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The Optimal Intertemporal Decision on Industrial Production and Harvesting a Renewable Natural Resource

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1. The Problem

The standard approach in the theory of *environmental economics* uses as analytical tool the concept of a damage function. This connects the emissions caused by industrial production with a welfare loss of the economy. In real life problems we observe that the carrying capacity of nature for external disturbances is not constant over time but changes due to varying ecological dynamics resulting from the emissions of the industrial process. Therefore the dimension of the environment as a renewable natural resource deserves closer inspection. In such a framework we will expect that the cost of emissions, measured as welfare loss, are not constant over time.

1.1 Two Damage Mechanisms

The philosophy of the following paper is to look at *"nature" as a regenerative natural resource*. The damage done to "nature" is then described by a downward shift in the regeneration function

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due to the waste emission of the industry. There are several possible ways to model this shift:

- If one unit of waste causes a loss of α units of "nature" then waste emissions are formally equivalent to harvesting these α units. The intertemporal aspects of harvesting a natural resource are well known in the framework of a partial equilibrium theory of natural resources. (Cf. for instance Siebert, 1983, or Dasgupta and Heal, 1979, Ch. 5.) Therefore the job would be to reconcile the aspect of optimal waste emission (caused by the production of a good Y) with the change in the regeneration function (which causes a loss in the amount of nature which can be harvested forever). A first model of this type was developed by Siebert (1982). The simple proportionality between the emissions and the shift in the regeneration function downwards presupposes a type of damage mechanism that is not the most common or at least not the only one: one unit of emission behaves like a beetle flying around and looking for a tree that it can eat up.
- Most emissions cause their damage by another mechanism: they diffuse after leaving the factory and come down more or less uniformly over the environment. It is not unreasonable to assume that the damage caused to nature in terms of destroyed ("harvested") units of the regenerative resource is then proportionate to the stock of nature. If there are only ten trees on an area where acid rain comes down, one might with good reasons suspect only half of the damage measured in terms of trees lost compared with a situation when there are twenty.

A second aspect to distinguish the effects of environmental disruption may be derived from the relevant variables in the utility function of the economy: one may measure the damage done to "nature" in foregone amounts of resource *flows* that can be harvested, or in terms of a reduced *stock* of the natural resource. In the following paper it will be assumed that the economy does not evaluate the stock of "nature as such" but merely in terms of resource flows that can be harvested.

1.2 The Model Without Industrial Emissions

In the following we will sketch the standard case of optimal use of a regenerative natural resource in order to obtain a system of reference. There is no industrial output changing the regeneration function of the resource. We assume there is only one good

W which can be harvested at constant unit cost g from nature. Consumption of W units yields a utility $V(W)$, where V is a strictly concave utility function with a positive first derivative for all $W \in (0, \infty)$:

$$V'(W) > 0, \quad V''(W) < 0 \quad \text{and} \quad \lim_{W \to 0} V'(W) = \infty.$$

The user cost μ of harvesting W units is derived from the changed regenerative behaviour of nature. The stock of the resource is Z. For a more differentiated discussion concerning harvesting costs see Ströbele (1987, Ch. 6).

The economy wishes to maximize

$$\max \int_0^\infty e^{-\delta \cdot t} \cdot [V(W) - g \cdot W] \cdot dt, \tag{1.1}$$

s. t.

$$\dot{Z} = f(Z) = a \cdot Z - b \cdot Z^2 - W. \tag{1.2}$$

(1.2) is a standard regeneration function for "nature", say "fish". As it is well known, there are two possibilities for a steady-state solution:

a) The natural resource is not "scarce" as given by nature.

The static maximisation of net utilities in each period without assigning any positive shadow price to the resource stock is the best solution, as long as there is resource "in abundance" given by nature. This case is shown in Fig. 1.1.

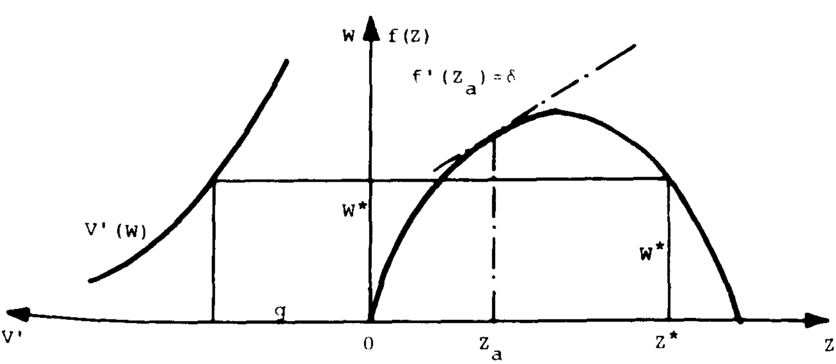

Fig. 1.1. No natural scarcity of the resource

b) Nature is a scarce asset.

If the harvesting cost parameter g is sufficiently low in relation to the utility evaluation and/or the natural regeneration function, we observe that the productivity of the resource stock should equal the rate of time preference in the steady-state. Simple calculations show that this is fulfilled for:

$$Z^* = \frac{a - \delta}{2 \cdot b}, \tag{1.3}$$

$$W^* = \frac{a^2 - \delta^2}{4 \cdot b}, \tag{1.4}$$

which obviously requires the unrestricted growth rate a to be larger than the rate of time preference δ. Otherwise no steady state exists and an asymptotical extinction of the species would be optimal in the long run. An extinction of the resource *in finite time* would not be optimal: this result simply stems from the assumed utility function with $V'(0) = \infty$, not from any cost considerations (cf. Sinn, 1982; Smith, 1977).

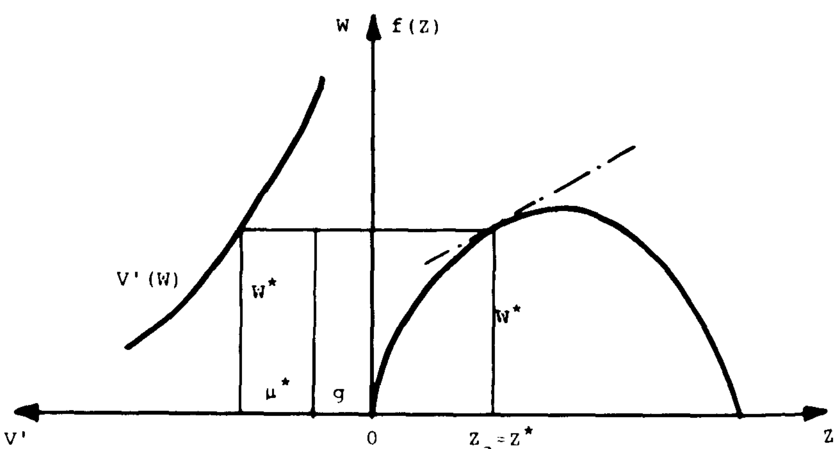

Fig. 1.2. Natural scarcity of the resource

2. Damage Mechanism I

2.1 The Model

The economy has two goods: one good Y (the industrial output, call it "car") may be produced at constant unit production cost c. Of course, this assumption is only made to simplify the analysis and to isolate "cost due to environmental damage". The second good W (call it "fish") is harvested from "nature", i. e. it is taken from a stock of a regenerative natural resource Z (stock of fish). Harvesting one unit causes constant cost g. We assume an additive separable utility function

Utility $(Y; W) = U(Y) + V(W)$ with:

$$V'(W) > 0, \quad V''(W) < 0 \quad \text{and} \quad \lim_{W \to 0} V'(W) = \infty,$$

$$U'(Y) > 0, \quad U''(Y) < 0 \quad \text{and} \quad \lim_{Y \to 0} U'(Y) = \infty. \tag{2.1}$$

Then the model is described by the following equations:

$$\max \int_0^\infty e^{-\delta t} \cdot (U(Y) - c \cdot Y + V(W) - g \cdot W) \cdot dt$$

subject to

$$\dot{Z} = a \cdot Z - b \cdot Z^2 - W - \alpha \cdot Y. \tag{2.2}$$

Obviously, the regeneration function (1.2) has been changed by the last term which describes the "harvesting" of fish due to the use of the industrial good: per unit Y there are α units of fish destroyed.

The Hamiltonian of this problem becomes

$$H = e^{-\delta t} \cdot (U(Y) - c \cdot Y + V(W) - g \cdot W) + + \mu \cdot e^{-\delta t} \cdot (a \cdot Z - b \cdot Z^2 - W - \alpha \cdot Y), \tag{2.3}$$

$$\frac{\delta H}{\delta Y} = 0: \; U'(Y) = c + \mu \cdot \alpha, \tag{2.4}$$

$$\frac{\delta H}{\delta W} = 0: \; V'(W) = g + \mu, \tag{2.5}$$

$$e^{\delta t} \cdot \frac{\delta H}{\delta Z} = -\dot{\mu} + \delta \cdot \mu: \; \dot{\mu} = \mu \cdot (\delta - a + 2 \cdot b \cdot Z). \tag{2.6}$$

(a) No scarcity of nature

As may be easily verified, there is now again the possibility that the optimal solution is given by

$$\mu = 0, \tag{2.7}$$

$$U'(Y) = c, \tag{2.8}$$

$$V'(W) = g. \tag{2.9}$$

This case implies that the natural resource is not scarce as given by nature but can be treated like an ordinary good without any limits imposed by a regeneration function. Of course, this result depends on the fact that the net marginal utility (i. e. marginal utility minus unit production cost) becomes negative for sufficiently large Y and W. This is the traditional result of microeconomics where all goods are producible without restrictions imposed by nature. Figure 2.1 shows how the optimal trajectory looks like: the optimal path cuts straight into the regeneration "mountain" with constant (Y^*, W^*) given by (2.8) and (2.9).

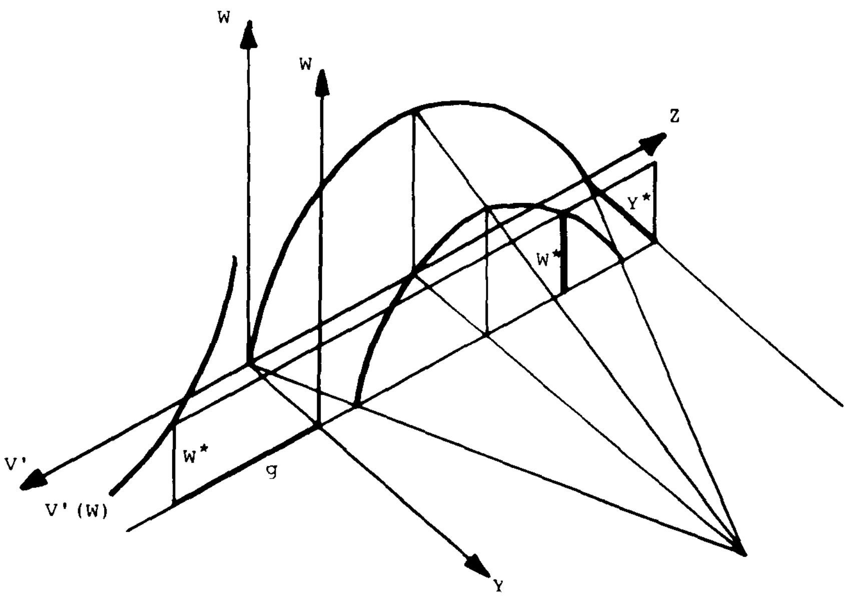

Fig. 2.1. Production and harvesting without limits imposed by nature

(b) Scarcity of nature

Alternatively the case of $\mu > 0$ may be possible: if c is sufficiently low, the point Y^* moves in direction of higher Y, i. e. where the regeneration "mountain" is already very flat, so that W^* according to (2.9) is no longer sustainable. The same holds if g is sufficiently low and W becomes "too large".

To simplify the exposition, this case is illustrated by assuming the extreme case $c = g = 0$. As can be read from (2.8) the case $\mu = 0$ can then not be optimal. The optimal path of (Y, W) is sketched in Fig. 2.2.

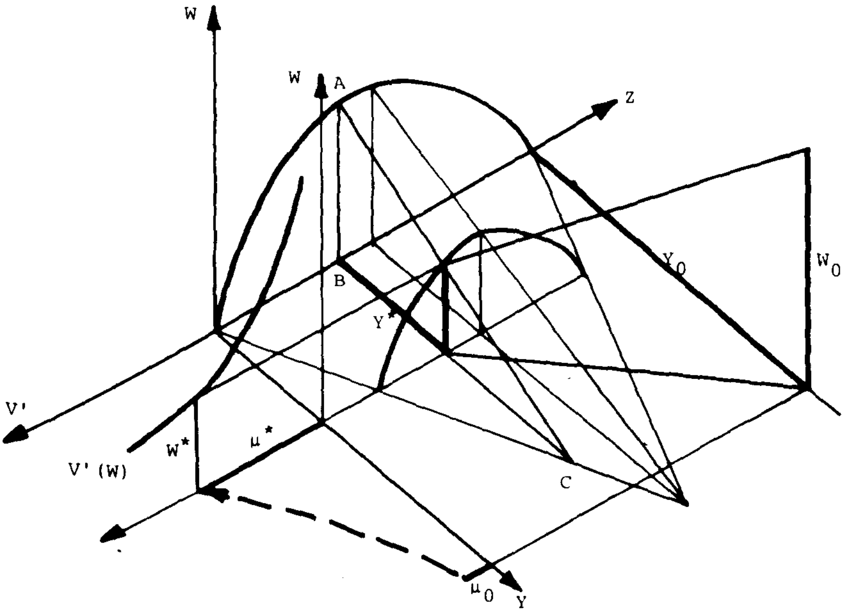

Fig. 2.2. Zero production and harvesting costs lead to natural scarcity

Along the optimal path we then must always have

$$U'(Y) = \alpha \cdot V'(W) \tag{2.10}$$

and

$$\hat{W} = -\hat{\mu} \cdot \frac{1}{\eta(W)}. \tag{2.11}$$

The first steady state condition describing the plausible requirement that the marginal productivity of the "asset" nature should be identical with the rate of time preference in a steady state is fulfilled by

$$Z^* = \frac{a - \delta}{2 \cdot b}. \tag{2.12}$$

Here, the steady state stock of natural resources Z^* is independent of the amount Y^*, i. e. it does not matter how many cars the economy produces: the long run optimum is always reached at the same stock Z^*. Of course, this is due to the fact that the *flow* of Y causes a damage to the growth of fish *independent of the size of the stock* Z. Without any stock effects between the two goods there is no reason to get a steady state result different from (1.3). This is also a characteristic of the model of Siebert (1982). The optimal amount W^* harvested in the long run optimum does of course depend on Y^* as is given by (2.13):

$$W^* = \frac{a^2 - \delta^2 - 4 \cdot b \cdot \alpha \cdot Y^*}{4 \cdot b} \quad \text{for } 0 \leq Y^* \leq \frac{a^2 - \delta^2}{4 \cdot \alpha \cdot b}. \qquad (2.13)$$

This condition is derived from the necessary condition $\dot{Z} = 0$ in (2.2) and (2.12). In Figs. 2.2 and 2.3 the steady-states must be situated on the plane ABC. The *steady state transformation curve* (2.13) is linear in (Y^*, W^*). Since the indifference curves are strictly convex, existence and uniqueness of an optimal steady state solution are ensured.

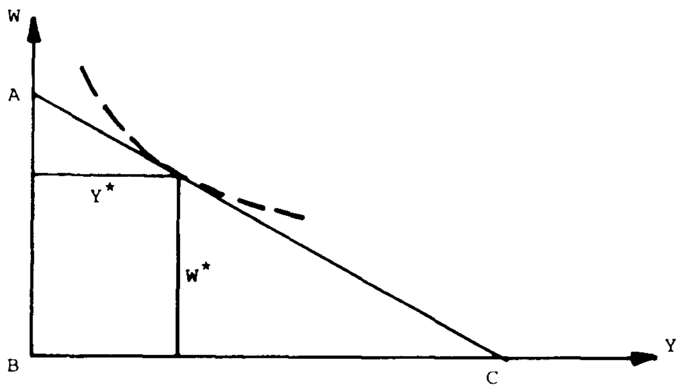

Fig. 2.3. Linear steady-state transformation curve (damage function I)

The *dynamics of the system* along an optimal path is very simple: if we start at $Z_0 = a/b$, i. e. a virgin natural system in its ecological equilibrium, there is "plenty of nature": from (2.6) one immediately derives that $\hat{\mu}$ is positive for $Z_0 = a/b$. Eq. (2.11) shows that the harvest W must fall over time. Since also condition (2.10) must be fulfilled on an optimal path we also know that then Y must fall over time. The advantage of a natural system still in abundance with fish (compared to the steady state) leads to low user costs μ. Starting in Y_0 these user costs increase at a growth rate $a+\delta$, which then decreases asymptotically to zero. In Fig. 2.2 the changing shape of the regeneration function due to "pollution" at different levels of Y is shown as well as the optimal trajectory leading into the steady state (Z^*, Y^*, W^*).

In case of $c>0$ and $e>0$ the optimal solutions may be a synthesis of the cases (a) and (b).

2.2 The Danger of Sudden Overexploitation Due to "Good Custom"

This analysis allows for an interesting interpretation: if the economy takes the natural resource W from a stock Z, it might be in the situation of Fig. 2.1 for many decades: Relatively high costs of industrial production and/or relatively high costs of harvesting W from nature prevent the economy from exploiting nature "too intensive". The shadow price of nature correctly is zero, since it is not scarce. But if due to technological progress in production or harvesting technologies and/or growth of demand (for example due to population growth of mankind) the conditions (2.8) and (2.9) no longer reflect the optimum, a positive shadow price of nature μ is necessary. But as is well known, the optimal solution with a positive shadow price of the natural resource is not ensured in a competitive equilibrium *without property rights* (cf. Levhari, Michener and Mirman, 1981). Since the economy may have become used to a seemingly well-functioning production scheme (with $\mu=0$), there is the danger of a sudden break-down of the natural system due to overexploitation: the regeneration "mountain" is cut through without stop and the resource Z extincts, even if this is not optimal. Of course, there is the chance to adapt to the "true" scarcity of nature by looking at the changing stock Z and implement some management scheme, but that demands a clear understanding and a quick reaction of a management authority.

3. Damage Type II

Now we analyse the second type of damage done to nature. The utility functions are assumed to be the same as in section 2. The only difference stems from a different treatment of the damage caused to the regeneration function.

Since the more interesting case is given when the natural resource has positive user costs, we assume $c = g = 0$.

The model then becomes

$$\max \int_0^\infty e^{-\delta t} \cdot (U(Y) + V(W)) \cdot dt$$

subject to

$$\dot{Z} = a \cdot Z - b \cdot Z^2 - W - \alpha \cdot Y \cdot Z. \tag{3.2}$$

The growth of fish $\dot{Z}$ is diminished by the harvest W and the emission $\alpha \cdot Y$ connected with the production of cars. But now this emission does not work as an absolute constant "harvest" as in section 2, but merely changes the growth rate of fish, i. e. there is a stock effect upon Z. As above we assume the unrestricted growth parameter $a > \delta$.

The standard Hamiltonian yields

$$H = e^{-\delta \cdot t} \cdot (U(Y) + V(W)) + \\ + \mu \cdot e^{-\delta \cdot t} \cdot (a \cdot Z - b \cdot Z^2 - W - \alpha \cdot Y \cdot Z). \tag{3.3}$$

The canonical equations of this dynamic optimization problem (3.1) and (3.2) are given by:

$$\frac{\delta H}{\delta Y} = 0: \ U'(Y) = \mu \cdot \alpha \cdot Z, \tag{3.4}$$

$$\frac{\delta H}{\delta W} = 0: \ V'(W) = \mu, \tag{3.5}$$

$$e^{\delta t} \cdot \frac{\delta H}{\delta Z} = -\dot{\mu} + \delta \cdot \mu: \ \dot{\mu} = \mu \cdot (\delta - a + 2 \cdot b \cdot Z + \alpha \cdot Y). \tag{3.6}$$

The canonical equations (3.4)—(3.6) show that for this case of damage mechanism we do not have $\mu = 0$ on an optimal path.

Let us first look at *possible steady states*. Obviously, in the very long run the economy reaches a situation where the welfare gain of one additional car is offset by a welfare loss measured in terms of marginal utility of reduced sustainable yields of fish caused by the industrial emissions and production Y^* and harvest W^* are constant. This steady-state is obtained by setting $\dot{Z} = 0$ in (3.2) and $\dot{\mu} = 0$ in (3.6):

$$Z^*(Y^*) = \frac{a - \delta - \alpha \cdot Y^*}{2 \cdot b}, \quad 0 \leq Y^* \leq \frac{a - \delta}{\alpha}, \tag{3.7}$$

$$W^*(Y^*) = \frac{(a - \alpha \cdot Y^*)^2 - \delta^2}{4 \cdot b}, \quad 0 \leq Y^* \leq \frac{a - \delta}{\alpha}. \tag{3.8}$$

The *stock* Z^* allowing for a steady-state is now *a linear function of the steady-state production of the industrial good* Y^*. The endpoints of this straight line (3.7), namely $Z^* = \frac{a - \delta}{2 \cdot b}$ for $Y^* = 0$ and $Z^* = 0$ for $Y^* = \frac{a - \delta}{\alpha}$ describe the bounded set of possible solutions for a steady-state (Y^*, Z^*). Different from our analysis in section 2, we obtain here that the steady-state stock Z^* depends upon the steady-state amount of industrial production Y^*. Of course this complicates the analysis a little bit.

The equation (3.8) is the steady-state transformation curve $W^*(Y^*)$. One easily derives

$$\frac{dW^*}{dY^*} = \alpha \cdot \frac{(\alpha \cdot Y^* - a)}{2 \cdot b} < 0, \tag{3.9}$$

$$\frac{d^2 W^*}{dY^{*2}} = \frac{\alpha^2}{2 \cdot b} > 0. \tag{3.10}$$

The steady-state transformation curve is monotonically falling but it is strictly convex, which means that the transformation block in the steady-state is not a convex set.

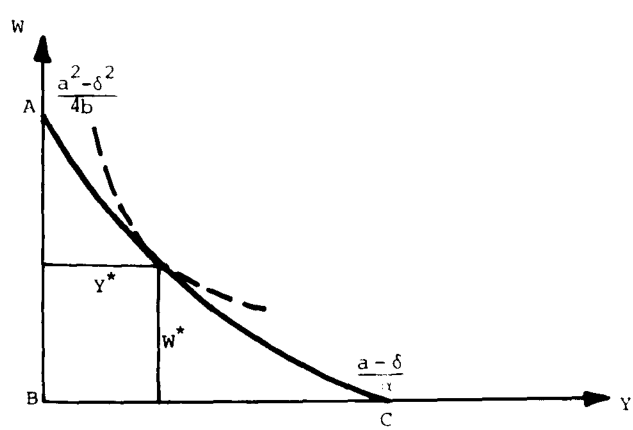

Fig. 3.1. Convex steady-state transformation curve (damage function II)

Whether there still can be ensured uniqueness and existence of a steady-state equilibrium, depends on the curvature of the transformation curve in relation to the indifference curves implied by the utility function. By analysing the static optimisation problem

$$\max\ U(Y^*) + V(W^*) \tag{3.1}$$

s. t. condition (3.8)

we observe that the bordered Hessian det (H^*) is strictly positive, where

$$H^* = \begin{bmatrix} 0 & -2\cdot\alpha\cdot(a-\alpha\cdot Y) & -4\cdot b \\ -2\cdot\alpha\cdot(a-\alpha\cdot Y) & U''(Y)-2\cdot\alpha\cdot Y & 0 \\ -4\cdot b & 0 & V''(W) \end{bmatrix}. \tag{3.12}$$

Of course, this can be derived only on the basis of our special separable utility function and the quadratic transformation curve (3.8), which leads to elimination of "disturbing" terms. Therefore we can be sure that a steady-state equilibrium exists.

In order to analyse the *dynamics leading to this steady-state* we obtain from (3.6) for $Z_0 = a/b$:

$$\hat{\mu} = \delta + a + \alpha\cdot Y > 0. \tag{3.13}$$

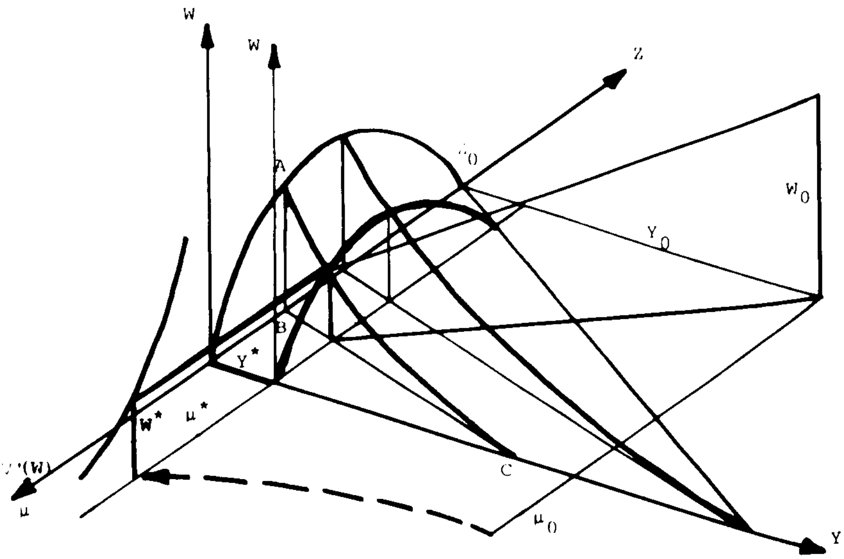

Fig. 3.2. Optimal path for the control variables (Y, W), Damage Type II

Again, we observe an increasing shadow price of the natural resource.

The optimal path is strictly monotonically falling in (Y, W) and asymptotically approaching the steady-state solution. Therefore the optimal path looks like the one in the (Y, W)-plane set up vertically in the right part of Fig. 3.2: since the natural resource is relatively abundant in the first periods the economy can afford to harvest more than W^* and produce more of Y than Y^*.

4. Conclusions

The hitherto very popular concept of a static damage function in environmental economics cannot deal with the very interesting aspects of the dynamic changes of "scarcity" of nature as a life support system. The economic theory of natural resources and environmental economics may get a deeper theoretical and conceptual integration if one looks at nature as a dynamic regenerative natural resource. The models above clarify some basic questions connected with such an attempt. The benefits of this integrative approach may give us a better understanding of the economic rationale which may be behind advertisements like "Come and pollute us": the comparative advantage of a country still relatively "clean" necessarily diminishes over time. So the next step of the analysis may be to look at the dynamics of industrial investments (in face of an expected "clean" environment) compared with the dynamics of nature whose regeneration function is shifted over time as a consequence of emissions. Another interesting question for further research may refer to the adaptive mechanisms by which the economy learns the "true" scarcity in case of different constellations of damage and/or cost parameters.

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