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Entanglement Concentration of Entangled Coherent States

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Abstract: With linear optical components to manipulate quantum states of optical fields, the quantum entanglement control and quantum communication can be implemented remotely. By analyzing optical splitter and its use of coherent states, it was found that when the initial state was a direct product of two partially entangled coherent states; another two modes would collapse to a new entangled state using optical splitter and photon-detectors on two modes. Based on this feature, the scheme on entanglement concentration was presented. In this scheme, the two partially entangled coherent states were utilized as the quantum channel, when zero photon and odd photons were detected in two modes separately, and another two modes would collapse to the maximum entangled state, thereby completing the process of the entanglement concentration. It was proved that, no matter how small the initial entanglement is, to distill some maximally entangled states from partially entangled pure states is possible.

Key words: Entangled coherent state; Entanglement concentration; Beam-splitters; Detection of photon

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0 Introduction

Quantum entanglement is a fundamental resource in quantum information, ensuring security of key distribution and efficiency of quantum computing[1-7]. Most of the quantum information protocols work best with maximally and pure entangled states, but entanglement is extremely sensitive to decoherence. Such difficulty imposes an extensive effort in order to recover a fairly good degree of entanglement after its interaction with noise, which is the main cause of the entanglement deterioration. In the last few years various schemes have been implemented in order to overcome this problem.

In an efficient realization of quantum information processing, highly entangled states play a key role. When an entangled state is initially prepared in a non-maximally entangled state, it needs to be distilled to a highly entangled state before using it for quantum information processing. To obtain highly entangled states from less entangled states, researchers have proposed the idea of entanglement concentration and purification, and proved that it is possible to convert two copies of a less entangled state into one copy of a more entangled state by using only local operations and classical communication[8]. Also, some entanglement concentration and purification protocols have been investigated[9-11]. Here we show that the entanglement concentration of entangled coherent states can be realized in a very simple way using only linear optical elements and photon detectors.

1 Entangled coherent states

The coherent state can be used to encode quantum information on continuous variables[12]. In quantum information processing, the entangled coherent state is normally categorized into a two mode continuous variable state. However, there was a suggestion to implement a logical qubit encoding through treating a coherent superposition state, which can make a single mode continuous variable state as a qubit in two dimensional Hilbert space[13] Thereupon, many schemes have been proposed for realizing quantum information processing by using the coherent state.

We first briefly review the bipartite entangled coherent states of the form

$$|\varphi\rangle_{12}=N_\vartheta(\cos\theta|\alpha\rangle_1|\alpha\rangle_2-\sin\theta|-\alpha\rangle_1|-\alpha\rangle_2) \quad (1)$$

where $N_\vartheta = (1-2\mathrm{e}^{-4|\alpha|^2}\cos\theta\sin\theta)^{-1/2}$ is the normalization factor and

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$$|\alpha\rangle = \exp\left(-|\alpha|^2/2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|0\rangle \quad (2)$$

is a coherent state. When $\theta = \pi/4$, Eq. (1) becomes

$$|\varphi\rangle_{12} = \frac{1}{\sqrt{2(1-e^{-4|\alpha|^2})}}(|\alpha\rangle_1|\alpha\rangle_2 - |-\alpha\rangle_1|-\alpha\rangle_2) \quad (3)$$

Now we introduce the even and odd coherent states

$$|\alpha\rangle^{\pm} = \frac{1}{\sqrt{2(1\pm e^{-2|\alpha|^2})}}(|\alpha\rangle \pm |-\alpha\rangle) \quad (4)$$

The Eq. (3) can be rewritten as the form

$$|\varphi\rangle_{12} = \frac{1}{\sqrt{2}}(|\alpha\rangle_1^+|\alpha\rangle_2^- - |-\alpha\rangle_1^-|-\alpha\rangle_2^+) \quad (5)$$

Because $|\alpha\rangle^+$ and $|\alpha\rangle^-$ are orthogonal, we can introduce them as a pair of orthogonal bases to span a two-dimensional subspace of the Hilbert space. Then Eq. (5) is a bipartite entangled pure state, its reduced density matrices is

$$\rho^{(1)} = \rho^{(2)} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \quad (6)$$

and it has the maximally partial entropy

$$S(\rho^{(1)}) = S(\rho^{(2)}) = -\mathrm{tr}(\rho^{(1)}\log\rho^{(1)}) = 1 \quad (7)$$

It is a maximally entangled state.

Now we consider how to generate the entangled coherent state. The experimental setup consists of a lossless 50/50 beam-splitter which is described by

$$\hat{B}_{12} = \exp\left[\mathrm{i}\frac{\pi}{4}(\hat{a}_1^+\hat{a}_2 + \hat{a}_2^+\hat{a}_1)\right] \quad (8)$$

here $\hat{a}_i$ and $\hat{a}_i^+$ (i denote 1, 2) are the bosonic annihilation and creation operators of system i, respectively. It transforms the coherent state $|\alpha\rangle_1|\beta\rangle_2$ as

$$\hat{B}_{12}|\alpha\rangle_1|\beta\rangle_2 = \left|\frac{\alpha+\mathrm{i}\beta}{\sqrt{2}}\right\rangle_1\left|\frac{\beta+\mathrm{i}\alpha}{\sqrt{2}}\right\rangle_2 \quad (9)$$

where the normalization factor is omitted on the right-hand side. Further equipping the beam splitter by a pair of $-\pi/2$ phase shifters described by the unitary operator $\hat{P}_2 = \exp\left(-\mathrm{i}\frac{\pi}{2}\hat{a}_2^+\hat{a}_2\right)$, we can have the operator $BS_{12} = \hat{P}_2\hat{B}_{12}\hat{P}_2$ that transforms the state $|\alpha\rangle_1|\beta\rangle_2$ into

$$BS_{12}|\alpha\rangle_1|\beta\rangle_2 = \left|\frac{\alpha+\beta}{\sqrt{2}}\right\rangle_1\left|\frac{\alpha-\beta}{\sqrt{2}}\right\rangle_2 \quad (10)$$

This transformation plays an important role in the realization of quantum information processing which involves coherent states.

Suppose the input state of the beam splitter BS(1) is a direct product of a coherent superposition states

$$|\Psi\rangle = \frac{1}{\sqrt{N_\alpha}}(|\sqrt{2}\alpha\rangle_1 - |-\sqrt{2}\alpha\rangle_1) \quad (11)$$

and a vacuum state $|0\rangle_2$, where N_α is a normalization factor, and the superposition $|\Psi\rangle$ of two coherent states $|\sqrt{2}\alpha\rangle$ and $|-\sqrt{2}\alpha\rangle$ may be generated by exploiting a coherent state $|\sqrt{2}\alpha\rangle$ propagating through a nonlinear medium[14]. Then the input state of the beam splitter BS(1) is

$$|\Psi\rangle_1 \otimes |0\rangle_2 = \frac{1}{\sqrt{N_\alpha}}(|\sqrt{2}\alpha\rangle_1 - |-\sqrt{2}\alpha\rangle_1)\otimes|0\rangle_2 \quad (12)$$

after interaction with the beam-splitter, the output state emerging from the beam splitter BS(1) is given by

$$|\Phi\rangle_{12} = \frac{1}{\sqrt{N_\alpha}}(|\alpha\rangle_1|\alpha\rangle_2 - |-\alpha\rangle_1|-\alpha\rangle_2) \quad (13)$$

This is a maximally entangled coherent state; we have proven its entanglement entropy is 1.

2 Entanglement concentration

In quantum information, researchers need maximally entangled states for an efficient realization of quantum information processing. If the initially prepared entangled coherent state is in a pure but not maximally entangled state, this state may be distilled to a maximally entangled state before using it for quantum information processing through entanglement concentration. Here we show that for an entangled coherent state the entanglement concentration may be realized by using a beam-splitter and photo-detectors.

Suppose two people, Alice and Bob, share a partially entangled coherent state

$$|\varphi\rangle_{12} = \frac{1}{\sqrt{N_\eta}}(\cos\eta|\alpha\rangle_1|\alpha\rangle_2 - \sin\eta|-\alpha\rangle_1|-\alpha\rangle_2) \quad (14)$$

Here, N_η is a normalization factor given by

$$N_\eta = 1 - \sin(2\eta)\exp(-4|\alpha|^2) \quad (15)$$

the real phase factor η, $0<\eta<\pi/2$, determines the degree of entanglement for state (14), from which we want to distill the maximally entangled state.

After sharing the quantum channel with Bob, firstly Alice prepares another a pair of particles in state $|\varphi\rangle_{34}$ which is in the same entangled state as the quantum channel $|\varphi\rangle_{12}$. The initial state of the whole system consisting of the four subsystems is then given by

$$|\psi\rangle_{1234} = |\varphi\rangle_{12}\otimes|\varphi\rangle_{34} \quad (16)$$

For convenience, now we assume mode 1 on Bob's side and modes 2, 3, and 4 on Alice's side, then state $|\psi\rangle_{1234}$ can be explicitly written as

$$|\psi\rangle_{1234} = \frac{1}{N_\eta}\left[\cos^2\eta|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4 - \frac{1}{2}\sin(2\eta)|\alpha\rangle_1|\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4 - \right.$$

$$\frac{1}{2}\sin(2\eta)|-\alpha\rangle_1|-\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4+$$
$$\sin^2\eta|-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3|-\alpha\rangle_4] \quad (17)$$

Secondly Alice lets modes 2 and 3 enter the input ports of the beam-splitter $BS(2)$. After interacting with the beam-splitter $BS(2)$, the stat of the whole system becomes

$$|\psi'\rangle_{1234}=\frac{1}{N_\eta}[\cos^2\eta|\alpha\rangle_1|\sqrt{2}\alpha\rangle_2|0\rangle_3|\alpha\rangle_4-$$
$$\frac{1}{2}\sin(2\eta)|\alpha\rangle_1|0\rangle_2|\sqrt{2}\alpha\rangle_3|-\alpha\rangle_4-$$
$$\frac{1}{2}\sin(2\eta)|-\alpha\rangle_1|0\rangle_2|-\sqrt{2}\alpha\rangle_3|\alpha\rangle_4+$$
$$\sin^2\eta|-\alpha\rangle_1|-\sqrt{2}\alpha\rangle_2|0\rangle_3|-\alpha\rangle_4] \quad (18)$$

Subsequently Alice makes the two-mode photon number measurement on modes 2 and 3 at her side to finish the entanglement concentration process. When zero photon is detected at 2 mode and n (nonzero) photons are detected at 3 mode simultaneity, the state consist of modes 1 and 4 collapses into

$$|\psi'\rangle_{14}\rightarrow|\alpha\rangle_1|-\alpha\rangle_4+(-1)^n|-\alpha\rangle_1|\alpha\rangle_4 \quad (19)$$

If n being odd, Bob and Alice get an entangled coherent state which is defined by the following

$$|\psi''\rangle_{14}=\frac{1}{\sqrt{N_-}}(|\alpha\rangle_1|-\alpha\rangle_4-|-\alpha\rangle_1|\alpha\rangle_4) \quad (20)$$

where $N_-=2[1-\exp(-4|\alpha|^2)]$ is normalization factor. For state (20), we have proved it is a maximally entangled state, its amount of entanglement is exactly one ebit and the entanglement is independent of the parameters involved. From Eq. (18) we can obtain the probability of finding zero photon in modes 2 and odd photon in modes 3 simultaneity

$$P(0,\text{odd})=|{}_2\langle 0|{}_3\langle\text{odd}|\psi'\rangle_{1234}|^2=$$
$$\frac{\sin^2 2\eta e^{-2|\alpha|^2}}{4N_\eta^2}\frac{(2\alpha^2)^n}{n!} \quad (21)$$

This leads the probability of obtaining the maximally entangled coherent states to be

$$P=\sum_{n=\text{odd}}P(0,\text{odd})=\frac{\sin^2 2\eta e^{-2|\alpha|^2}}{4N_\eta^2}\sum_{n=\text{odd}}\frac{(2\alpha^2)^n}{n!}=$$
$$\frac{1}{4N_\eta^2}\sin^2 2\eta e^{-2|\alpha|^2}\sinh(2\alpha^2) \quad (22)$$

which implies that no matter how small the initial entanglement is, it is possible to distill some maximally entangled states from partially entangled pure states.

3 Conclusion

In summary, we have presented a simple scheme to distill the maximally entangled coherent states from partially entangled pure states. In this scheme, we exploited the important features of the beam splitter transforming entangled coherent state. According to this schemes, we have proved, no matter how small the initial entanglement is, to distill some maximally entangled states from partially entangled pure states is possible. In our entanglement concentration scheme, the key factor is photon number measurement which must be sensitive enough to measure the number of photons and determine whether the number is even or odd. In practice this is difficult, especially for large numbers of photons, but in principle this can be done. With the recent developments in experimental techniques for generating and manipulating the single photon, such possibility is increasing. Besides, in Ref. [15], Xiao-Guang Wang described one method to distinguish even and odd n by coupling the field to a two-level atom through dispersive interaction, which can enhance the possibility to realize this scheme.

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纠缠相干态的纠缠浓缩

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摘　要：利用线性光学元器件对光场量子态进行操纵，可以实现远程的量子纠缠调控和量子通讯. 通过分析光学分束器对相干态光场的作用，发现当初始光场态是两个两部分纠缠态的直乘时，让其中的两模通过光学分束器作用后再对其进行光子计数，另外两模将会塌缩到新的纠缠态. 基于这个特点，提出了一个实现部分纠缠相干态纠缠浓缩的方案. 在这个方案中，两个部分纠缠相干态被用来作为量子信道，通过光学分束器作用后对光场进行光子数探测时，如果测量到光场的两模分别处于奇光子数态和零光子数态，则光场另外的两模将塌缩到最大纠缠态，从而完成纠缠浓缩的过程. 计算结果表明，对于纠缠相干态，无论其初始的纠缠是多么微弱，利用这种方法总有一定的几率可以从中提纯出最大纠缠态.

关键词：纠缠相干态；纠缠浓缩；光束分离器；光子探测