Nonlocal gravity and its cosmological manifestations

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We suggest a class of generally covariant ghost-free nonlocal gravity models generating de Sitter or Anti-de Sitter background with an arbitrary value of the effective cosmological constant and featuring a mechanism of dark matter simulation.

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Introduction. In the vast list of efforts aimed to reconcile evidences for cosmic acceleration [1] with gravity theory (based on quintessence type models [2], $f(R)$ and braneworld theories [3, 4], massive gravity [5] and nonlocal cosmology [6]), etc.) this paper represents a one more attempt to achieve this goal by a nonlocal infrared modification of the metric sector of the theory. A distinctive feature of our approach is that the realization of the old idea of a scale-dependent gravitational coupling – nonlocal Newton constant [7–9] – amounts to the construction of the class of stable, ghost-free models compatible with the general relativistic (GR) limit and generating the dS or AdS background with an *arbitrary value* of the effective cosmological constant Λ. The driving force of our approach is the understanding that, to resolve such issues as cosmic coincidence problem, this scale cannot be encoded in the fundamental or effective action of the theory (like, for instance, massive graviton), but should arise dynamically by the analogue of symmetry breaking. However, absolutely new feature of the model will be that neither local nor global scaling invariance, to be broken at this scale, will be present in the model. Moreover, as a bonus for the construction of the ghost-free cosmic acceleration we will get a possible dark matter (DM) simulation.

Flat-space background setup. Our starting point will be the observation [8] that the Einstein action in the vicinity of flat-space background can be written down as

$$S_E=\frac{M_P^2}{2}\int dx\, g^{1/2}\left(-R^{\mu\nu}\frac{1}{\Box}G_{\mu\nu}\,+\mathrm{O}[R^3_{\mu\nu}]\right), \qquad (1)$$

where $G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor and $1/\Box$ is the Green's function of the covariant d'Alembertian acting on symmetric tensor. Thus, the idea of a nonlocal scale dependent Planck mass [7] can be realized as the replacement of M_P^2 by a nonlocal operator – a function $M^2(\Box)$ of $\Box$, $M_P^2R^{\mu\nu}(1/\Box)G_{\mu\nu}\Rightarrow R^{\mu\nu}(M^2(\Box)/\Box)G_{\mu\nu}$. If we adopt this strategy, then the search for $M^2(\Box)$ should be encompassed by the correspondence principle according to which nonlocal terms of the action should form a correction to the Einstein Lagrangian arising via the replacement $R\Rightarrow R+R^{\mu\nu}F(\Box)G_{\mu\nu}$. The nonlocal form factor of this correction $F(\Box)$ should be small in the GR domain, but it must considerably modify dynamics at the DE scale. Motivated by customary spectral representations for nonlocal quantities like $F(\Box)=\int dm^2\,\alpha(m^2)/(m^2-\Box)$ we might try the following ansatz, $F(\Box)=\alpha/(m^2-\Box)$, corresponding to the situation when the spectral density $\alpha(m^2)$ is sharply peaked around some m^2. As we will see, for $m^2\neq 0$ this immediately leads to a serious difficulty. Schematically the inverse propagator of the theory – the kernel of the quadratic part of the action in metric perturbations $h_{\mu\nu}$ – becomes $\sim -\Box+\alpha\Box^2/(m^2-\Box)$. Then its physical modes are given by the two roots of this expression – the solutions of the corresponding quadratic equation $\Box=m^2_\pm$. In addition to the massless graviton with $m_-^2=0$ massive modes with $m_+^2=O(m^2)$ appear and contribute a set of ghosts which cannot be eradicated by gauge transformations (for the latter were already expended on the cancelation of ghosts in the massless sector – longitudinal and trace components of $h_{\mu\nu}$).

Therefore, only the case of $m^2=0$ remains, and as a first step to the nonlocal gravity we will consider the action

$$S=\frac{M^2}{2}\int dx\, g^{1/2}\left(-R+\alpha\, R^{\mu\nu}\frac{1}{\Box}\,G_{\mu\nu}\right). \qquad (2)$$

On the flat-space background this theory differs little from GR provided the dimensionless parameter α is small, $|\alpha|\ll 1$. Upper bound on $|\alpha|$ should follow from post-Newtonian corrections in this model. The additional effect of α is a small renormalization of the effective Planck mass. In the linearized theory we have an obvious relation $S=-\frac{M^2(1-\alpha)}{2}\int dx\, g^{1/2}R+\alpha O[h^3_{\mu\nu}]$. which allows one to relate the constant M to M_P,

$$M^2=\frac{M_P^2}{1-\alpha}. \qquad (3)$$

Treatment of nonlocality. At this point we have to discuss the treatment of nonlocality in (2). Nonlocal $1/\Box$ requires the specification of boundary conditions which generically violate causality in variational equations of motion for the nonlocal action. However, causality is recovered as follows. We assume that (2) is the quantum effective action whose nonlocality originates from quantum effects. There is a theorem based on Schwinger-Keldysh technique [10] that for an appropriately defined initial quantum state $|\mathrm{in}\rangle$ the effective equations for the mean quantum field $g_{\mu\nu}=\langle\mathrm{in}|\,\hat g_{\mu\nu}|\mathrm{in}\rangle$ originate from the *Euclidean* quantum effective action $S=$

$S_{\rm Euclidean}[g_{\mu\nu}]$ by the following procedure [11]. Calculate nonlocal $S_{\rm Euclidean}[g_{\mu\nu}]$ and its variational derivative. In the Euclidean signature spacetime nonlocal quantities, relevant Green's functions and their variations are generally uniquely determined by their trivial (zero) boundary conditions at infinity, so that this variational derivative is unambiguous in Euclidean theory. Then make a transition to the Lorentzian signature and impose the *retarded* boundary conditions on the resulting nonlocal operators,

$$\frac{\delta S_{\rm Euclidean}}{\delta g_{\mu\nu}(x)}\Bigg|^{\,\rm retarded}_{\,++++\;\Rightarrow\;-+++}=0. \qquad (4)$$

Theese equations are causal ($g_{\mu\nu}(x)$ depending only on the field behavior in the past of the point x) and satisfy all local gauge and diffeomorphism symmetries encoded in the original $S_{\rm Euclidean}[g_{\mu\nu}]$.[1]

We will assume that our model falls into the range of validity of this procedure, which implies a particular vacuum state $|{\rm in}\rangle$ and the one-loop approximation (in which it was proven to the first order of perturbation theory in [12] and to all orders in [11]). The extension of this range is likely to include multi-loop orders and the $|{\rm in}\rangle$-state on the (A)dS background considered below, for which this state apparently coincides with the Euclidean Bunch-Davies vacuum.

Thus, the action (2) is understood as the Euclidean one (this explains our sign choice in the Einstein term) with zero boundary conditions for $1/\Box$ at infinity. The nonlocal part can be localized in terms of the auxiliary tensor field $\varphi^{\mu\nu}$ subject to the same boundary conditions, and the theory can be equivalently described by the action

$$S[\,g,\varphi\,]=\frac{M^2}{2}\int dx\,g^{1/2}\Big\{-R-2\alpha\,\varphi^{\mu\nu}R_{\mu\nu} \\ -\alpha\left(\varphi^{\mu\nu}-\frac12\,g^{\mu\nu}\varphi\right)\Box\,\varphi_{\mu\nu}\Big\}. \qquad (5)$$

The field $\varphi^{\mu\nu}$ satisfies the variational equation $\Box\,\varphi^{\mu\nu}=-G^{\mu\nu}$ and formally carries ghosts with a wrong sign of the kinetic terms, which cannot be eradicated by diffeomorphism symmetry (because the latter at maximum can remove eight unphysical components of $g_{\mu\nu}$).

However, these ghosts are harmless because in view of boundary conditions $\varphi^{\mu\nu}$ exists only in the intermediate states. In the Lorentzian context of (4) this means that $\varphi^{\mu\nu}$ is given by a retarded solution of its equation of motion $\varphi^{\mu\nu}=-(1/\Box)_{\rm ret}G^{\mu\nu}$ and does not include free waves coming from asymptotic infinity. The actual particle content of the theory is determined in terms of the original metric field $g_{\mu\nu}$, and it indeed turns out to be ghost-free, because the quadratic part of the action coincides with that of Einstein one modulo a small renormalization of the Planck mass (3).[2]

In the local representation (5) our model could be directly applied to the FRW cosmology, which easily yields a (quasi) de Sitter point of the cosmological evolution. With the natural Lorentz-invariant ansatz $\varphi^{\mu\nu}\simeq\Phi\,g^{\mu\nu}/4$, which is supposed to be valid close to a certain moment t_0 corresponding to the present epoch, the cosmological evolution can be compatible with the current DE data. By the appropriate choice of initial conditions the Hubble factor $H=\dot a/a$, the field Φ and the parameter of the effective equation of state $w=-1-2\dot H/3H^2$ can satisfy at t_0 the following relations $\dot\Phi_0=-4\,H_0/\sigma$, $w_0=-1$, $\dot w_0=-16\,H_0(2\sigma-1)/(2+3\sigma^2)$, where $\sigma=(2\alpha/3)^{1/2}(2+\alpha\Phi_0-3\alpha)^{-1/2}$ is determined by the value of the field $\Phi_0=\Phi(t_0)$. If σ is chosen to satisfy $\sigma=O(1)>1/2$ we have $\dot w_0=O(1)\times H_0<0$ which makes the model qualitatively compatible with the observable cosmic acceleration.

These very rough estimates could have served as a starting point for a serious quantitative comparison with the DE scenario, if a formal application of (2) to the FRW setup would not disregards nontrivial boundary conditions in cosmology. On the de Sitter background (which is a zeroth-order approximation for the cosmic acceleration) the Ricci curvature $R_{\mu\nu}=\Lambda g_{\mu\nu}$ is covariantly constant but nonzero, and the nonlocal part of (2) is divergent, because $g_{\mu\nu}$ is a zero eigenvector of $\Box$. This means that the action (2) should be modified to circumvent this difficulty.

Nonlocal gravity on the (A)dS background. We will regulate the action (2) by adding to the covariant $\Box$ the matrix-valued potential term built of a generic combination of tensor structures linear in the curvature,

$$S=\frac{M^2}{2}\int dx\,g^{1/2}\,\left(-R+\alpha\,R^{\mu\nu}\frac{1}{\Box+\hat P}\,G_{\mu\nu}\right), \qquad (6)$$

$$\hat P\equiv P_{\alpha\beta}^{\;\;\;\mu\nu}=aR^{\;(\mu\;\;\nu)}_{(\alpha\;\;\beta)}+b\big(g_{\alpha\beta}R^{\mu\nu}+g^{\mu\nu}R_{\alpha\beta}\big) \\ +cR^{(\mu}_{(\alpha}\delta^{\nu)}_{\beta)}+dR\,g_{\alpha\beta}g^{\mu\nu}+eR\delta^{\mu\nu}_{\alpha\beta}. \qquad (7)$$

Here the hat denotes the action of the matrix (or its inverse) on the second rank symmetric tensor field, and a, b, c, d and e represent arbitrary parameters to be restricted by the requirement of a stable (A)dS solution in the model. Of course, such a modification of the original action (2) leaves its linear approximation on a flat background intact, because it deals with $O[h^3_{\mu\nu}]$-terms.

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[1] A similar treatment of a nonlocal action in [13] was very reservedly called the "integration by parts trick" needing justification from the Schwinger-Keldysh technique. However, this technique only provides the causality of effective equations, but does not guarantee the Euclidean-Lorentzian relation (4). The latter is based, among other things, on the choice of the $|{\rm in}\rangle$-state.

[2] Analogous elimination of ghosts by boundary conditions in conformal gravity [14] differs from our case because these ghosts are higher-derivative and essentially nonlinear. Therefore, the nonghost nature of the theory requires further verification even after integrating these ghosts out.

Now the Green's function $1/(\Box + \hat{P})$ acting on the Einstein and Ricci tensors in (6) is well defined even for the (A)dS background with the covariantly constant $R_{\mu\nu} = \Lambda g_{\mu\nu}$ and $R_{\alpha\mu\beta\nu} = \frac{\Lambda}{3}(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu})$, for which

$$P_{\alpha\beta}^{\;\;\mu\nu} = -C\Lambda\delta^{\mu\nu}_{\alpha\beta} + \left(\frac{A}{4} + B + \frac{C}{4}\right)\Lambda g_{\alpha\beta}g^{\mu\nu}, \quad (8)$$

$$\hat{P}\, g_{\mu\nu} \equiv P_{\alpha\beta}^{\;\;\mu\nu}\, g_{\mu\nu} = (A+4B)\Lambda g_{\mu\nu}, \quad (9)$$

$$A = a + 4\,b + c, \quad B = b + 4\,d + e, \quad (10)$$

$$C = a/3 - c - 4e, \quad (11)$$

Now we show that under a certain restriction on parameters of $\hat{P}$ the model (6) has (A)dS solution with an *arbitrary* value of the cosmological constant Λ. Indeed, introduce the local conformal variation with the parameter $\delta\sigma = \delta\sigma(x)$, $\delta_\sigma = \int d^4x\, \delta\sigma\, g_{\alpha\beta}\, \delta/\delta g_{\alpha\beta}$. It acts on various quantities in (6) according to their conformal weights, $\delta_\sigma g_{\mu\nu} = \delta\sigma\, g_{\mu\nu}$,

$$\begin{aligned}
&\delta_\sigma R_{\mu\nu} = O(\nabla), \quad \delta_\sigma R = -\delta\sigma\, R + O(\nabla),\\
&\delta_\sigma R^{\mu\nu} = -2\delta\sigma\, R^{\mu\nu} + O(\nabla), \quad \delta_\sigma \hat{P} = -\delta\sigma\, \hat{P} + O(\nabla),\\
&\delta_\sigma \Box = O(\nabla),
\end{aligned} \quad (12)$$

modulo the derivatives $O(\nabla)$ acting on $\delta\sigma(x)$ and these quantities themselves. The conformal variation of (6) *on the (A)dS background* then reads

$$\begin{aligned}
\frac{\delta S}{\delta\sigma}\bigg|_{\rm (A)dS} &= \frac{M^2}{2} g^{1/2}\left(-R + \alpha\, R^{\alpha\beta}\hat{P}^{-1}\, G_{\alpha\beta}\right)\\
&= -2M^2\Lambda\left(1 + \frac{\alpha}{A+4B}\right) g^{1/2}.
\end{aligned} \quad (13)$$

Since all tensor quantities on this background algebraically express via $g_{\mu\nu}$ the metric variational derivative of the action reduces to this variation, and the equation of motion $\delta S/\delta g_{\mu\nu}\,|_{\rm (A)dS} = \frac{1}{4}\, g^{\mu\nu}(\delta S/\delta\sigma)\,|_{\rm (A)dS} = 0$ holds with an arbitrary value of Λ when

$$\alpha = -A - 4B. \quad (14)$$

Note that the existence of the (A)dS solution with an arbitrary Λ is neither the result of the local Weyl invariance of the theory, nor even its global scale invariance. Rather this is the corollary of the relation (14) which, in particular, guarantees the vanishing on-shell value of the action $S\,|_{\rm (A)dS} = 0$. Thus, this solution is another vacuum – a direct analogue of the flat-space one.

Another remarkable consequence of Eq.(14) is the stability of the (A)dS solution against ghost and tachyon excitations. In principle, the hope to eradicate ghosts and tachyons from the quadratic part of the action $S_{(2)}$ on the (A)dS background is based on the observation that in the DeWitt gauge, $\chi^\mu \equiv \nabla_\nu h^{\mu\nu} - \frac{1}{2}\nabla^\mu h = 0$, $S_{(2)}$ contains only two contractions $h^2_{\mu\nu}$ and h^2 ($h \equiv g^{\mu\nu}h_{\mu\nu}$), their nonlocal parts being given by $h^{\mu\nu}(\Box + \hat{P})^{-1}h_{\mu\nu}$ and $h(\Box - \alpha\Lambda)^{-1}h$. (The form of the Green's function in the trace sector follows from the equation $(\Box + \hat{P})g_{\mu\nu}h = g_{\mu\nu}(\Box - \alpha\Lambda)h$ also based on (14)). As in the discussion above, the ghosts necessarily appear if these nonlocalities are nonvanishing in $S_{(2)}$, because the dispersion equation for $\Box$ becomes quadratic and generates doubled set of physical modes with $\Box = m^2_\pm$ ($\hat{P}$ playing the role of nonvanishing m^2 above). Therefore it is a priori possible to cancel these two nonlocalities and provide the right signs of the remaining local terms by an appropriate choice of five parameters in (7).

Curious fact is that in the DeWitt gauge $S^{(2)}$ reads (and this is the main technical result of this Letter)

$$\begin{aligned}
S^{(2)} = \frac{M^2_{\rm eff}}{2}\int d^4x\, g^{1/2}\bigg\{&-\frac{1}{4}\, h^{\mu\nu}\Box\, h_{\mu\nu} + \frac{1}{16}\, h\,\Box\, h\\
&-\frac{1}{4}\left(C - \frac{4}{3}\right)\Lambda\, h^2_{\mu\nu} + \frac{1}{16}\left(C - \frac{4}{3}\right)\Lambda\, h^2\\
&-\frac{\Lambda^2}{4}\left(C - \frac{2}{3}\right)^2\bigg(h^{\mu\nu}\frac{1}{\Box + \hat{P}}\, h_{\mu\nu}\\
&-\frac{1}{4}\, h\,\frac{1}{\Box - \alpha\Lambda}\, h\bigg)\bigg\},
\end{aligned} \quad (15)$$

$$M^2_{\rm eff} = \frac{8B\,(2B+\alpha)}{\alpha\,(1-\alpha)}\, M^2_P. \quad (16)$$

Unexpected property is that all tensor squared terms here differ from their trace squared counterparts by the same factor -4, which leads to a single equation for C, $C = 2/3$, and the positivity requirement for $M^2_{\rm eff}$. With $|\alpha| \ll 1$ this requirement selects the range of the parameter B, $B < -\alpha/2$ and $B > 0$ for $\alpha > 0$, and even more interesting compact range of B for a negative α,

$$0 < B < -\frac{\alpha}{2}, \quad \alpha < 0. \quad (17)$$

The action without gauge-fixing can be obtained from (15) by representing $h_{\mu\nu}$ in the DeWitt gauge as the projection of the non-gauged field, $h_{\mu\nu}|_{\chi^\alpha=0} = h_{\mu\nu} - 2\left[\nabla_{(\mu}\delta^\alpha_{\nu)}/(\Box + \Lambda)\right]\chi_\alpha$. The result reads

$$\begin{aligned}
S_{(2)} = \frac{M^2_{\rm eff}}{2}\int d^4x\, g^{1/2}\bigg\{&\frac{1}{4}\, h^{\mu\nu}\left(-\Box + \frac{2}{3}\Lambda\right) h_{\mu\nu}\\
&-\frac{1}{8}\, h\left(-\Box - \frac{2}{3}\Lambda\right) h - \frac{1}{2}\,\chi^2_\mu\\
&-\frac{1}{16}\left[\,2\,\nabla_\mu\chi^\mu - (\Box + 2\Lambda)\, h\,\right]\\
&\times\frac{1}{\Box + 2\Lambda}\left[\,2\,\nabla_\nu\chi^\nu - (\Box + 2\Lambda)h\right]\bigg\}.
\end{aligned} \quad (18)$$

Interestingly, the first two lines here coincide with the quadratic part on the (A)dS background of the Einstein-Hilbert action with Λ-term.

The variation of (18) with respect to $h^{\mu\nu}$ gives the nonlocal equation, which after imposing the DeWitt gauge $\chi^\mu = 0$ simplifies to

$$\left(-\Box + \frac{2}{3}\Lambda\right) h_{\mu\nu} + \frac{1}{2}\nabla_\mu\nabla_\nu h - \frac{\Lambda}{6} g_{\mu\nu}h = 0, \quad (19)$$

and $\Box h = 0$. The residual gauge transformations $\Delta^f h_{\mu\nu} = 2\nabla_{(\mu} f_{\nu)}$ with the parameter f_μ satisfying the equation $(\Box + \Lambda) f_\mu = 0$ can be used to select two polarizations – non-ghost physical modes. In particular, the boundary conditions for h can be nullified, so that h identically vanishes and makes in view of the DeWitt gauge the propagating free modes transverse and traceless as in the Einstein theory with Λ-term.

In the presence of matter sources the right hand side of (19) gets replaced by $8\pi G_{\rm eff} T_{\mu\nu}$, where $G_{\rm eff} \equiv 1/8\pi M_{\rm eff}^2$ is the effective gravitational constant vs the Newton one $G_N = 1/8\pi M_P^2$,

$$G_{\rm eff} = \frac{\alpha(1-\alpha)}{8B(2B+\alpha)}\, G_N. \qquad (20)$$

After careful commutation of covariant derivatives with $(-\Box + \frac{2}{3}\Lambda)^{-1}$ the gravitational potential of matter sources takes modulo the gauge transformation the following form

$$h_{\mu\nu} = \frac{8\pi G_{\rm eff}}{-\Box + \frac{2}{3}\Lambda} \left(T_{\mu\nu} + g_{\mu\nu} \frac{\Box - 2\Lambda}{\Box + 2\Lambda} \frac{\Lambda}{3\Box} T \right). \qquad (21)$$

The tensor structure here differs from the GR analog $T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T$, which for non-relativistic sources gives $O(1)$ correction. What is much more interesting, it yields an unexpected bonus in the form of the dark matter (DM) simulation – $1/|\alpha|$-amplification of the gravitational attraction due to the replacement of the Newton gravitational constant G_N by $G_{\rm eff} \sim G_N/|\alpha|$ with $|\alpha| \ll 1$. This is possible with $|B| \sim \alpha$ for a positive α and necessarily happens in the case (17) of a negative α, because the factor $\alpha/8B(2B+\alpha) \geq 1/|\alpha|$ and

$$G_{\rm eff} \geq \frac{1-\alpha}{|\alpha|}\, G_N \gg G_N. \qquad (22)$$

Eq.(21) is valid for the perturbation range $|\delta R_{\mu\nu}| \sim |\nabla\nabla h_{\mu\nu}| \ll \Lambda |g_{\mu\nu}|$ and $|h_{\mu\nu}| \ll |g_{\mu\nu}|$ equivalent to very small matter densities $|T_{\mu\nu}| \ll M_P^2 \Lambda$. For stronger sources the theory reduces to the GR regime with the Planck mass (3). Thus it interpolates between GR theory and its strongly coupled infrared modification which is likely to generate a stable ghost-free stage of cosmic expansion and, perhaps, even simulate the DM effect on rotation curves.

Of course, prospective nature of this model should not be exaggerated. It should undergo tests on consistency of post-Newtonian corrections, the effect of nonlocal stress-tensor trace contribution in (21) should be analyzed for vacuum stability purposes. Moreover, the mechanism should be found, by which the model picks up a concrete scale $|T_{\mu\nu}| \sim M_P^2 \Lambda$ at which it undergoes a crossover from the GR regime to cosmic acceleration, as well as other issues that go beyond this Letter.

In conclusion we mention that serendipity of ghost-free nonlocal gravity models (6) satisfying the relation (14) might not be exhausted by applications in cosmology. In particular, without the Riemann term in (7) ($a = 0$) they admit generic (not maximally symmetric) Einstein space solutions, $R_{\mu\nu} = \Lambda g_{\mu\nu}$, also with an *arbitrary* Λ, which might have implications for zero entropy black holes [15] and be an alternative to the conformal gravity model of [14], whereas with a negative Λ they become a new testing ground for AdS/CFT correspondence perhaps promising other exciting consequences.

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