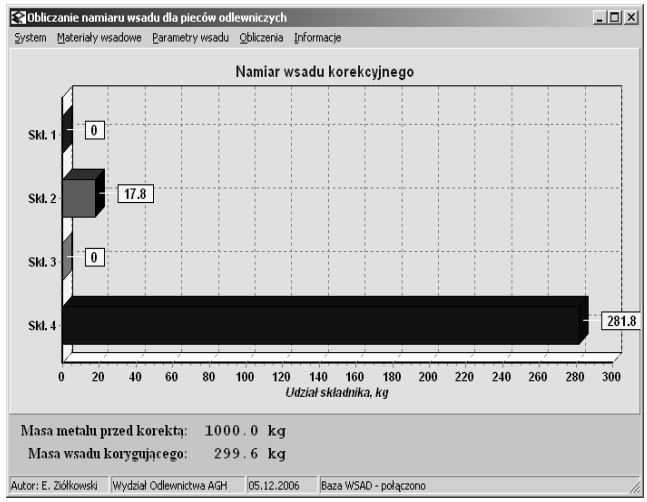

Fig. 2. The results of calculations of the correcting charge burden

Having calculated the correcting charge burden it is necessary to check if the operation of correcting the chemical composition of molten metal can be carrried out in a given foundry furnace. Very important is here the information to what level the correcting charge will fill the furnace and what time is available for the operation of making corrections. If the correcting procedure cannot be carried out in one single cycle, the correction of the successive batches of the molten metal (e.g. in a foundry ladle) should be taken into consideration.

4. Conclusions

The mathematical model described in this article and an algorithm for determination of the charge burden correcting the misadjusted chemical composition of molten metal have been developed to help the melt operator in taking proper decisions. The algorithm operating in a specially designed computer program may considerably reduce the cost of molten metal production when charge materials characterised by uncertain (fuzzy) chemical composition are used.

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