Pre-big bang collapsing universe from modern Kaluza-Klein theory of gravity.

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Abstract

We study the collapse of the universe described by a scalar field spherically symmetric collapse of a system described by a massless scalar field from a 5D Riemann-flat canonical metric, on which we make a dynamical foliation on the extra space-like dimension. The asymptotic universe (absent of singularities) results to be finite in size and energy density, with an vacuum dominated equation of state. The important result here obtained is that the asymptotic back-reaction effects are given by a negative constant: $\frac{1}{2}\left[\frac{1}{1+\dot{\psi}^2}+\frac{1}{\dot{\psi}^2}\right]\left\langle\left(\delta\dot{\bar{\varphi}}\right)^2\right\rangle+\frac{1}{2a^2}\left\langle\left(\vec{\nabla}\delta\varphi\right)^2\right\rangle\Big|_{t\rightarrow\infty}=\frac{-8\Lambda_0}{3\pi G}$.

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I. INTRODUCTION, BASIC EQUATIONS AND MOTIVATION

The cosmological models based on classical General Relativity (GR) predict that the Big Bang singularity is the true beginning of the universe (boundary of the spacetime). It has been long expected that the existence of singularity in the classical general relativity which has been shown to be quite generic thanks to the singularity theorems, will be removed when classical framework of gravity is extended to a quantum framework of gravity. When one describes the early universe, GR is applied beyond its domain of validity. The quantum effects which dominate in this epoch are expected to resolve the singularity. In particular, the existence of the cosmological singularity in the framework of Loop Quantum Gravity (LQG)[1] has been subject of study in the last years. During the inflationary big bang expansion, the universe suffered an exponential accelerated expansion driven by a scalar (inflaton) field with an equation of state close to a vacuum dominated one. The most conservative assumption is that the energy density $\rho = P/\omega$ is due to a cosmological parameter which is constant and the equation of state is given by a constant $\omega = -1$, describing a vacuum dominated universe with pressure P and energy density ρ. On the other hand, exists a kind of exotic fluids that may be framed in theories with matter fields that violate the weak energy condition[2], such that $\omega < -1$. These models were called phantom cosmologies, and their study represents a currently active area of research in theoretical cosmology[3].

On the other hand, the spherically symmetric collapse of a massless scalar field has been of much interest towards understanding the dynamical evolutions in general relativity. Both, analytical[4] and numerical[5] investigations, have been undertaken by various authors to gain more insight into the formation of black holes. One remarkable finding of these numerical investigations is the demonstration of criticality in gravitational collapse. Specifically, it was found that for a range of values of the parameter characterizing the solution, black hole forms and there was a critical value of the parameter beyond which the solutions are such that the scalar field disperses without forming any black hole. However, this result has been obtained mainly through numerical studies and a proper theoretical understanding of this phenomenon is still lacking (see e.g. [6] and the references therein).

In this letter we study the cosmological collapse of a pre-big bang universe driven by a massless scalar field in the framework of the 5D Modern Kaluza-Klein theory of gravity[1], also known as the Space-Time-Matter theory of gravity[7, 8]. We shall use a dynamical foliation, $\psi \equiv \psi(t)$, of the noncompact space-like extra dimension ψ. This topic was explored previously by Ponce de León[9]. He showed that the FRW line element can be reinvented on a dynamical four-dimensional hypersurface, which is not orthogonal to the extra dimension, without any internal contradiction. The effective 4D hypersurface is selected by the requirement of continuity of the metric and depends explicitly on the evolution of the extra dimension. More recently, was demonstrated that phantom scenarios[10], warm inflation[14] and super exponential inflationary scenarios[12] can be obtained through this mechanism from a 5D Riemann-flat.

In order to study the dynamics of a scalar field φ on a 5D vacuum, we consider the canonical metric

$$dS^2 = g_{\mu\nu}(y^\sigma, \psi) dy^\mu dy^\nu - d\psi^2. \tag{1}$$

Here the 5D coordinates are orthogonal: $y \equiv \{y^a\}$[2]. The geodesic equations for an relativistic observer are

$$\frac{dU^a}{dS} + \Gamma^a_{bc}\, U^b U^c = 0, \tag{2}$$

where $U^a = \frac{dy^a}{dS}$ are the velocities and Γ^a_{bc} are the connections of (1). Now we consider a parametrization $\psi(x^\alpha)$, where $x \equiv \{x^\alpha\}$ are an orthogonal system of coordinates, such that the effective line element (1), now can be written as

$$dS^2 = h_{\alpha\beta}\, dx^\alpha dx^\beta. \tag{3}$$

It is very important to notice that S will be an invariant, so that derivatives with respect to S will be the same on 5D or 4D. In other words, in this paper we shall consider that spacetime lengths that remain unaltered when we move on an effective 4D spacetime.

We are interested to study how is the effective 4D dynamics of φ, obtained from a dynamical foliation of a 5D Ricci-flat canonical metric. We consider a classical massless scalar field $\varphi(y^a)$ on the metric (1). In order to make a complete description for the dynamics of the scalar field, we shall consider its energy momentum tensor. In order to describe a true 5D physical vacuum we shall consider that the field is massless and there is absence of

[1] This topic has been studied earlier by Liu and Wesson[16].
[2] Greek letters run from 0 to 3, and arabic letters run from 0 to 4.

interaction on the 5D Ricci-flat manifold, so that

$$T^a_{\ b} = \Pi^a \Pi_b - g^a_{\ b}\, \mathcal{L}\left[\varphi, \varphi_{,c}\right], \tag{4}$$

where $\mathcal{L}\left[\varphi, \varphi_{,c}\right] = \frac{1}{2}\varphi^a \varphi_a$ is the lagrangian density for a free and massless scalar field on (1) and the canonical momentum is $\Pi^a = \frac{\partial \mathcal{L}}{\partial \varphi_{,a}}$. Notice that we are not considering interactions on the 5D vacuum, because it is related to a physical vacuum in the sense that the Einstein tensor is zero: $G^a_b = 0$. The effective 4D energy momentum tensor will be

$$\bar{T}_{\alpha\beta} = e^a_{\ \alpha}\, e^b_{\ \beta}\, T_{ab}\Big|_{\psi(x^\alpha)}. \tag{5}$$

In other words, using the fact that $\mathcal{L}$ is an invariant it is easy to demonstrate that

$$\bar{T}^\alpha_{\ \beta} = \bar{\Pi}^\alpha\, \bar{\Pi}_\beta - h^\alpha_{\ \beta} \mathcal{L}, \tag{6}$$

where $\mathcal{L}$ is an invariant of the theory

$$\mathcal{L} = \frac{1}{2}\varphi^{,a}\varphi_{,a} = \frac{1}{2}\left(e^a_\alpha \bar{\varphi}^{,\alpha}\right)\left(\bar{e}^\beta_a \bar{\varphi}_{,\beta}\right). \tag{7}$$

II. COLLAPSING UNIVERSE: BASIC EQUATIONS

We consider the 5D Riemann-flat metric[17]

$$dS^2 = \psi^2 \left[\frac{\Lambda(t)}{3} dt^2 - e^{-2\int \left(\frac{\Lambda(t)}{3}\right)^{1/2} dt}\, dr^2\right] - d\psi^2, \tag{8}$$

which describes 5D extended universe with variable cosmological function $\Lambda(t) > 0$, which is contracting with the time. The noncompact extra coordinate ψ has spatial unities (we shall consider unities $c = \hbar = 1$). Furthermore, $dr^2 = dx^2 + dy^2 + dz^2$ is the 3D Euclidean metric, t is the cosmic time and ψ is the space-like noncompact extra dimension. Since the metric (1) is Riemann-flat (and therefore Ricci-flat), hence it is suitable to describe a 5D vacuum vacuum ($G_{ab} = 0$) in the framework of Space-Time-Matter (STM) theory of gravity[18]. With this aim we shall consider the 5D action

$${}^{(5)}I = \int d^4x\, d\psi \sqrt{\left|\frac{{}^{(5)}g}{{}^{(5)}g_0}\right|}\left(\frac{{}^{(5)}R}{16\pi G} + \frac{1}{2} g^{ab}\varphi_{,a}\varphi_{,b}\right), \tag{9}$$

where ${}^{(5)}g$ is the determinant of the covariant metric tensor g_{AB}:

$${}^{(5)}g = \psi^8 \left(\frac{\Lambda}{3}\right) e^{-6\int \sqrt{\frac{\Lambda}{3}} dt}, \tag{10}$$

and ${}^{(5)}g_0 = \psi_0^8\left(\frac{\Lambda_0}{3}\right)$ is a constant to make dimensionless the expression $\left|{}^{(5)}g/{}^{(5)}g_0\right|$.

In order to describe a 5D vacuum on (8), we shall consider φ as a massless test classical scalar field, which is minimally coupled to gravity. For this reason, the φ-contribution to the Lagrangian will be considered as purely kinetic and free of any interaction on (8). From the mathematical point of view, the second term in the action (9) is constructed by using monogenic fields φ, which have null D´Alambertian on the 5D Riemann-flat metric (8)[22].

Now we consider a dynamical foliation: $\psi \equiv \psi(t)$, on the metric (8). If we require that t to be a cosmic time, we need

$$\psi^2 \frac{\Lambda(t)}{3} - \dot{\psi}^2 = 1, \tag{11}$$

and hence

$$\Lambda(t) = 3\left[\frac{1+\dot{\psi}^2}{\psi^2}\right]. \tag{12}$$

In this paper we shall study the particular case where $\Lambda(t)$ is a constant of time: $\Lambda(t) = \Lambda_0$. In this particular case the solutions are

$$\psi_{(D,I)}(t) = \frac{1}{6}\left[\frac{9 + e^{\mp 2\sqrt{\frac{\Lambda_0}{3}}t}}{\sqrt{\frac{\Lambda_0}{3}}\, e^{\mp\sqrt{\frac{\Lambda_0}{3}}t}}\right], \tag{13}$$

where the solution $\psi_{(D)}(t)$ decreases with time and $\psi_{(I)}(t)$ increases. The expanding version of this solution was considered in[19]. In this letter we shall consider the model generated with $\psi_{(D)}(t)$. To simplify the notation we shall call it: $\psi_{(D)}(t) \equiv \psi(t)$[3]. In this case the scale factor of the universe on the 4D hypersurface described by the effective 4D metric

$$dS^2 = dt^2 - \psi^2(t)\, e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\, dr^2, \tag{14}$$

is

$$a(t) = \psi(t)\, e^{-\sqrt{\frac{\Lambda_0}{3}}t}. \tag{15}$$

Notice that this scale factor tends to a constant as t tends to infinity

$$a_0 \equiv \lim_{t\to\infty} a(t) = \frac{3}{2}\sqrt{\frac{3}{\Lambda_0}}. \tag{16}$$

This is a very interesting behavior because the model describes a contracting universe which has an asymptotic finite size a_0. This effect is due to the fact in General Relativity the

[3] Note that the large times value of $\psi(t)$ is $\psi_0 \equiv \psi(t)|_{t\to\infty} \to 3/2$

action is invariant under time reflections. Thus, to any standard cosmological solution $H(t)$, describing decelerated expansion and decreasing curvature ($H > 0$, $\dot{H} < 0$), time reversal associates a reflected solution, $H(-t)$, describing a contracting Universe. In a string cosmology context, this solutions are called dual[20]. In this letter we are dealing with an extra dimensional cosmological model where the extra dimension is non-compact. However, this duality is preserved and the interpretation of the results obtained by Gasperini and Veneziano in [20] are preserved.

The effective Hubble parameter is given by

$$H(t) = \frac{\dot{a}}{a} = -2\sqrt{\frac{\Lambda_0}{3}}\, \frac{e^{-2\sqrt{\frac{\Lambda_0}{3}}t}}{\left[9 + e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right]} < 0, \tag{17}$$

which is negative in agreement with that one expects for a collapsing universe. Notice that the asymptotic Hubble parameter is $\lim_{t\to\infty} H(t) \to 0$.

On the other hand, the relevant components of the Einstein tensor are (we use cartesian coordinates)

$$G^0_{\ 0} = -\frac{4\Lambda_0 e^{-4\sqrt{\frac{\Lambda_0}{3}}t}}{\left(9 + e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right)^2}, \tag{18}$$

$$G^x_{\ x} = -\frac{4\Lambda_0 e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\left[6 + e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right]}{\left(9 + e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right)^2}, \tag{19}$$

so that, using the fact that the Einstein equations are respectively $G^0_{\ 0} = -8\pi G\,\rho$ and $G^x_{\ x} = G^y_{\ y} = G^z_{\ z} = 8\pi G\, P$, we obtain the equation of state for the universe

$$\frac{P}{\rho} = \omega(t) = -e^{2\sqrt{\frac{\Lambda_0}{3}}t}\left[6 + e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right], \tag{20}$$

which always remains with negative values $\omega(t) < -1$, and tends to negative infinity values for large asymptotic times. This is because the energy density tends to zero very much rapidly than the pressure. Notice that in a pre-big bang followed with a post-big bang with time reflexion $t \to -t$ in the equation of state (20), such that equation would describe an inflationary post-big bang expansion with $\omega|_{t\to-\infty} \to -1$, which assures a asymptotic spatially flat universe, in agreement with observations[21]. The effective 4D scalar curvature

$$\bar{\mathcal{R}} = 8\Lambda_0 \frac{e^{-2\sqrt{\frac{\Lambda_0}{3}}t}}{\left[9 + e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right]}, \tag{21}$$

decreases with the time and has a null asymptotic value $\bar{\mathcal{R}}\big|_{t\to\infty} \to 0$.

The expectation values for the energy density and the pressure, written in terms of the scalar field $\varphi(t,\vec{r},\psi(t)) \equiv \bar{\varphi}(t,\vec{r})$, are

$$\rho = \left\langle 0|\bar{T}^0_{\ 0}|0\right\rangle \ = \ \left\langle \frac{3}{2\Lambda_0\psi^2(t)}\dot{\bar{\varphi}}^2 + \frac{1}{2a^2(t)}\left(\vec{\nabla}\bar{\varphi}\right)^2 + \frac{1}{2}\left(\frac{\partial\bar{\varphi}}{\partial\psi}\right)^2\right\rangle, \tag{22}$$

$$P = -\left\langle 0|\bar{T}^i_{\ j}|0\right\rangle \ = \ -\delta^i_{\ j}\left\langle \frac{3}{2\Lambda_0\psi^2(t)}\dot{\bar{\varphi}}^2 - \frac{1}{6a^2(t)}\left(\vec{\nabla}\bar{\varphi}\right)^2 - \frac{1}{2}\left(\frac{\partial\bar{\varphi}}{\partial\psi}\right)^2\right\rangle. \tag{23}$$

Here, the notation $\langle 0|...|0\rangle$ denotes the quantum expectation value calculated on a 4D vacuum state. Because we are considering a spatially isotropic and homogeneous background, we shall consider an averaging value with respect to a gaussian distribution on a Euclidean 3D volume.

III. BACKGROUND DYNAMICS AND BACK-REACTION EFFECTS

We consider the semiclassical expansion for the scalar field $\bar{\varphi}(\vec{r},t) = \bar{\phi}(t,\psi(t)) + \delta\bar{\varphi}(\vec{r},t)$, such that the expectation value for the effective 4D scalar field fluctuations becomes null: $\langle\delta\bar{\varphi}\rangle = 0$. In general, since the averaging is considered as gaussian, the momentums of odd order in the fluctuations $\left\langle \left[\delta\dot{\bar{\varphi}}(\vec{r},t)\right]^{(2n+1)}\right\rangle$, with n integer, will be zero.

Taking into account the fact that $\frac{\partial\bar{\varphi}}{\partial\psi}\Big|_{\psi(t)} = \dot{\bar{\varphi}}/\dot{\psi}$, we obtain the effective background Energy-Momentum components on the 4D effective hypersurface (14)

$$\left\langle \bar{T}^0_{\ 0}\right\rangle = \rho \ = \ \frac{3}{2\Lambda_0\psi^2(t)}\dot{\bar{\phi}}^2 + \frac{3}{2\Lambda_0\psi^2(t)}\left\langle\left(\delta\dot{\bar{\varphi}}\right)^2\right\rangle + \frac{1}{2a^2(t)}\left\langle\left(\vec{\nabla}\delta\bar{\varphi}\right)^2\right\rangle + \frac{1}{2}\left(\frac{\dot{\bar{\phi}}}{\dot{\psi}}\right)^2 + \frac{1}{2}\frac{\left\langle\left(\delta\dot{\bar{\varphi}}\right)^2\right\rangle}{\dot{\psi}^2}, \tag{24}$$

$$-\left\langle \bar{T}^x_{\ x}\right\rangle = P \ = \ \frac{3}{2\Lambda_0\psi^2(t)}\dot{\bar{\phi}}^2 + \frac{3}{2\Lambda_0\psi^2(t)}\left\langle\left(\delta\dot{\bar{\varphi}}\right)^2\right\rangle - \frac{1}{6a^2(t)}\left\langle\left(\vec{\nabla}\delta\bar{\varphi}\right)^2\right\rangle - \frac{1}{2}\left(\frac{\dot{\bar{\phi}}}{\dot{\psi}}\right)^2 - \frac{1}{2}\frac{\left\langle\left(\delta\dot{\bar{\varphi}}\right)^2\right\rangle}{\dot{\psi}^2}. \tag{25}$$

Using the Einstein equations with the expression (11), we obtain the important result that describes the temporal evolution for the background back-reaction effects

$$\frac{1}{2}\left[\frac{1}{1+\dot{\psi}^2}+\frac{1}{\dot{\psi}^2}\right]\left\langle\left(\delta\dot{\bar{\varphi}}\right)^2\right\rangle + \frac{1}{2a^2}\left\langle\left(\vec{\nabla}\delta\bar{\varphi}\right)^2\right\rangle = \frac{36\Lambda_0}{\pi G}\left[9+e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right]^2 \times \left\{e^{-4\sqrt{\frac{\Lambda_0}{3}}t} - \frac{\left(81e^{2\sqrt{\frac{\Lambda_0}{3}}t}+e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right)\left(9+e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right)}{2\left(81e^{2\sqrt{\frac{\Lambda_0}{3}}t}-126+e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right)}\right\} \tag{26}$$

Finally, we obtain the temporal evolution for the background field

$$\dot{\bar{\phi}}^2 = \frac{\Lambda_0}{12\pi G}\frac{\left(e^{-\sqrt{\frac{\Lambda_0}{3}}t}-9e^{\sqrt{\frac{\Lambda_0}{3}}t}\right)^2\left(9+e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right)}{\left(81e^{2\sqrt{\frac{\Lambda_0}{3}}t}-126+e^{-2\sqrt{\frac{\Lambda_0}{3}}t}\right)}, \tag{27}$$

which, for very large times tends to $\left.\dot{\bar{\phi}}^2\right|_{t\to\infty} \to \frac{3\Lambda_0}{4\pi G}$. On the other hand, using the equation (26), we obtain the large times rugosity term

$$\frac{1}{2}\left[\frac{1}{1+\dot{\psi}^2}+\frac{1}{\dot{\psi}^2}\right]\left\langle\left(\delta\dot{\bar{\varphi}}\right)^2\right\rangle + \frac{1}{2a^2}\left.\left\langle\left(\vec{\nabla}\delta\bar{\varphi}\right)^2\right\rangle\right|_{t\to\infty} = -\frac{8\Lambda_0}{3\pi G}, \tag{28}$$

which takes a constant negative value and provide us the contribution of the squared field fluctuations and their gradients to the cosmological constant.

IV. FINAL COMMENTS

Starting from a 5D Riemann-flat canonical metric on which we make a dynamical foliation, in this letter we have studied a model for a massless scalar field which drives a gravitational collapse of the universe, which finally acquires a finite size: $a_0 = \frac{3}{2}\sqrt{\frac{3}{\Lambda_0}}$ related to the cosmological constant Λ_0, but with null asymptotic scalar curvature. An important result here obtained is that the asymptotic universe (absent of singularities) results to be finite and absent of matter, with $\left.\omega\right|_{t\to\infty} \to -\infty$. This is because the pressure is negative (opposes the collapse) along all the contraction and its asymptotic value tends to zero, but more slowly than does the energy density. This seems puzzling but is the apparent result of the continuous dispersal[23] of the universe during its collapse. This topic deserves a further study. But the more notorious result is that the asymptotic back-reaction effects are given by a negative constant [see eq. (28)]. Finally, a remarkable difference of this model with phantom scenarios is that here the kinetic component of energy density related to the massless scalar field remains always positive. The negative contribution to the energy density, which in our model is the responsible of the deceleration of the collapse, being given by the back-reaction effects described in eq. (26), with an asymptotical contribution given by (28).

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