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A mathematical programming approach to crop mix problem

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Problems normally faced by farmers include what to plant, how much to plant and when to plant. In this study, a linear programming crop mix model for a finite-time planning horizon was proposed. Given limited available resources such as budget and land acreage, the crop-mix planning model was formulated and transformed into a multi-period linear programming problem. The objective was the maximization of the total returns at the end of the planning horizon. The model was applied to selected single-harvest crops and solved by LINDO, a linear programming package. The results were analyzed and discussed.

Key words: Crop mix, single-harvest, linear programming.

INTRODUCTION

There is no denying fact that agriculture and agricultural products play an important role in sustaining life on planet earth. Studies on agricultural farm production planning or management normally focus on crop rotation and crop mix. This crop planning is related to many factors; measurable and non-measurable. These include factors such as types of land available for cultivation, yield rates of the cultivated crops, weather conditions and rainfall, irrigation system, availability of the agricultural inputs such as machinery, fertilizer, capital and labour and the cost of production.

The crop rotation concept involves the exploitation of jointly beneficial interrelationship among individual crop (El-Nazer and McCarl, 1986; Heady, 1948). This is characterized by a cultivation of a sequence of crops, for example soybeans, maize and fallow, while satisfying the crop succession requirements when applied on the same piece of land. Perhaps one of the best examples is the most cited 8-course rotation cycle of cotton-fallow-fallow-cotton-fallow-sorghum-fallow-fallow which has been applied for many years in the Gezira scheme in Sudan, and later replaced by a 4-course cycle of cotton-wheat-sorghum (or groundnuts)-fallow (Haneveld and Stegeman, 2005). One of the earliest utilization of linear

programming approach in modelling crop rotation planning was treated by Hildreth and Reither (1951) where sequences of crops (or rotations) were transformed into a set of linear programming activities. Since then, crop rotation problems became common themes in most linear programming literatures on agriculture (El-Nazer and McCarl, 1986; Haneveld and Stegeman, 2005; Musser et al., 1985).

Mixed-cropping (or crop mix), on the other hand, is a cropping system involving more than one crop being cultivated simultaneously on a farming space in the same cropping period or season (Jolayemi and Olaomi, 1995). High crop yield, better spread of production over the growing period, improved quality of product and reduced risk of total crop failure have been identified as some of the main advantages of mixed-cropping. The principal objective in any crop mix problem is to search for an optimal combination of crops amongst those considered such that it maximizes the total overall contributions while satisfying a system of constraints such as land availability, capital and others. The success of mixed-cropping scheme is therefore dependent on the adoption and the integration of mathematical techniques and management of all the components of the production systems. Jolayemi and Olaomi (1995) extended an earlier version of a linear programming model for selecting crops for a mixed-cropping scheme. Unlike the earlier model, the revised version involves no enumeration

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or computational process and therefore requires less computational effort. The superiority of the procedure is further highlighted by simulating and solving many hypothetical crop-selection problems. Sarker et al. (1995) developed a linear programming model to determine the areas to be allocated to different crops to maximize the total contributions from agricultural activities in Bangladesh. The results revealed that an annual contribution can be increased by 1594510 million Taka (US\$1.00 ~ 41.00 Taka). In addition to land and capital constraints, the model also included the country's food demand and trade bounds which limit the total crops imported and/or exported. More than 100 crops cultivated were divided into 10 major groups. The model was then solved on PC using a package programme Quantity Software for Operation Management.

Other mathematical programming tools that are utilized for modelling and analyzing agriculture production planning include integer and goal programming. Butterworth (1985), employed a linear integer programming technique to a problem area of 500 acres in solving the mixed cropping model to determine the selection of crops, particularly vegetables and livestock herds subject to resource constraints of land, labour, machinery and building capacity. The total gross margin for the new farming scheme showed an increase of 38000 pound-sterling. The applicability of linear goal programming model in agricultural sector in Egypt was addressed by Bazaraa and Bouzaher (1981). The model was illustrated using data from regions with three cropping seasons. The agricultural activities covered by the model consisted of 10 winter crops, seven summer crops, three nili crops, two permanent crops, two types of pastures, four types of livestock and three types of fertilizers. The results generated suggested the need for using improved farming techniques through labour intensive means to obtain a relatively higher degree of specialization and a relatively lower cotton production, a conclusion consistent with that reached by an independent study conducted by the U.S Agency for International Development and Ministry of Agriculture in Egypt.

In general, different crops mature at different ages. This means the age at first harvest differs for each crop under investigation. In other words, the crops are harvested at different times although they are initially planted simultaneously. However, more often than not, most studies on crop management modelling overlook this aspect, thereby casting doubts on the practical applicability of the results generated by the models.

This work attempted to minimize this possibility by incorporating the age of maturity (corresponding to the time of first harvest). Afterwards, formal definitions and basic assumptions were detailed, followed by the formulation of multi-period crop-mix model for a finite-time planning horizon. The study goes further to illustrate the applicability of the model for the selected vegetables

by utilizing the annual data published by the Ministry of Agriculture in Malaysia. Results and sensitivity analysis are discussed together with a collection of real life success stories. Finally, the study concluded and highlighted the avenues and directions for future research.

MATERIALS AND METHODS

Formulation of the multi-period crop-mix model

The following definitions were made;

Definition 1

Single-harvest crop was defined as a crop that can be harvested once only. Each new harvest requires new replanting. This is true for most vegetables.

Definition 2

Maturity age or growth time period was defined as the length (or the number) of time periods that elapsed between planting and harvesting. This normally differs for most crops.

Basic assumptions

Not all factors affecting crop planning can be incorporated into the model. To ease the formulation, the following assumptions were adopted:

1. The availability of physical resources of the farm (such as land, water, labour, etc.) does not change during the planning horizon, and neither does the technology nor the level of management.
2. The location of area for each crop is immaterial in the sense that it had no influence on the crop's productivity (yield per acre). In other words, the land was assumed to be equally suitable for all the selected crops.
3. The crop-mix depended only on the types of crops that were grown and not on the methods of cultivation that were used.
4. An acre of crop production can be substituted for an acre of another type of crop production and
5. The crop prices and yields per acre do not change during the planning horizon.

Notations and variables

Considering a piece of land on which different selected combination of single-harvest crops of different maturity age may be cultivated; A was the area (in acres) of the farmland available for mix-cropping scheme, T the number of time periods in the planning horizon, K the amount of capital (or budget) available at the beginning of the planning period which can be regarded as an initial investment and S was the monthly administrative expenditure required to manage the farm which included wages, salaries and other expenses.

For a planning horizon of T periods the decision variable X (m, C, t) can be let, in general to denote the acreage allocated to crop C of maturity age m, planted in period $t = 1, 2, \dots, T-m+1$. This will generate $T-m+1$ decision variables for crop C. Without loss of generality and for illustrative purposes, crops with maturity ages of 1, 2 and 3 time periods was considered. Thus, three sets of decision variables were gotten:

Table 1. Acreage allocations to crop of maturity age p=1, 2, 3 time periods, for planning horizon, T=12 time periods.

Crop types	Time periods in the planning horizon					
	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
J	X(1,J,1)	X(1,J,2)	X(1,J,3)	X(1,J,4)	X(1,J,5)	X(1,J,6)
K		X(2,K,1)		X(2,K,3)		X(2,K,5)
K			X(2,K,2)		X(2,K,4)	X(2,K,6)
L		X(3,L,1)			X(3,L,4)	
L			X(3,L,2)			X(3,L,5)
L				X(3,L,3)		X(3,L,6)

Crop types	Time periods in the planning horizon					
	t = 7	t = 8	t = 9	t = 10	t = 11	t = 12
J	X(1,J,7)	X(1,J,8)	X(1,J,9)	X(1,J,10)	X(1,J,11)	X(1,J,12)
K		X(2,K,7)		X(2,K,9)		X(2,K,11)
K	X(2,K,6)		X(2,K,8)		X(2,K,10)	
L		X(3,L,7)			X(3,L,10)	
L	X(3,L,5)		X(3,L,8)			
L		X(3,L,6)		X(3,L,9)		

(1) X(1,J,t) to denote acreage allocated to crop J of maturity age 1 time period, planted in period t = 1,2,...,T. This generated T decision variables for crop J.

(2) X(2,K,t) to denote acreage allocated to crop K of maturity age 2 time periods, planted in period t = 1,2,...,T - 1. This generated T - 1 decision variables for crop K, and

(3) X(3,L,t) to denote acreage allocated to crop L of maturity age 3 time periods, planted in period t = 1,2,...,T - 2. This generated T - 2 decision variables for crop L.

The pattern is illustrated in Table 1 for T = 12 time periods. Finally, R_i, the expected revenue (in RM) per acre with respect to crop I and C_i, the total cultivation cost (in RM) per acre with respect to crop I was required. This included all costs such as costs of seeds/seedlings, fertilizer, machinery, preparation and maintenance of the farmland from planting to harvesting, harvesting, post-harvesting, insurance and others, and W_t, cash in hand (excess or saving) in time period t = 1, 2, ..., T.

Linear programming model

The linear programming formulation for the crop-mix model can then be presented as follows.

Objective function: The objective function of the model is to maximize the total contributions or revenues accumulated at the end of the planning horizon, that is, maximize

$$Z = \sum_J R_J X(1,J,T) + \sum_K R_K X(2,K,T-1) + \sum_L R_L X(3,L,T-2) + W_T$$

Where,

$\sum_J R_J X(1,J,T)$ is revenues generated by all crops of type J planted in period T,

$\sum_K R_K X(2,K,T-1)$ is revenues generated by all crops of type K planted in period T-1,

$\sum_L R_L X(3,L,T-2)$ is revenues generated by all crops of type L planted in period T-2 and W_T is the cash in hand in period T.

Systems of constraints

Land constraints

The total acreage under cultivation at any time period must not exceed the total available land of A acres. Thus, specifically (Table1)

$$t = 1: \sum_J X(1,J,1) + \sum_K X(2,K,1) + \sum_L X(3,L,1) \leq A$$

$$t=2: \sum_J X(1,J,2) + \sum_K \{X(2,K,1) + X(2,K,2)\} + \sum_L \{X(3,L,1) + X(3,L,2)\} \leq A,$$

$$3 \leq t \leq T - 2:$$

$$\sum_J X(1,J,t) + \sum_K \{X(2,K,t-1) + X(2,K,t)\} + \sum_L \{X(3,L,t-2) + X(3,L,t-1) + X(3,L,t)\} \leq A$$

$$t = T - 1: \sum_J X(1,J,T-1) + \sum_K \{X(2,K,T-1) + X(2,K,T-2)\} + \sum_L \{X(3,L,T-2) + X(3,L,T-3)\} \leq A$$

$$t=T: \sum_J X(1,J,T) + \sum_K X(2,K,T-1) + \sum_L X(3,L,T-2) \leq A$$

Table 2. Data on selected vegetable crops 2000 to 2005.

Variable	Crop	Productivity (tones/hectare) ^(a)				Average productivity (kg/acre)	Revenue per acre, (RM) ^(b)	Cost per acre, (RM) ^(c)
		2000	2003	2004	2005			
X(1,A)	Spinach	9.20	11.5	14.3	14.0	4900	4900.00	3430.00
X(1,B)	Pak Choy	9.40	14.1	16.1	16.2	5580	6140.00	4300.00
X(2,A)	Lady's finger	12.9	13.1	12.9	12.3	5120	7680.00	5380.00
X(2,B)	Chinese kale	9.7	13.8	14.5	14.6	5260	11570.00	8100.00
X(2,C)	Lettuce	10.9	12.5	17.9	12.0	5290	14810.0	10370.00
X(2,D)	Cucumber	26.7	17.9	19.1	16.5	8020	5610.00	3930.00
X(3,A)	French bean	7.10	11.0	15.1	10.3	4350	11740.00	8220.00
X(3,B)	Long bean	11.6	13.1	13.9	13.9	5250	6300.00	4410.00

a, Agriculture Statistical Handbook 2006, Department of Agriculture, Ministry of Agriculture, Government of Malaysia; b, calculated using prices from famaxchange website <http://sdvi.fama.net.my>; c, estimated as 70.0% of the revenue per acre.

Balance equations

All revenues accumulated at time t and cash in hand at time $t-1$ (for $t > 1$) can be utilized for crop production cost, administrative requirements and cash in hand for period t ($1 < t \leq T-2$). Thus:

$$\sum_J R_J X(1, J, t-1) + \sum_K R_K X(2, K, t-2) + \sum_L R_L X(3, L, t-3) + W_{t-1}$$

$$= \sum_J C_J X(1, J, t) + \sum_K C_K X(2, K, t) + \sum_L C_L X(3, L, t) + S + W_t$$

However for $t = 1$, we had an initial capital, K acting as revenue, giving:

$$\sum_J C_J X(1, J, 1) + \sum_K C_K X(2, K, 1) + \sum_L C_L X(3, L, 1) + W_1 = K - S$$

while for $t = T - 1$, we had

$$\sum_J R_J X(1, J, T-2) + \sum_K R_K X(2, K, T-3) + \sum_L R_L X(3, L, T-4) + W_{T-2}$$

$$= \sum_J C_J X(1, J, T-1) + \sum_K C_K X(2, K, T-1) + S + W_{T-1}$$

and for $t = T$, we had

$$\sum_J R_J X(1, J, T-1) + \sum_K R_K X(2, K, T-2) + \sum_L R_L X(3, L, T-3) + W_{T-1}$$

$$= \sum_J C_J X(1, J, T) + S + W_T$$

Non-negativity constraints

All decision variables are non-negative. The general model formulated above involved $3T+9$ decision variables and 24 linear constraints.

EMPIRICAL IMPLEMENTATION

To illustrate the applicability and feasibility of the model, the annual time series data extracted from Agriculture Statistical Handbook 2006, published by The Department of Agriculture, Ministry of Agriculture, Government of Malaysia was utilized. On the revenue side, the prices were downloaded from Federal Agricultural Marketing Authority (FAMA) website <http://sdvi.fama.net.my> on 20 September 2010. From the collected data, ten vegetable crops of different maturity ages were selected for the study. These were spinach (bayam) and pak choy (sawi) for the approximately 1-month maturity age group, lady's finger (bendi), Chinese kale (kailan), lettuce (salad) and cucumber (timun) for the approximately 2-month maturity age group and French beans (kacang buncis) and long beans (kacang panjang) for the approximately 3-month maturity age group. Table 2 summarizes the relevant data and the variable names corresponding to the area allocated to each crop.

Using the above data the crop-mix model for planning horizon of 12 months (one calendar year) was run using a linear programming package LINDO (Linear Interface Discrete Optimizer). The model consisted of 100 decision variables and 24 constraints.

RESULTS AND DISCUSSION

Four separate linear programming models for a fixed land area of 10.0 acres were solved by varying the parameters associated with the initial available capital, K and the monthly requirement of administrative expenses, S . These were grouped under four scenarios:

Scenario 1: $K = \text{RM } 50,000.00$; $S = \text{RM } 10,000.00$ per month.

Scenario 2: $K = \text{RM } 50,000.00$; $S = \text{RM } 15,000.00$ per month.

Scenario 3: $K = \text{RM } 100,000.00$; $S = \text{RM } 10,000.00$ per month.

Scenario 4: $K = \text{RM } 100,000.00$; $S = \text{RM } 15,000.00$ per month.

Table 3. Optimal solution (scenario 1).

Crop	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
X(1,A,t)	3.448276					
X(1,B,t)	6.551724	10.00000	7.936715	7.492879	5.045210	4.838650
X(2,C,t)			2.063285	2.063285	4.510953	4.510953
X(2,C,t)				0.443836	0.443836	0.650397
Revenue (RM)		57124.14	61400.00	48731.43	76563.54	37550.80
Cost (RM)	40000.00	43000.00	55524.14	36821.96	68473.01	27550.80
S (RM)	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00
W_t (RM)		4124.14		1909.47		
	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
X(1,A,t)						
X(1,B,t)	1.719746	2.370142			10.0000	
X(2,C,t)	7.629858	7.629858			10.0000	10.0000
X(2,C,t)	0.650397		10.0000	10.0000		
Revenue (RM)	96516.54	20191.62	127550.87		148100.00	
Cost (RM)	86516.54	10191.62	103700.00		103700.00	
S (RM)	10000.00	10000.00	10000.00	10000.00	10000.00	10000.00
W_t (RM)			13850.87	3850.87	38250.87	28250.87

Result for scenario 1

Scenario 1 acted as basic model. The available initial capital was limited to RM50000.00, a sum synonymous with personal loan provided by most commercial banks. The choice of RM10000.00 for monthly requirement was not unrealistic if the following expenses were considered for:

1. Hiring of 5 farm workers at RM1200.00 = RM 6000.00
 2. Farmer's own salary at RM2000.00 = RM 2000.00
 3. Estimated expenses on water and electricity = RM 2000.00
- Total monthly expenses = RM10000.00

The optimal mix-cropping for this scenario is depicted in Table 3. The scheme was capable of generating a maximum return of RM176350.87 at the end of the planning horizon. If the initial capital was deducted, on the assumption of 10.0% interest rate, the net contribution amounted to RM121, 350.87.

This optimal cropping scheme involved the combined cultivations of spinach, pak choy and lettuce. The first month saw the mixing of the two relatively less productive candidates, spinach and pak choy (34.5 and 65.5% respectively). The inclusion of the main contributor, lettuce only started in the 3rd month and its allocation increased gradually from 20.63% to full domination in the 9th month and continued until the end of the planning horizon. Throughout the planning period, the monthly requirement of RM10000.00 was fully met. Excess cash in hand occurred in the 2nd month (RM4124.14), 4th month (1909.47) and from the 9th month onwards ($W_9 =$

RM13850.87 to $W_{12} =$ RM28250.87) due mainly to the full cultivation of the main contributor, lettuce. Thus, single cropping became optimal from the 9th month onwards. As can be seen, there was no revenue generated beginning from the 10th and 12th months. The monthly expenses were met by the surplus from their respective previous months. The final accumulated return at the end of the planning period was the sum of the yield from the 10 acres of lettuce (RM148100.00) and the RM 28250.87 cash in hand in the 12th month.

Sensitivity analysis

Scenario 2

In this scenario the monthly expenses was increased to RM15000.00, while retaining the values of all the other parameters. This was regarded as an increase in the wages of the farm workers or the number of workers or the personal income for the farmer or the water and electricity expenses. The associated optimal scheme resulted in a maximum return of RM49625.52 at the end of the planning horizon. This is of no surprise since most of the contributions were taken earlier at RM5000.00 per month. The mix-cropping scheme suggests the combinations of spinach and pak choy only. The monthly revenue generated was not sufficient to consider lettuce as an alternative crop. The land allocations between spinach and pak choy fluctuated from 91.9 and 8.05% in the first month to 94.96 and 5.04% in the 12th month, respectively. Under this scheme there was no monthly cash in hand, that is, $W_t = 0$, for all $t = 1, 2, \dots, 12$.

Scenario 3

In this scenario the initial capital was increased to RM100000.00, while retaining the values of all the other parameters as in scenario 1. This can be regarded as an increase in the amount of fund or loan available for investment, following the promising results of scenario 1. The associated optimal scheme resulted in a maximum return of RM244009.14 at the end of the planning horizon. The initial capital was deducted, on the assumption of 10.0% interest rate; the contribution amounted to RM134009.14. The mix-cropping scheme suggests the combinations of pak choy and lettuce for the first four months. From the fifth month onwards, single cropping of lettuce was optimal. The amount of cash in hand varied from RM32709.14 in month 5 to RM105909.14 in month 11. Thus, the final accumulated sum at the end of the planning horizon is due to the 10 acres of lettuce (RM148100.00) and RM95909.14 cash in hand in month 12.

Scenario 4

In this scenario the monthly expenses was increased to RM15000.00, while retaining the initial capital of RM100000.00. This was regarded as a combination of scenarios 2 and 3, relating to increase in monthly expenses and the amount of fund or loan available for investment. The resulting optimal scheme generated a maximum return of RM180626.52 at the end of the planning horizon. If the initial capital was deducted, on the assumption of 10.0% interest rate, the contribution amounted to RM70626.52. As in scenario 3, the mix-cropping scheme suggests the combinations of pak choy and lettuce, but for a longer initial period of six months. Single cropping of lettuce only occurred from month 7 onwards. The amount of cash in hand varied from RM3726.52 in the 8th month to RM47526.52 in the 11th month. Thus, the final accumulated sum at the end of the planning horizon was due to the 10 acres of lettuce (RM148100.00) and RM32526.52 cash in hand in the 12th month.

Real life success stories

Some real life success stories experienced by farmers are reported in the Department of Agriculture website <http://www.doa.gov.my/web/guest/tkpm>. A former government servant in the police force, Mohd Nasir retired in 2001 and embarked on farming, mainly vegetables such as park choy, spinach, cucumber, lady's fingers, lettuce and others. The annual revenue generated was 451 metric tonnes, valued at RM600,000.00. His marketing strategy includes export to Singapore. Another government servant, Ramli Idrus retired in 2004 and started vegetable farming, generated

an annual revenue of 600 metric tonnes, valued at RM900,000.00. A family man, Thiam Kong Seng operated a farm with the help of the family, mainly on spinach, park choy and kangkung (water spinach). Today, his annual revenue is reported to be 898 metric tonnes, valued at RM539,000.00. A small time farmer, Mohd Husin generated 100 metric tonnes, valued at RM236,000.00 annually. Another small time farmer Chu Wei Peng who started his project in 2007 with the help of family members managed to generate an annual revenue of 110 metric tonnes, valued at RM198,000.00. His marketing strategy was limited to local retailers only. Those are just a few success stories of real people reported in the local website. However, the size of each farm was not reported.

Conclusion

In this study, a crop-planning scheme was formulated as a linear programming problem and solved for selected vegetable crops using LINDO. The results indicate promising returns even for a relatively short planning horizon of 12 months and if properly implemented will enhance farm income and provide beneficial contribution to the farming societies.

The results of this study are short run in nature and subjected to the assumptions and limitations imposed. This opens avenues and directions for further exploration. A longer planning horizon of several years might be appropriate if a sufficiently large initial capital is available. A farm size of a few hundred acres can shift farming into a commercial business entity. Incorporating multi-harvest crops into the model increased the complexity but results generated will provide more promising alternatives to the farmers. Of equal interests are the effects of discounting over time, crop rotation for crops with different maturity ages and risk management.

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