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An effective theory of accelerated expansion

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Abstract: We work out an effective theory of accelerated expansion to describe general phenomena of inflation and acceleration (dark energy) in the Universe. Our aim is to determine from theoretical grounds, in a physically-motivated and model independent way, which and how many (free) parameters are needed to broadly capture the physics of a theory describing cosmic acceleration. Our goal is to make as much as possible transparent the physical interpretation of the parameters describing the expansion. We show that, at leading order, there are five independent parameters, of which one can be constrained via general relativity tests. The other four parameters need to be determined by observing and measuring the cosmic expansion rate only, $H(z)$. Therefore we suggest that future cosmology surveys focus on obtaining an accurate as possible measurement of $H(z)$ to constrain the nature of accelerated expansion (dark energy and/or inflation).

Keywords: dark energy theory; inflation.

Contents

1. Introduction

It has now been experimentally established [1, 2, 3, 4] that the Universe is currently accelerating and current knowledge indicates that eventually will enter a deSitter phase. There is also experimental evidence that large–scale structures in the early Universe were seeded by non-casual perturbations [5]; the best theoretical model to explain this is inflation, which consist of an accelerating de-Sitter phase in the early Universe. Therefore, there is now strong experimental evidence that the Universe has experienced periods of accelerated expansion, however we have no satisfactory explanation of what has been driving it and of the physical mechanism behind it.

Progress can arise both from the theoretical and observational front. It can happen that a fundamental theory is found that tells us what the nature of inflation and/or dark energy are. On the other hand, it could be that we need to exploit astronomical observations in order to zoom in into the detailed properties of the expansion before we can discriminate among competing theories and shed light on the mechanism driving the expansion.

If the latter scenario is realized the question that arises is: how can we determine in the most model-independent way, the nature of acceleration? Can this be done in such a way that observational constraints can have an easy and transparent interpretation in terms of the properties of a possible underling physical mechanism? The most widely used approach is to assume that the expansion is driven by a scalar field with constant equation of state $p = w\rho$ and use observations to find w (p, ρ are the pressure and energy density of the scalar field). Of course, in this framework, a cosmological constant corresponds to $w = -1$ while scalar fields with dynamics deviate from this value. It is clear that one cannot assume w to be constant with time, and that a more general scenario would require a reconstruction of $w(t)$. It is important to keep in mind that this description is a drastic simplification as it does not cover all possible scenarios (see e.g.,[6]), and there could be models where p, ρ are not even defined, but from observations of any expansion history one could always reconstruct an effective $w(t)$. In this case the $w(t)$ would have no physical meaning in terms of properties of a fluid or a scalar field and would be of difficult or ambiguous theoretical interpretation. Even with this drastic simplification, it is very challenging observationally to determine $w(t)$ [6]. To circumvent this problem, the community has proposed several parameterizations: some more closely related to the underlying physics (e.g.,[6]) and others purely phenomenological ([7, 8] and refs therein). All these parameterizations make compromises in order to adjust the number of parameters to the observables in the sky. Even if we do not consider this limitation, it is unclear how closely related they are to tell us something about the underlying physics driving the expansion. Moreover degeneracies can be found such that a constant equation of state can be mimicked by a variable one in these parameterizations (e.g.,[9]).

Given the above limitations, it is worth asking the question of how to build a general theory of accelerated expansion that captures as much as possible the physics and thus avoids arbitrary parameterizations. This is what we set-up to do in this paper: we build an effective theory of cosmological accelerated expansion. While the philosophy of our approach can be recognized in the works of e.g., Ref [10, 11] we go beyond their approach and propose a physically–motivated way to truncate the effective theory of expansion.

2. A generic minimal set up

The simplest theory of expansion involves, besides gravity, a single canonically normalized expansion field described by the leading Lagrangian density

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}A(\varphi)m_p^2R - \frac{1}{2}B(\varphi)m^2g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - V(\varphi)\,, \tag{2.1}$$

where m_p is the Planck mass and m is the scalar field mass. This minimal setup follows from the principle of general covariance and locality for both sectors: gravity and matter. In addition, to preserve the fundamental features of the scalar theory in flat space-time limit, we also require gauge invariance to hold.

As is customary in the construction of effective field theories, we have written Eq. (2.1) as a function of a dimensionless scalar field, φ, with $A(\varphi), B(\varphi)$ being unknown, generic, functions. This facilitates the classification of the derivative terms but, as we shall show, spoils any predictability.

2.1 Lagrangian corrections and power–counting

Effective field theories have become a standard tool for the phenomenological description of physical processes involving pseudo-Goldstone bosons. This approach is grounded in a few simple assumptions: *i)* There is a mass gap in the spectrum, μ, for all states except for the

Goldstone bosons themselves. From now on, μ will denote the intrinsic cut off of the theory, i.e., the scale where we expect the break down of the effective theory. *ii)* The underlying theory has an exact symmetry that is spontaneously broken. This breaking allows us to construct a Lagrangian whose terms are ordered in powers of derivatives, $\Delta\varphi \ll \mu$, and *iii)* A remaining symmetry is explicitly broken giving rise to the potential term modulated by the coefficients $\lambda_i \ll \mu$.

Under these assumptions, one can construct a low-energy effective theory for the pseudo-Goldstone bosons organized in powers of p/μ and λ_i/μ, where p is the typical momentum of the low-energy processes.

The use of effective theories can be split into two categories. In one class the spectrum and the symmetries of the theory are known but the degrees of freedom are strongly correlated and thus it is impossible to tackle their dynamics by perturbation method. Here we find the case of QCD or superconductivity systems. Then there is a second class where we only have partial knowledge of the physical picture, as in the case of the electroweak physics at very high-energy, where our knowledge of the spectrum can be incomplete. The case of the expansion of the Universe lies in the second of these two main categories, in the context of cosmology see Ref. [10, 11].

The philosophy of our approach can be recognized in the works of e.g., Ref. [10, 11], although we attempt to go beyond their treatment. Our setup is based on two unrelated ansatze from where our treatment will differ from the above references: *i)* we start discussing the natural choice of the power–counting in a *predictive theory of expansion*. We stress first of all that the power–counting in an effective theory is an *assumption* that must be phenomenologically validated because any a priori, judicious choice in the counting is plausible. And *ii)* we shall assume the existence of an energy scale in the problem where an explicit symmetry is broken [12]. The potential that we obtain afterwards is assumed to be almost flat and for such reason any point is valid as a vacuum. This is motivated by the fact that a $LCDM$ model is a good fit to the data and that observational constraints indicate that the Hubble parameter is very close to being constant during inflation; so far there is no evidence for deviations from a flat potential (slow–roll for inflation). In fact the effective theory approach is built to describe the range of time and scales accessible by observations, not e.g., the full potential. Thus, for example, for inflation, it will describe the $\sim 10-20$ efoldings accessible to observations and for dark energy only the few efoldings since dark energy started affecting the expansion.

Eq. (2.1) is a complete description to leading order and involves different generic functions which one could try to approximate with truncated expansions. In the case of accelerated cosmic expansion it is not however clear a priori how to implement the truncation. We thus begin by motivating our choice of power counting. To do so we shall bear in mind the construction of the low-energy QCD effective theory. Although we restrict our argument to the leading-order approximation, this can be extended to higher-order terms. Contrary to QCD, in a theory of expansion one lacks a physical observable directly related to the scattering of the scalar field. This has some shortcomings for the construction of the effective theory: in low-energy QCD the coefficients modulating the operators in the Lagrangian density e.g., those equivalent to $A(\varphi)$ and $B(\varphi)$ in low-energy QCD, can be expanded around its vacuum as an infinite polynomial in the dimensionless pseudo-Goldstone boson field. All these terms have the same weight in the series and therefore they can not be truncated without any further criteria. In the case of low-energy QCD the truncation of the polynomial is automatically achieved by computing the contribution to a given scattering S matrix element with a fixed number of external fields [13], this pins down a finite number of terms in the expansion. Due to the lack of experimental information for the scalar scattering processes in the accelerated cosmic expansion, one can not use the same procedure and without any further insight one has to deal with an infinite number of terms. A natural way to avoid this situation is to maintain the power–counting $[\varphi] = \mathcal{O}(\mu)$. Note that Refs.[10, 11] adopted the treatment of would be pions, using a pseudo-Goldstone boson field their "natural" choice is instead $[\varphi] = \mathcal{O}(1)$.

Adopting the power–counting $[\varphi] = \mathcal{O}(\mu)$ we obtain an expression where the derivative terms are just a reordering of those in the series obtained in [10]. In addition some of the effects due to the scalar field are suppressed and thus push their effect to higher orders w.r.t. the expansion in [10].

Applying this rationale, Eq. (2.1) leads directly to

$$\frac{\mathcal{L}_0}{\sqrt{-g}} = \frac{1}{2}m_p^2 R - \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - V(\varphi)\,. \tag{2.2}$$

One can argue that Eq. (2.2) can be obtained from Eq. (2.1) by a conformal transformation and redefinitions of the scalar field, without mentioning any explicit power–counting. However, in this case, the problematic just explained above is shifted to the new potential that will contain an infinite number of equally important terms and we have no control over how to truncate a possible expansion. On the other hand, in the approach presented here, Eq. (2.2) acquires a different interpretation: we have a self consistent and physically motivated way to expand the potential and corrections to the Largangian.

In the remainder of this section we shall organize the corrections to Eq. (2.2). We can interpret our results by comparing with Ref. [10] and then identifying the relevant derivative terms consistent with our power–counting. As we have suppressed, via the power–counting, the corrections due to the scalar field, it is not surprising to obtain that the main effects are due to gravity:

$$\frac{\Delta\mathcal{L}_1}{\sqrt{-g}} = f_7 R^2 + f_8 R_{\mu\nu}R^{\mu\nu} + f_9 C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \dots \tag{2.3}$$

where ellipsis stand for operators further suppressed by powers of $1/\mu$ and we have discarded parity violating operators. The natural values of the constants are $f_7 \sim f_8 \sim f_9 \sim \mathcal{O}(1)$. Notice that the coefficients in Eq. (2.3) are just numerical factors and not functions of φ (as it happens in [10]), but when one computes next-to-leading corrections they acquire a logarithmic dependence on the scale μ. At this point we have two *equivalent* ways to handle the problem : *i)* if we were after the equation of motion themselves and not the perturbations to them, we could substitute the last term in Eq. (2.3) involving the Weyl tensor by a linear combination of curvature bilinears that appears in the Gauss-Bonnet identity. This, in turn, is a total derivative and does not affect the field equation of motion

[14]. The final result amounts to substitute the term modulated by f_9 for those of f_7 and f_8. We would obtain, in this case, a generalized gravity theory which is equivalent to a multi-scalar-tensor gravity action with four derivatives. It is known, in some specific examples, that the associated vacuum is unstable [15]. Or *ii)* we can choose the f_9 term over the terms modulated by f_7 and f_8 (because our effective theory is only valid to describe small deviations around a flat potential). This choice allows us to perform a conformal transformation disentangling the gravity and the scalar field in a neat fashion. Unfortunately, as the Weyl tensor will vanish in de-Sitter like spaces, this will entail the disadvantage that only corrections and not gravity effects itself can be computed directly. We will adopt the second option to make easier the comparison of our results with previous work in the literature [10].

The next term we focus on is the potential. In the absence of any hint on the symmetries realized by the scalar field we are forced to write the most general polynomial functional for the potential. Hence, in full generality we can write

$$V(\varphi) = \lambda_1\varphi + \lambda_2\varphi^2 + \lambda_3\varphi^3 + \lambda_4\varphi^4 + \ldots \tag{2.4}$$

where the ellipses stand for subleading terms in $1/\mu$. The natural values for the coupling constants in four dimensions are $\lambda_n \sim \mathcal{O}(\mu^{4-n})$. Although it could be included, here we refrained from using a constant term in the potential that serves as a cosmological constant: in fact an effective theory is useful if we do not have a priori a model for the expansion, such as e.g., a cosmological constant. In the absence of such theory, the effects of a constant terms in the potential must be mimicked by the scalar interaction itself and not added beforehand. The stability of the model is ensured at $\varphi \approx 0$ by the analysis of [16]. Note that choosing some ad-hoc specific values for the parameters the potential can become that of the chaotic inflation [17], the Standard Model Higgs [18] or the minimal inflation scenario [19, 20].

Following this logic, we look for all the possible terms which give rise to mixed product of derivatives with monomials in the scalar field

$$\frac{\Delta\mathcal{L}_2}{\sqrt{-g}} = R(\alpha_1\varphi + \alpha_2\varphi^2) + g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}(\beta_1\varphi + \beta_2\varphi^2) \tag{2.5}$$

where, for completeness, we have displayed the leading and subleadings contribution, $\alpha_1 \sim \mathcal{O}(\mu)\,, \alpha_2 \sim \mathcal{O}(1)\,, \beta_1 \sim \mathcal{O}(1/\mu)\,, \beta_2 \sim \mathcal{O}(1/\mu^2)$.

Collecting the leading terms from Eqs. (2.2), (2.3) and (2.5), we obtain

$$\frac{\Delta\mathcal{L}}{\sqrt{-g}} = W(\varphi)\frac{m_p^2}{2}R + f_9 C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} - \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - V(\varphi)\,, \tag{2.6}$$

with $W(\varphi) := 1 + \frac{2}{m_p^2}\alpha_1\varphi + \frac{2}{m_p^2}\alpha_2\varphi^2$ and $V(\varphi)$ given in Eq.(2.4).

Due to the power–counting, this expression has the virtue that corrections to the scalar kinetic energy are a subleading effect with respect to the potential, thus only if one is compelled to include higher-order corrections to Eq. (2.6) can formally find terms similar to those obtained in a kinetically driven inflation [21] as

$$\frac{\Delta\mathcal{L}_3}{\sqrt{-g}} = (g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu})^2\ . \tag{2.7}$$

Moreover, if we demand that the accelerated expansion field enjoys a shift symmetry, $\varphi \to \varphi + \text{const}$, the coefficients λ_i and α_i responsible of the explicitly symmetry breaking must vanish and the action involves only $\partial_\mu \varphi$ terms. Conversely, if the symmetry breaking terms are small, the radiative corrections will stay under control keeping the potential sufficiently flat.

2.2 A trip to the Einstein frame

The addition of the explicitly symmetry breaking terms has brought the initial Lagrangian density written in the Einstein frame to the final form, Eq. (2.6), that corresponds to a Lagrangian density in the Jordan frame. This physical system can be mapped onto a scalar tensor gravity in addition to the field φ. We do not attribute any special role to the frame in which the theory is formulated [22] and for simplicity reasons we would like to undo this non-minimal coupling to gravity [23]. To do this one performs a conformal transformation at the action level [24] , $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ where $\Omega^2 = W(\varphi)$. That leads to the Lagrangian density

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{m_p^2}{2}\tilde{R} + f_9 \widetilde{C}_{\mu\nu\alpha\beta}\widetilde{C}^{\mu\nu\alpha\beta} - \frac{1}{2}\left[\frac{1}{\Omega^2} + \frac{6(\alpha_1 + 2\alpha_2\varphi)^2}{m_p^2\Omega^4}\right]\tilde{g}^{\mu\nu}\widetilde{\partial}_\mu\varphi\widetilde{\partial}_\nu\varphi - \frac{V(\varphi)}{\Omega^4}\,, \quad (2.8)$$

where we have used the fact that under the action of the conformal transformation the Weyl tensor transform as ${C_{\mu\nu\alpha}}^{\beta} = {\widetilde{C}_{\mu\nu\alpha}}{}^{\beta}$.

One can make a further step and normalize the kinetic term for the scalar field in Eq. (2.8) by the differential redefinition

$$\Omega^4\left(\frac{dq}{d\varphi}\right)^2 = \Omega^2 + \frac{6}{m_p^2}(\alpha_1 + 2\alpha_2\varphi)^2\,. \quad (2.9)$$

With this one obtains finally the familiar form

$$\mathcal{S} = \int d^4\,x\sqrt{-\tilde{g}}\left(\frac{m_p^2}{2}\tilde{R} + f_9\widetilde{C}_{\mu\nu\alpha\beta}\widetilde{C}^{\mu\nu\alpha\beta} - \frac{1}{2}\tilde{g}^{\mu\nu}\widetilde{\partial}_\mu q\widetilde{\partial}_\nu q - U(q)\right)\,, \quad (2.10)$$

with $U(q) := \frac{V(\varphi)}{\Omega^4}$. In the limit of exact shift symmetry, in the previous expression the quantities with tilde become the quantities before conformal transformation and the potential goes to zero; we thus recover the expression of [10]. Thus the α_i parameters can be seen as the responsible for the spin-0 deformations of the metric; if one writes the corrections to the metric field as $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \simeq \eta_{\mu\nu} + \rho_{\mu\nu} = (1+\Omega^2)\eta_{\mu\nu} + h_{\mu\nu}$, the canonical action for the spin-2 field is not obtained from the field $h_{\mu\nu}$, but it is instead given by $\rho_{\mu\nu}$, i.e., the conformal transformation mixes the degrees of freedom corresponding to the scalar and the tensor modes. As mentioned above, we expect the parameters α_i to be small quantities implying that the spin-0 deformations are due to the symmetry breaking. In addition small values ensure the stability of the solution [25].

After integrating Eq. (2.9) one gets a dependence for the new field in terms of the initial one, which is reminiscent of what happens sometimes in string theory e.g., [26]. Although appealing by itself, this behavior is generic and comes into play just because of the conformal transformation. As we are after the first corrections due to the λ_i, α_i, f_i coefficients, we discard, after integration, any non-linearities due to them in the potential, obtaining that, from the initial seven free parameters in Eq. (2.6) ($\lambda_{1,\ldots,4}$, $\alpha_{1,2}$, f_9), only five of them must be taken into account: four in the potential, λ_i, and one in the pure gravity side f_9. All other parameters are subleading or only appear as non-linear corrections.

The numerical values for the coefficients λ_i and f_i at a given scale are arbitrary and not constrained by symmetry. Their actual value can only be found by matching the theoretical expression for some specific observables with its experimental value. This will be our next task.

3. Reconstruction procedure

If we had chosen f_7 and f_8 over the f_9 term, in principle constraints could be obtained by some pure test of general relativity. For instance, the Eöt-Wash experiments provide the strictest bound, $|f_7| \lesssim 10^{-9}m^2$, while astronomical bounds are weaker $|f_7| \lesssim 0.6 \times 10^{18}m^2$ derived from the orbit of Mercury [27] and $|f_7| \lesssim 1.7 \times 10^{17}m^2$ from binary pulsars moving in a circular orbit [28]. In cosmological tests of gravity it is customary to test for deviations from GR at different scales and not to assume any relation between solar system constraints and cosmological constraints. One of the consequences of our approach is that these parameters describing modifications of GR, have only a logarithmic dependence on scale. In particular $f_7 \sim f_{7,0} + \log(k/\mu)$ where $f_{7,0}$ denotes local measurement and $k \sim 1/r$ with r denoting separation. To the best of our knowledge there are no estimates of the values of f_8.

Here, we have chosen instead to consider f_9, which could also, in principle, be constrained by tests of gravity. In particular the f_9 term affects the non-linear bispectrum of density perturbations and thus could be constrained from analysis of forthcoming large-scale structure surveys (Gil-Marin et al., in preparation).

What we have achieved is a direct connection between the parameters describing the physics of the effective theory and the observables. In fact, let us start from the Friedman equation and the Klein- Gordon equation. To derive them we assume that the components of $\tilde{g}_{\mu\nu}$ correspond to that of the physical space that, as usual, would be described by an isotropic, flat homogeneous space-time

$$ds^2 = -N(t)^2dt^2 + a(t)^2\delta_{ij}dx^idx^j\ , \tag{3.1}$$

where $N(t)$ is the lapse function and for the moment we set it to unity. We also restrict the analysis to classical field configurations that do not break neither homogeneity nor isotropy, $q = q(t)$. From Eq. (2.10) one can obtain the energy-momentum tensor

$$\tilde{G}_{\mu\nu} = \tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{R}\,\tilde{g}_{\mu\nu} = q_{,\mu}q_{,\nu} - \left[\frac{1}{2}q_{,\alpha}q_{,}^{\ \alpha} + U(q)\right]\tilde{g}_{\mu\nu} + T^B_{\mu\nu}, \tag{3.2}$$

where $T^B_{\mu\nu}$ is the background energy-momentum tensor truncated consistently with the effective order we work. Comparing the matter content of Eq. (3.2) with the energy-momentum tensor of a perfect fluid in thermodynamic equilibria one realizes that the pressure and the energy density can be cast as $\frac{1}{2}q_{,\alpha}q_{,}^{\alpha} - U(q) = p_q$, $\frac{1}{2}q_{,\alpha}q_{,}^{\alpha} + U(q) = \rho_q$, respectively. Notice that the extrema of p_q with respect to the kinetic energy correspond to the same equation of state as that of a cosmological constant: $\rho_q + p_q = 0$.

The Friedman's equations are obtained straightforward once we identify the rhs of Eq. (3.2) as $T^q_{\mu\nu} + T^B_{\mu\nu}$. The resulting field equations are the standard for a scalar-tensor cosmology

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\rho_m + \rho_q\right) , \quad \frac{\ddot{a}}{a} = \frac{1}{6}(\rho_m + 3p_m + \rho_q + 3p_q) , \tag{3.3}$$

and

$$\ddot{q} + 3H\dot{q} - U' = 0 . \tag{3.4}$$

Dotted quantities stand for derivatives w.r.t. time and prime quantities denote derivatives w.r.t. the field q; ρ_m denotes the matter density parameter and we assume that the Universe is dominated by collisionless, pressureless matter, $p_m = 0$.

In the case of inflation, since $\rho_m = 0$ the set of equations simplifies and one reproduces the standard approach. In the case of dark energy the presence of the matter density complicates finding a solution.

Nevertheless, given a set of parameters λ_i, note that the first Friedman equation and the Klein Gordon equation can be rewritten in terms of redshift using the fact that $H(z) = -1/(1+z)dz/dt$ and then can be combined in a single differential equation for the field as a function of redshift $q(z)$:

$$\overline{16\left(\frac{dq}{dz}\right)^2 (1+z)^2 U' - \left[U(1+z) + \rho_{m,0}(1+z)^4\right]\frac{dq}{dz} - U' = 0,} \tag{3.5}$$

where $\rho_{m,0}$ denostes the present day matter density.

Once the solution of this equation, $q(z)$ (where only the positive solution is the relevant one) has been obtained, it can be substituted in the expressions for U' and thus of $H(z)$. $H(z)$, being the Universe expansion rate, is the key observable which can be obtained from galaxy surveys via e.g., the standard clocks approach [9, 30] or baryon acoustic oscillations measurements ([29] and references therein).

In practice however the λ_i are not known and one would like to constrain them from $H(z)$ measurements, in other words, in practice, one wants to solve the "inverse problem" (from $H(z)$ constraints to constraints on λ_i) while we have shown so far that the "direct problem" has a solution (from the λ_i to the observable $H(z)$). There are two approaches to solve the "inverse problem": *i)* Markov Chain Monte Carlo. This is the standard workhorse in cosmology today for parameter inference. This problem is perfectly suited to be addressed with this technique. The actual constraints will be presented elsewhere (Jimenez et al. in preparation) *ii)* An exact analytical approach which we present below.

3.1 Analytical solution

In this approach, and similarly to [6], we use equivalent quantities to the inflationary–flow parameters $\{\epsilon_n\}$ [21] that, in turn, can be cast as functions of H and its derivatives. They are defined recursively as $\epsilon_{n+1} = \frac{d\log|\epsilon_n|}{dN}$, $n \geq 0$, $N = \log[a(t)/a(t_0)]$ is the number of efoldings since the time t_0 and $\epsilon_0 := H(N_0)/H(N)$.

In the case of inflation, although they are theoretical quantities, they can be expressed in terms of directly measurable quantities as the scalar density perturbation index, n, and the tensor gravitational perturbation index, n_g, through the relations $n = 1-4\epsilon_1-\epsilon_2$, $n_g = -2\epsilon_2$ [32]. In the case of dark energy they can be related to the Universe expansion (Hubble) rate [6]. For the time being we shall make use only of the first two parameters ϵ_1, ϵ_2. One can, obviously consider higher-order terms, although in reality experimental data will be increasingly insensitive to higher derivatives of H; thus we refrain from using them and look for other quantities that contain only ϵ_1 and ϵ_2.

We shall split the theoretical input into two classes: the first one which are obtained directly from the definitions of the inflationary–flow parameters in terms of the potential

$$\epsilon_1 = \frac{m_p^2}{2}\left(\frac{U'}{U}\right)^2, \quad \epsilon_2 = m_p^2\left(\frac{(U')^2-2UU''}{U^2}\right), \tag{3.6}$$

and a second more elaborated class of relations obtained following the lines of [6] as we shall outline here.

Using both Friedman equations one can rewrite the first inflationary–flow parameter in terms of the energy density and pressure as $\epsilon_1 = 1 - \frac{\ddot{a}}{a}H^{-2} = \frac{3}{2}\frac{\rho_m+\rho_q+p_m+p_q}{\rho_m+\rho_q}$. The above expression together with the first Friedman equation leads to

$$\frac{1}{2}\dot{q}^2 = \epsilon_1\frac{H^2}{\kappa^2} - \frac{1}{2}(\rho_m + p_m)\,, \quad U(q) = (3-\epsilon_1)\frac{H^2}{\kappa^2} + \frac{1}{2}(\rho_m - p_m)\,, \quad \kappa^2 = \frac{8\pi}{m_p^2}\,. \tag{3.7}$$

Integrating the former of these relations with respect to time we can obtain the field in terms of the observables.

The last relation we need makes use of the scalar equation of motion. Rewriting Eq. (3.4) as $U' = -(3H\dot{q}^2 + \dot{q}\ddot{q})/\dot{q}$ and using Eq. (3.7), the derivative of the potential w.r.t. the field becomes

$$\begin{aligned} U'(q) &= -\frac{3}{2\sqrt{\pi}}m_p H^2\epsilon_1^{1/2}\left[1 - \frac{\kappa^2}{2H^2\epsilon_1}(\rho_T + p_T)\right]^{-1/2} \\ &\times \left\{1 - \frac{\epsilon_1}{3} + \frac{\epsilon_2}{6} - \frac{\kappa^2}{6H^3\epsilon_1}\left[3H(\rho_T + p_T) + \frac{1}{2}(\dot{\rho}_T + \dot{p}_T)\right]\right\}. \end{aligned} \tag{3.8}$$

This expression together with Eq. (3.7) and (3.6) determine the system of five equations from where we determine the set $\{\lambda_1, \ldots, \lambda_4, q(t)\}$. Compared with the "Monte Carlo" approach where only the $H(z)$ determination is needed, in this approach one needs to determine observationally H, dH/dz and d^2H/dz^2 which is very challenging.

Although an analytical treatment for the full system can be done, the results are too cumbersome to be presented in a reasonably manageable way. Instead we report an implicit system of equations (where we have used $m_p = 1$):

$$
\begin{aligned}
\lambda_1 &= 4\sqrt{2}\frac{q^2(2\epsilon_1-\epsilon_2)-4}{16+q^4(2\epsilon_1-\epsilon_2)^2}q^3\epsilon_1^{1/2}(\lambda_3+3q\lambda_4)+\frac{16q^2(3\lambda_3+8q\lambda_4)}{16+q^4(2\epsilon_1-\epsilon_2)^2}\,,\\
\lambda_2 &= -4\sqrt{2}\frac{q^4\epsilon_1^{1/2}(\lambda_3+3q\lambda_4)(2\epsilon_1-\epsilon_2)}{16+q^4(2\epsilon_1-\epsilon_2)^2-8q^2(2\epsilon_1+\epsilon_2)}\\
&\quad +4q\frac{\left[q^2(14\epsilon_1+5\epsilon_2)-12\right]\lambda_3+3q\left[q^2(10\epsilon_1+3\epsilon_2)-8\right]\lambda_4}{16+q^4(2\epsilon_1-\epsilon_2)^2-8q^2(2\epsilon_1+\epsilon_2)}\,,\\
\lambda_3 &= \frac{3}{8\sqrt{\pi}q^2}\frac{\dot{p}_m+\dot{\rho}_m+6H(p_m+\rho_m)}{p_m+\rho_m-2H^2\epsilon_1}\epsilon_1^{-1/2}(4H^2\epsilon_1-2p_m-2\rho_m)^{1/2}\\
&\quad +\frac{3}{\sqrt{2\pi}q^2}H^3\epsilon_1^{1/2}+\frac{1}{q^3}\left\{2p_m-3q\lambda_1-2[2H^2(\epsilon_1-3)+q^2\lambda_2+\rho_m]\right\}\,,\\
\lambda_4 &= \frac{1}{4\sqrt{2\pi}q^3}\frac{3\dot{p}_m+3\dot{\rho}_m+2H[H^2\epsilon_1(2\epsilon_1-\epsilon_2-6)+9p_m+9\rho_m]}{\epsilon_1^{1/2}(2H^2\epsilon_1-p_m-\rho_m)^{1/2}}\\
&\quad +\frac{3}{q^2}H^2\epsilon_1+\frac{1}{2q^4}(4q\lambda_1+2q^2\lambda_2+3\rho_m-18H^2-3p_m)\,.
\end{aligned}
\tag{3.9}
$$

4. Constraints on the parameter space

Although it might look that there is total freedom in choosing the values for the unknown parameters this is not the case. There are several consistency relations that the effective theory must fulfill; these restrict the range of the parameters numerical values.

4.1 The fate of gravitational higher-order corrections

One particular case of accelerated expansion is inflation. This is believed to be a semiclassical effect. In that case quantum gravity corrections can be neglected because they are small in the following sense: curvature corrections must be suppressed at the Planck scale. This is not always the case and in fact some inflationary models based on pseudo-Goldstone fields suffer from this pathology. We do not dwell more on this issue but instead estimate roughly some constraints that the pure gravitational part must fulfill in order to make sense for the perturbative expansion. In particular, we shall demand that the contribution to the energy density coming from higher orders terms is subleading. For that purpose we focus on the pure gravitational part of Eq. (3.2) viz. $\tilde{G}_{\mu\nu}-T^B_{\mu\nu}=0$ with

$$
\begin{aligned}
-T^B_{\mu\nu} = &\; 2f_7R\left(\tilde{R}_{\mu\nu}-\frac{1}{4}\tilde{R}\tilde{g}_{\mu\nu}\right)+2f_7(\tilde{g}_{\mu\nu}\Box\tilde{R}-\nabla_\mu\nabla_\nu\tilde{R})+2f_8\left(\tilde{R}_{\mu\rho}\tilde{R}^\rho_\nu-\frac{1}{4}\tilde{R}^{\rho\sigma}\tilde{R}_{\rho\sigma}\tilde{g}_{\mu\nu}\right)\\
&+f_8\left(\Box\tilde{R}_{\mu\nu}+\frac{1}{2}\Box\tilde{R}\tilde{g}_{\mu\nu}-2\nabla_\mu\nabla_{(\rho}\tilde{R}_{\nu)}{}^\rho\right)\,.
\end{aligned}
\tag{4.1}
$$

Under the previous requirement of absolute convergence for the effective series the $\{0,0\}$ component leads to the inequality

$$
(26f_7-6f_8)\dot{H}^2+2(4f_7-f_8)\dddot{H}+2(16f_7-f_8)H\ddot{H}-6(2f_7+f_8)H^2\dot{H}\ll H^2\,. \tag{4.2}
$$

If observational constraints can be obtained from cosmological observations that lead to values of $H, \dot{H}, \ddot{H}, \dddot{H}$, then, from the above equation we can constraint the values of f_7 and f_8.

4.2 Inflation

Most inflation models require a transplankian scale for the scalar field $q \gg m_p$. To circumvent this problem some non-minimal scalar-gravity coupling has been advocated recently [31]. Although this approach is promising for Higg-like potential models, we want to stress that it can not be implemented in our effective field theory because *it mixes gravitational terms of different effective orders* within our power-counting set up.

In order to ascertain the correctness of the framework developed in this paper, we check whether this situation can be present. Using Eq. (3.1) into Eq. (2.10) and varying the action with respect to the lapse function and then setting it afterwards to 1 by time reparameterization invariance, we obtain the Hamiltonian constraint $H^2 = \frac{1}{6m_p^2}\left(\dot{q}^2 + 2U\right)$. The slow–roll conditions can be read automatically from the previous expression and Eq. (3.4): $\dot{q}^2 \ll 2U$ and $|\ddot{q}|^2 \ll 3H|\dot{q}|$. These two expressions imply $H^2 \approx \frac{1}{3m_p^2}U\,, \dot{q} \approx -\frac{V'}{3H}$ from where one can deduce

$$-\frac{\dot{H}}{H^2} \approx \frac{m_p^2}{2q^2}\left(\frac{\lambda_1 + 2\lambda_2 q + 3\lambda_3 q^2 + 4\lambda_4 q^3}{\lambda_1 + \lambda_2 q + \lambda_3 q^2 + \lambda_4 q^3}\right)^2 \ll 1\,. \tag{4.3}$$

The standard constraint, for example in chaotic inflation, to obtain enough number of efoldings, implies that the inequality can be only achieved if $q \gg m_p$. However, from the above equation, it is now clear how to obtain sub-planckian inflation, for instance, $-\lambda_1 \approx \lambda_2 \approx \lambda_3 \gg \lambda_4$ will produce enough number of efoldings and inflate without having to deal with transplanckian scales, $q \ll m_p$.

5. Conclusions

We have developed an effective theory of accelerated expansion with the aim of making a direct connection between cosmological observables and parameters describing the physics behind the expansion. We have adopted what we propose as the "natural" power counting to expand (and truncate) the expression for the Lagrangian describing the accelerating Universe. Our choice is motivated by being natural for scalar fields.

In doing so we have discovered that only five parameters are needed to fully described the (effective) theory at leading order. These parameters are readily connected to the physics behind expansion and thus are the most natural set an observer should attempt to determine. In particular one parameter describes deviations from standard –general relativity– gravity and the other four completely determine the expansion history. Although our treatment is general to both inflation and dark energy, we have concentrated on the case for dark energy which yields less standard results.

Our conclusion is that one should do observations that test deviations from general relativity (via e.g., constraints on the growth of cosmological structure or higher-order correlations of the matter density field), and measure the Hubble parameter $H(z)$ with the best possible accuracy in at least four redshift bins. Once $H(z)$ has been measured, Eq. (3.5) can be used to obtain the physical parameters of the Lagrangian that describes the accelerated expansion.

Note that, even neglecting modifications to General Relativity, if from observations one wanted to reconstruct in a non-parametric way the potential of the accelerating field, both the Hubble parameter $H(z)$ and is derivative $dH(z)/dt$ would be needed as shown in [6]. Here, the effective theory expansion already provides a parameterization of the potential as a function of the field, and thus only a measurement of $H(z)$ and the matter density are needed.

A priori we have no restrictions on the actual values of the unknowns in the theory, beside imposing the validity of the effective theory expansion. If, as an outcome of confronting the theory with experimental data in Eq. (3.5) some of these parameters turns to be numerically suppressed, we would have learned something about the pattern of the explicit symmetry breaking for the scalar field. If, on the contrary, it turns out their numerical values do not match the a priori power–counting estimates, this would be an indication of a break down of the proposed approach and that an alternative power–counting must be implemented. Although we have tackled only the leading contribution, perturbations on any of both fields can be computed in a straightforward manner along the lines of Ref. [10]. For instance, the gravitational corrections turns to be identical to those found by Ref. [10]. In our case the speed of sound equals unity, $c_s = 1$. Corrections to this value come from higher order operators as in Eq. (2.7).

Future cosmology surveys promise to provide measurements of the expansion rate – either directly measuring H(z), or closely related quantities such as the angular diameter distance or the luminosity distance as a function of redshift– with percent precision over a wide range of redshift ($z \sim 3$) and over tens of redshift bins. This will offer a unique opportunity to gain insight into the mechanism of cosmic acceleration.

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