SUBSTITUTION POSSIBILITIES FOR UNPRICED NATURAL RESOURCES: RESTRICTED COST FUNCTIONS FOR THE CANADIAN METAL MINING INDUSTRY

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Abstract—The effects of resource depletion on economic growth depend critically on the elasticities of substitution between non-renewable natural resources and reproducible inputs. Estimation of the elasticities of substitution for natural resources has been hindered by the absence of data on their prices, which results from the prevalence of vertical integration in natural resource industries. In this paper we use the theory of restricted cost functions to develop a general procedure for estimating substitution possibilities for unpriced inputs. Estimation of the model with data for the Canadian metal mining industry indicates that the elasticities of substitution for the natural resource, metallic ore, are equal to unity.

I. Introduction

THE effects of natural resource depletion on economic growth have been shown to depend critically on the elasticities of substitution between natural resources and reproducible inputs. For example, Nordhaus (1973) showed that the pessimistic simulation results of the "Limits to Growth" models (Meadows et al., 1972) could be reversed by assuming a unitary elasticity of substitution between natural resources and reproducible capital. Similarly, Dasgupta and Heal (1979) demonstrated that economic decay could be avoided even in the very long run if the asymptotic elasticity of substitution were greater than unity, or if it were equal to unity and the output elasticity of natural resources were less than that of reproducible capital.

The elasticities of substitution for natural resources also affect the impacts of government tax and regulatory policies. For example, Slade (1984) has shown that the effect of taxes on the rate of extraction of a natural resource can depend critically on the elasticity of substitution between the resource and reproducible inputs in producing natural resource products.

Despite their importance, very little empirical information is currently available on the elasticities of substitution for natural resources. The most important obstacle to empirical research in this area is the prevalence of vertical integration in natural resource industries, which results in the unavailability of market price data for natural resource inputs. This has forced existing empirical studies either to rely on the use of proxy variables for natural resource prices (Anders et al., 1980) or to limit the investigation of substitution possibilities to those between natural resource products and other inputs in later stages of production (e.g., Moroney and Trapani, 1981).

In this paper we use the theory of restricted cost functions to develop a methodology for estimating substitution possibilities for unpriced natural resources. Estimation of the model with data for the Canadian metal mining industry indicates that the elasticities of substitution between reproducible inputs and the natural resource, metallic ore, are equal to unity. The model also provides statistical tests of the characteristics of the production process in this industry.

The econometric model is described in the next section and tests of hypotheses are discussed in section III. The formulas for the elasticities of substitution and demand are discussed in section IV and the empirical results are presented in section V. Section VI contains concluding remarks.

II. The Econometric Model

The output of the Canadian metal mining industry is specified to be a function of metallic ore, N, and three reproducible inputs, capital, K, labor, L, and energy, E. The general form of the production function, allowing for technical change, can be written

$$Q = Q(\mathbf{X}, N, T) \qquad (1)$$

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where Q is the quantity of final output, $\mathbf{X}$ is the vector of reproducible inputs, and T is time.

With the development of duality theory in recent years, the standard approach to estimating the characteristics of a production function has become the estimation of its dual cost function.[1] The total cost function dual to equation (1) can be written

$$TC = TC(Q, \mathbf{P_X}, P_N, T), \tag{2}$$

where TC is minimum total cost, $\mathbf{P_X}$ is the vector of reproducible input prices, and P_N is the price of the natural resource input.

The prevalence of vertical integration in natural resource industries results in the general unavailability of data on P_N, and therefore the inability to estimate the total cost function directly. However, this problem can be circumvented using the theory of restricted cost functions.

A restricted cost function corresponds to the solution to the problem of minimizing the cost of some subset of the inputs subject to the choice of quantities of the remaining inputs.[2] In the present case, the relevant restricted cost function is that corresponding to the minimization of the cost of the reproducible inputs given the quantity of the natural resource input, N, which is itself assumed to be set equal to its cost minimizing level.[3] The solution of this cost minimization problem yields the restricted cost function,

$$CR = CR(Q, \mathbf{P_X}, N, T), \tag{3}$$

where CR is the minimum total expenditure on reproducible inputs given Q, $\mathbf{P_X}$, N, and T.

Estimation of the restricted cost function, (3), can provide as much information on the characteristics of the production process as would estimation of the total cost function (2). Of particular importance in the present context, the results obtainable using the restricted cost function include estimates of all cross-elasticities of substitution between the natural resource input and the reproducible inputs.

We use a translog approximation for the restricted cost function,

$$\begin{aligned}\ln CR = {} & \alpha_0 + \alpha_Q \ln Q + \sum_i \alpha_i \ln P_i + \alpha_N \ln N \\ & + \alpha_T T + \frac{1}{2}\gamma_{QQ}(\ln Q)^2 \\ & + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j + \frac{1}{2}\gamma_{NN}(\ln N)^2 \\ & + \frac{1}{2}\gamma_{TT}T^2 + \sum_i \gamma_{iQ} \ln P_i \ln Q \\ & + \sum_i \gamma_{iN} \ln P_i \ln N + \sum_i \gamma_{iT}(\ln P_i)T \\ & + \gamma_{QN} \ln Q \ln N + \gamma_{QT}(\ln Q)T \\ & + \gamma_{NT}(\ln N)T, \qquad i, j = K, L, E \end{aligned} \tag{4}$$

where, without loss of generality, $\gamma_{ij} = \gamma_{ji}$ for $i \neq j$. Linear homogeneity in prices is imposed on the restricted cost function by the restrictions

$$\begin{aligned} & \sum_i \alpha_i = 1.0 \\ & \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \sum_i \gamma_{iN} = \sum_i \gamma_{iT} = \sum_i \gamma_{iQ} = 0 \\ & \qquad i, j = K, L, E. \end{aligned} \tag{5}$$

Given the large number of parameters to be estimated, it is desirable to obtain additional effective degrees of freedom by estimating input cost share equations jointly with the restricted cost function. Using Shephard's lemma, the cost share equations can be derived by logarithmic differenti-

[1] The reasons for choosing to estimate the dual cost function include the ability to derive, by simple differentiation, systems of demand equations that are consistent with cost minimizing behavior; the probable greater exogeneity of prices than of input quantities as regressors; and the much simpler formulas for elasticities of demand and substitution, which make it possible to calculate estimated standard errors for them.

[2] The restricted cost function is a special case of the restricted profit function, in which profit is maximized subject to the choice of quantities of some inputs. The concepts were introduced by Samuelson (1953–1954) and their implications for econometric research were developed by Lau (1976) and McFadden (1978). Applications of restricted cost and profit functions have to date been rather limited. Studies using the translog functional form include Atkinson and Halvorsen (1976), Brown and Christensen (1981), and Caves, Christensen, and Swanson (1981).

[3] Of course, firms will not explicitly solve the restricted cost minimization problem considered here, but instead will solve simultaneously for the wealth maximizing quantities of Q and N together with the quantities of reproducible inputs that minimize total costs. However, the optimal quantities of reproducible inputs given by the solution to the restricted cost minimization problem will be identical to the quantities implied by the more general wealth maximization problem, see Lau (1976) It should be noted that if capital were a quasi-fixed factor, the relevant restricted cost function would be that corresponding to the minimization of the cost of the other reproducible inputs given the quantities of capital and N. The null hypothesis that capital is a variable factor was tested and could not be rejected at the 0.01 level.

ation of the cost function,

$$\frac{\partial \ln CR}{\partial \ln P_i} = \frac{\partial CR}{\partial P_i}\frac{P_i}{CR} = \frac{P_i X_i}{CR}$$
$$\equiv M_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \gamma_{iQ} \ln Q + \gamma_{iN} \ln N + \gamma_{iT} T, \quad i, j = K, L, E. \quad (6)$$

A classical additive disturbance term is appended to each of equations (4) and (6) to reflect errors in cost minimizing behavior. Because Q and N are endogenous variables, the system of equations is estimated by iterative three-stage least squares (Berndt et al., 1974).[4] Since the cost shares sum to unity, one of the cost share equations is dropped. The estimation results are invariant to the choice of equation to be dropped.

The translog functional form provides a second-order approximation to any continuously twice-differentiable cost function. To test for specific characteristics of the production function that are of interest, the corresponding restrictions on the translog cost function are imposed and tested using F tests. The characteristics of the production function to be tested include homogeneity, separability, and Hicks' neutrality. The restrictions for each of the hypothesis tests are discussed in the next section.

III. Hypothesis Tests

The production function is homogeneous of degree $1/\theta$ in all inputs if the reproducible cost function satisfies the restrictions

$$\alpha_Q = \theta(1 - \alpha_N)$$
$$\gamma_{kQ} = -\theta\gamma_{kN} \qquad k = K, L, E, N, Q, T. \quad (7)$$

Conditional on homogeneity, linear homogeneity is tested by imposing the additional restriction $\theta = 1$.

If the production function is not homogeneous, the degree of returns to scale will vary across observations. The estimated degree of returns to scale in all inputs can be calculated for each observation as (Caves, Christensen, and Swanson, 1981),

$$\frac{1 - \partial \ln CR/\partial \ln N}{\partial \ln CR/\partial \ln Q}. \quad (8)$$

The production function is characterized by logarithmic strong separability of the reproducible inputs, K, L, E, from the natural resource, N, if

$$\gamma_{iN} = 0 \qquad i = K, L, E. \quad (9)$$

Logarithmic strong separability in all inputs is tested conditional on (9) by imposing the additional restrictions,

$$\gamma_{KE} = \gamma_{KL} = \gamma_{LE} = 0. \quad (10)$$

Hicks' neutral technical change in the reproducible inputs is tested by imposing the restrictions

$$\gamma_{kT} = 0 \qquad k = K, L, E. \quad (11)$$

Neutrality of technical change with respect to scale is tested by imposing the restrictions

$$\gamma_{QT} = \gamma_{NT} = 0. \quad (12)$$

Conditional on the acceptance of restrictions (11) and (12), the absence of technical change is tested by imposing the additional restrictions

$$\alpha_T = \gamma_{TT} = 0. \quad (13)$$

If the absence of technical change is rejected, the estimated rate of technical change can be calculated for each observation. The rate of technical change, defined as the rate at which output could grow over time with all inputs held fixed, is calculated as (Caves, Christensen, and Swanson, 1981)

$$-\frac{\partial \ln CR/\partial T}{\partial \ln CR/\partial \ln Q}. \quad (14)$$

If all inputs were reproducible, the rate of technical change would be expected to be positive, since adverse changes in technology would presumably not be adopted.[5] However, the quality of the ore input can be expected to decrease over time because, other things equal, the highest quality ore will be extracted first. Exponentially declining ore quality is equivalent to negative factor augmentation and could result in the overall mea-

[4] The set of instruments includes the log of the wholesale price index for metal mining as well as the functions of time and of the prices of reproducible inputs that appear in equations (4) and (6).

[5] Unless they were imposed from outside the industry, e.g., in the form of stricter safety or environmental regulations.

sured rate of technical change in a natural resource industry being negative.[6]

IV. Elasticities of Substitution and of Demand

Uzawa (1962) has shown that Allen elasticities of substitution, σ_{km}, can be derived from a total cost function as

$$\sigma_{km} = \frac{CT \cdot CT_{km}}{CT_k \cdot CT_m} \qquad k, m = K, L, E, N \tag{15}$$

where CT is the total cost function and the subscripts indicate partial derivatives with respect to input prices, e.g.,

$$CT_{km} = \partial^2 CT/\partial P_k \partial P_m.$$

The output-constant price elasticities of demand can then be calculated (Allen, 1938) as

$$E_{km} = \sigma_{km} MT_m \tag{16}$$

where MT_m is the share of input m in total costs, $P_m X_m/CT$.

Using the theoretical relationships between the derivatives of CR and CT first established by Lau (1976) and discussed further by Brown and Christensen (1981), all the terms entering (15) can be derived from the restricted cost function, CR. The resulting formulas for the cross elasticities of substitution are

$$\sigma_{ij} = \frac{(1 - M_N)(M_i M_j + \gamma_{ij})}{M_i M_j} - \frac{(1 - M_N)(M_j M_N + \gamma_{jN})(M_i M_N + \gamma_{iN})}{(M_N^2 - M_N + \gamma_{NN}) M_i M_j} \qquad i, j = K, L, E, i \neq j \tag{17}$$

$$\sigma_{iN} = -\frac{(1 - M_N)(M_i M_N + \gamma_{iN})}{(M_N^2 - M_N + \gamma_{NN}) M_i} \qquad i = K, L, E \tag{18}$$

where $M_N \equiv \partial \ln CR/\partial \ln N = -P_N N/CR$ by Hotelling's lemma.

[6] If it were possible to measure the quantity of ore entering the restricted cost function in "efficiency units," rather than natural units (tons) as used here, the overall measured rate of technical change would be expected to be positive. Although it clouds the interpretation of the technical change parameters, exponentially declining ore quality will not bias the estimates of the other parameters because, as in the case of factor augmenting technical change, it will be controlled for by the inclusion of the time variable, T, in the restricted cost function.

Using (16), the formulas for the own and cross-price elasticities of demand are

$$E_{ii} = \frac{M_i^2 - M_i + \gamma_{ii}}{M_i} - \frac{(M_i M_N + \gamma_{iN})^2}{(M_N^2 - M_N + \gamma_{NN}) M_i} \qquad i = K, L, E \tag{19}$$

$$E_{NN} = \frac{M_N}{M_N^2 - M_N + \gamma_{NN}} \tag{20}$$

$$E_{ij} = \frac{M_i M_j + \gamma_{ij}}{M_i} - \frac{(M_j M_N + \gamma_{jN})(M_i M_N + \gamma_{iN})}{(M_N^2 - M_N + \gamma_{NN}) M_i} \qquad i, j = K, L, E \quad i \neq j \tag{21}$$

$$E_{iN} = \frac{M_N (M_i M_N + \gamma_{iN})}{(M_N^2 - M_N + \gamma_{NN}) M_i} \qquad i = K, L, E \tag{22}$$

$$E_{Ni} = -\frac{M_i M_N + \gamma_{iN}}{M_N^2 - M_N + \gamma_{NN}} \qquad i = K, L, E. \tag{23}$$

It is interesting to note the relationships between the elasticities derived from the restricted cost function including N, and those that would be provided by a cost function including only the reproducible inputs. If the reproducible inputs were separable from the natural resource input, the latter cost function would provide consistent parameter estimates. However, the estimated elasticities for the reproducible inputs would be conditional on the given quantities of the natural resource input and so would not allow for substitution occurring through induced changes in N.

The formula for the conditional own-price elasticities for reproducible inputs would be identical to the first term in equation (19). Since convexity of the restricted cost function in N (Lau, 1976) implies that the second term in (19) is negative, the conditional own-price elasticities would be smaller in absolute value than the estimates obtained with the model used here, consistent with the Le Chatelier principle. Similarly, the conditional Allen elasticity of substitution would be proportional to the first term in equation (17). Because the second term in (17) may be either positive or negative, the signs of the estimated conditional elasticities of substitution might differ from the estimates obtained when N is included in the model.[7]

V. Empirical Results

The restricted cost function, equation (4), and cost share equations, (6), are estimated with annual time series data for the Canadian metal mining industry for 1954 through 1974.[8] Final output, Q, is the dollar value of ore concentrate deflated by the wholesale price index for metal mining. The quantity of capital is calculated using the perpetual inventory method and the price of capital, P_K, is a modified Christensen-Jorgenson (1969) service price index reflecting acquisition cost, the rate of interest, and the rate of depreciation. The quantity of labor is the total number of workers and the price of labor, P_L, is equal to average wages plus indirect benefits. Data on the energy input are for electricity only. The price of energy, P_E, is equal to expenditure per kilowatt hour. All input prices are expressed in nominal terms. The quantity of ore input, N, is equal to the total number of tons of metallic ore hoisted. All variables entering in log form are normalized such that their values for the median year, 1964, are equal to unity. The time variable, T, is normalized to have the value zero in 1964.

The system of equations was estimated in its most general form and with restrictions imposed that correspond to the tests of hypotheses discussed in section III. A significance level of 0.025 was used for each of the hypothesis tests. The null hypothesis of homogeneity of the production function is rejected. Logarithmic strong separability of the reproducible inputs from the natural resource is accepted, but logarithmic strong separability with respect to all inputs is rejected. Hicks' neutrality and scale neutrality are accepted, but the absence of technical change is rejected.[9]

We also tested restrictions on the second-order terms in the endogenous variables, Q and N, because their parameters were invariably large but very imprecisely estimated, and their inclusion resulted in apparent violation of the regularity conditions for several observations.[10] The restrictions $\gamma_{QQ} = \gamma_{NN} = \gamma_{QN} = 0$ can not be rejected either singly or jointly and are imposed in the final form of the model.[11]

The parameter estimates for the final form of the model are shown in table 1 together with their estimated standard errors. Eight of the seventeen estimated parameters are significant at the 1% level. The values of R^2 for the individual equations range from 0.873 to 0.989.[12] The estimated cost function is nondecreasing and concave in the prices of reproducible inputs and nonincreasing and convex in the quantity of ore for all observations, as required for regularity (Lau, 1976).

From equation (18), acceptance of logarithmic strong separability of the reproducible inputs from the natural resource, restrictions (9), together with

TABLE 1.—PARAMETER ESTIMATES

Parameter	Estimate	Standard Error
α_0	0.0191	0.0185
α_Q	1.9943[a]	0.4093
α_T	0.0020	0.0148
α_N	−0.6535[a]	0.2693
α_K	0.2715[a]	0.0039
α_L	0.6852[a]	0.0044
α_E	0.0433[a]	0.0015
γ_{KK}	0.0314	0.0257
γ_{LL}	0.0514	0.0308
γ_{EE}	0.0021	0.0144
γ_{KL}	−0.0403	0.0266
γ_{KE}	0.0090	0.0127
γ_{LE}	−0.0111	0.0110
γ_{KQ}	0.1937[a]	0.0389
γ_{LQ}	−0.2107[a]	0.0450
γ_{EQ}	0.0170	0.0150
γ_{TT}	0.0035[a]	0.0007

[a] Significant at the 0.01 level.

[7] This is analogous to the case discussed by Berndt and Wood (1979) in which capital and energy may appear as substitutes in manufacturing if a model excluding materials is used, but as complements if a model including materials is used. The use of a restricted cost function for manufacturing industries when data on the quantity, but not the price, of materials are available would be an interesting avenue to explore in future research.

[8] We are grateful to G. Anders of the Ontario Ministry of Natural Resources for making the data available to us. The data are described in more detail in Smithson et al. (1979).

[9] The use of either 0.01 or 0.05 levels of significance for the individual tests would leave all results unchanged except that the absence of technical change could not be rejected at the 0.01 level.

[10] The imprecision of the estimates of these parameters is presumably the result of a high degree of collinearity among the second-order terms in Q and N.

[11] The full set of restrictions imposed in the final form of the model, corresponding to logarithmic partial strong separability, neutral technical change, and exclusion of second-order terms in Q and N, are also accepted when tested jointly.

[12] The values of R^2 are calculated as unity minus the ratio of the residual sum of squares to the total sum of squares.

$\gamma_{NN} = 0$, implies that the elasticities of substitution between the natural resource and each of the reproducible inputs is equal to unity for all observations.[13] The finding of unitary elasticities of substitution is especially encouraging in that the estimates reflect substitution possibilities only within the natural resource industry itself. Allowing for substitution possibilities in later stages of the production of final output would presumably result in still larger elasticities (Moroney and Trapani, 1981).

Estimates of all other elasticities of demand and substitution, evaluated for the median year, are shown in table 2 together with their estimated standard errors.[14] All the estimated own-price elasticities are highly significant but less than unity in absolute value. The demands for energy and capital are the most price-responsive, with estimated own-price elasticities of -0.92 and -0.72, respectively.

[13] The estimates of these elasticities when separability is not imposed are $\sigma_{KN} = 1.20$, $\sigma_{LN} = 0.94$, and $\sigma_{EN} = 0.76$.

[14] For the median year, which can be interpreted as the point of expansion for the translog approximation to the cost function, estimates of the elasticities, degree of returns to scale, and rate of technical change are functions of parameters only. Their standard errors are calculated using a first-order Taylor series approximation; see Kmenta (1971).

TABLE 2.—ELASTICITIES OF DEMAND AND SUBSTITUTION EVALUATED FOR MEDIAN YEAR

	Estimate	Standard Error
Elasticities of Demand		
E_{NN}	-0.6048^{b}	0.0985
E_{KK}	-0.7203^{b}	0.1240
E_{LL}	-0.5106^{b}	0.0814
E_{EE}	-0.9246^{b}	0.3662
E_{NK}	0.1642^{b}	0.0273
E_{KN}	0.3952^{b}	0.0644
E_{NL}	0.4144^{b}	0 0670
E_{LN}	0.3952^{b}	0.0644
E_{NE}	0.0262^{b}	0.0044
E_{EN}	0.3952^{b}	0.0644
E_{KL}	0.2659^{b}	0.0964
E_{LK}	0.1054^{b}	0.0357
E_{KE}	0.0592	0.0476
E_{EK}	0.3710	0.2887
E_{LE}	0.0100	0.0158
E_{EL}	0.1584	0.2525
Elasticities of Substitution		
σ_{KL}	$0\ 6417^{a}$	0.2930
σ_{KE}	2.2593	1.9579
σ_{LE}	0 3824	0.6337

[a] Significant at the 0 10 level
[b] Significant at the 0 01 level

Eight of the twelve estimated cross-price elasticities of demand are significant at the 1% level. The largest is the cross-price elasticity of demand for ore with respect to the price of labor, 0.41. Because of the restrictions corresponding to logarithmic strong separability of the reproducible inputs from ore, the cross-price elasticities for the reproducible inputs with respect to the price of ore are constrained to be equal. Their common value is 0.40.

The Allen elasticities of substitution between pairs of the reproducible inputs are all positive. Thus the finding of capital–energy complementarity in some studies of the manufacturing sector (Berndt and Wood, 1975, 1979; Field and Grebenstein, 1980) is not confirmed for the Canadian metal mining industry.[15]

Estimates of returns to scale are calculated using (8). For the median year, the estimate of returns to scale is 0.83 and its estimated standard error is 0.12. Thus the point estimate indicates decreasing returns to scale but is not significantly different from unity at the 10% level.[16]

Estimates of the rate of technical change are calculated using (14). The estimated rate of technical change evaluated for the median year is -0.1% with an estimated standard error of 0.7. Although the estimated rate of technical change is effectively equal to zero for the median year, the estimates for individual years indicate that the rate of technical change has decreased over time.[17]

The empirical results obtained here can be compared with those of Anders et al. (1980), who use the same basic data to investigate the characteristics of production in Canadian metal mining. Instead of using a restricted cost function as is done here, they deal with the nonexistence of market price data for the ore input by using a proxy

[15] Estimates of the conditional Allen elasticities of substitution can be calculated by setting all natural resource parameters equal to zero in equation (17). The estimated conditional elasticities differ in magnitude but not in sign from the estimates in table 2. The conditional estimates are $\sigma_{KL} = 0.78$, $\sigma_{KE} = 1.76$, and $\sigma_{LE} = 0.62$.

[16] The estimates of the degree of returns to scale for individual observations range from 0.79 to 0.90. It should be noted that returns to scale for an industry, as estimated here, are not necessarily equal to returns to scale for individual firms in the industry

[17] The estimated rate of technical change decreases from 1.59% in 1954 to -2.03% in 1974. As discussed in section III, negative rates of technical change in natural resource industries may arise from negative rates of factor augmentation for ore, reflecting decreasing ore quality over time.

variable, average profit royalty tax per ton, for the price of ore.[18]

Anders et al. reject Hicks' neutral technical change. Because they estimate only the share equations, not the cost function itself, they are not able to test for homogeneity of the production function or to calculate rates of technical change. Their results agree with ours in indicating that the production function is separable in the reproducible inputs.[19]

Their estimates of the elasticities of substitution between ore and the reproducible inputs indicate that all three reproducible inputs are complements with ore, $\sigma_{KN} = -0.40$, $\sigma_{LN} = -0.29$, and $\sigma_{EN} = -0.66$. These estimates have very different, and far more pessimistic, implications for the effects of resource depletion on economic growth than does our finding of unitary positive elasticities of substitution. However, the apparent finding that all three reproducible inputs are complements with ore is not consistent with economic theory. Also, their estimate of the own elasticity for ore is positive and significant at the 10% level providing further evidence that their choice of a proxy variable for ore price does not provide useful estimates of elasticities of substitution for ore.

VI. Concluding Comments

The use of a restricted cost function permits the estimation of the characteristics of production in natural resource industries in the absence of data on the market prices of natural resource inputs. The finding of unitary elasticities of substitution between the natural resource and reproducible inputs has encouraging implications for the sustainability of economic growth, especially since it only reflects substitution possibilities in the natural resource industry itself.

Because the data used here are at a high level of aggregation, conclusions concerning the characteristics of production in natural resource industries should be considered as only tentative. Estimation of the model with data for individual natural resources or, preferably, individual natural resource firms, should provide valuable new empirical information in this important area. The basic model should also prove of use in other production studies where data are available on quantities but not prices of some inputs.[20]

[18] Anders et al. also use somewhat different definitions of the variables P_L and T, estimate only the cost share equations, and treat Q as an exogenous variable.

[19] Anders et al. test for weak separability rather than logarithmic strong separability. Therefore their results imply that the elasticities of substitution between ore and the reproducible inputs are equal, but not necessarily equal to unity.

[20] The model can also be used to estimate the shadow price of natural resource inputs; see Halvorsen and Smith (1984).

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