Vainshtein mechanism in Gauss-Bonnet gravity and Galileon aether

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We derive field equations of Gauss-Bonnet gravity in 4 dimensions after dimensional reduction of the action and demonstrate that in this scenario Vainshtein mechanism operates in the flat spherically symmetric background. We show that inside this Vainshtein sphere the fifth force is negligibly small compared to the gravitational force. We also investigate stability of the spherically symmetric solution, clarify the vocabulary used in the literature about the hyperbolicity of the equation and the ghost-Laplacian stability conditions. We find superluminal behavior of the perturbation of the field in the radial direction. However, because of the presence of the non linear terms, the structure of the space-time is modified and as a result the field does not propagate in the Minkowski metric but rather in an "aether" composed by the scalar field $\pi(r)$. We thereby demonstrate that the superluminal behavior does not create time paradoxes thank to the absence of Causal Closed Curves. We also derive the stability conditions for Friedmann Universe in context with scalar and tensor perturbations.

I. INTRODUCTION

One of the most mysterious discoveries of modern cosmology is related to late time acceleration of universe [1, 2]. For past one decade or so, extensive efforts have been made to understand the underlying reason of this phenomenon (for a recent review see [3]). According to the standard lore , the acceleration is caused by an exotic form of matter with large negative pressure called dark energy. The cosmological constant and a variety of scalar field systems as representative of dark energy are consistent with observations. Though some of the scalar field systems with generic features, such as tracker solutions, are attractive but nevertheless they do not address the conceptual problems associated with cosmological constant.

There exist an alternative thinking which advocates the need for a paradigm shift, namely, that gravity is modified at large scales which might give rise to late time acceleration. We know that gravity is modified at small scales and thus it is quite plausible that modification also cures at large distance where it has not been varied directly. As for the small scale corrections, no deviations from Einstein's theory have been yet detected; perhaps we need to probe still higher energies to observe these effects. However, situation is quite different challenging at large scales. Indeed, Einstein's theory is consistent with observations to a high accuracy at solar scales thereby telling us that any modification to gravity should be confronted with tough constraints posed by solar physics. Secondly, as for the late time acceleration, the modification should also be distinguished from cosmological constant.

The aforementioned requirements are difficult to satisfy consistently in f(R) theories of gravity. These theories are equivalent to GR plus a scalar degree of freedom whose potential is uniquely constructed from space time curvature R [4]. The scalar degree of freedom should mimic quintessence and should hide its detection at small scales *à la chameleon* [5].

The investigations show that the generic $f(R)$ models either reduce to GR plus cosmological constant [6] or has a ϕMDE instead of a standard matter epoch [7] or hit curvature singularity [8] or give rise an ugly fine tuning [9] which is the price one has to pay for implementing the chameleon mechanism. One might try to remedy these theories by complementing them by quadratic curvature corrections [10]. However, as demonstrated by Lam Hui et al [11], theories based upon chameleon mechanism lead to a violation of equivalence principal of the order of one.

In view of the aforesaid, we need to look for an alternative mechanism of mass screening. The Vainshtein mechanism [12] is the one which gives rise to mass screening: Any modification of gravity in the neighborhood of a massive body within a radius dubbed Vainshtein radius are switched off kinematically. The mechanism was invented to address the discontinuity problem [13, 14] of massive gravity *à la* Pauli-Fierz [15]. In this theory, the zero helicity graviton mode ϕ is coupled to the stress of energy momentum tensor gives rise to serious violations of local gravity constraints in the limit of vanishing mass of graviton. Vainshtein pointed out that non-linear effects become crucial in this case. The non-linear derivative term added to Pauli-Fierz Lagrangian was shown to implement the mass screening thereby removing the problem of discontinuity.

It is interesting to note that the non-linear term naturally arises in DGP model [16] in the decoupling limit [17] such that Vainshtein mechanism is in built in the theory. The scalar degree of freedom obeys Galilean symmetry in flat space time and is free from ghosts is called *Galileon* [18]. There exist higher order Galileon Lagrangians in flat and curved space-time [19]. The higher order Lagrangian are necessary for realizing the late time de-Siter solution

[20].

We should also mention that Galileons are deeply related to massive gravity [21] and Dirac-Born-Infeld systems [22] in the framework of higher dimensional theories [23–26].

In this paper, we consider dimensional reduction of Gauss-Bonnet gravity [27] and look for the possibility of mass screening in the model. We also investigate causal structure of flat spherically symmetric and homogeneous and isotropic backgrounds.

II. GAUSS-BONNET GRAVITY AND ITS KALUZA-KLEIN REDUCTION

We shall consider the following action in $D+N$ dimensions,

$$\mathcal{S} = \int \mathrm{d}^{D+N}\mathrm{x}\sqrt{-g^{(D+N)}}\left(R + \alpha R_{\mathrm{GB}}\right) \qquad (1)$$

which is the simplest non trivial form of Lovelock theory.

In order to simplify the analysis, we use the metric anzatz,

$$\mathrm{ds}^2 = \mathrm{g}_{\mu\nu}\mathrm{dx}^\mu\mathrm{dx}^\nu + \mathrm{e}^\pi\gamma_{\mathrm{ij}}\mathrm{dx}^\mathrm{i}\mathrm{dx}^\mathrm{j} \qquad (2)$$

where the Greek letters run form 0 to $D-1$ and the Latin characters from D to $D+N-1$. The scalar field π appearing in the metric plays the role of the size of the extra dimensions and depends on the first D coordinates. It acquires a non trivial character in case the volume of the compactified dimensions becomes a variable.

Following a standard prescription for dimensional reduction on a compact flat space, we get

$$\mathcal{S} = \int \mathrm{d}^D\mathrm{x}\sqrt{-g^{(D)}}e^{N\pi/2}\Big[R^{(D)} + d_1(\partial\pi)^2 + \alpha\Big(R_{\mathrm{GB}}^{(D)}$$
$$+ d_2 G_{\mu\nu}\pi^\mu\pi^\nu + d_3(\partial\pi)^4 + d_4(\partial\pi)^2\Box\pi\Big)\Big] \qquad (3)$$

with

$$d_1 = \frac{N(N-1)}{4} \qquad (4)$$

$$d_2 = -N(N-1) \qquad (5)$$

$$d_3 = -\frac{N(N-1)^2(N-2)}{16} \qquad (6)$$

$$d_4 = -\frac{N(N-1)(N-2)}{4} \qquad (7)$$

It should be noticed that the reduced action does not depend on D-coefficients.

It is interesting to observe that the action in $D+1$ dimensions reduces to the same form as the original one with a global factor.

$$\mathcal{S} = \int \mathrm{d}^D\mathrm{x}\sqrt{-g^{(D)}}\,e^{\pi/2}\left[R + \alpha R_{\mathrm{GB}}\right] \qquad (8)$$

Next, we perform a conformal transformation (in D-dimensions) for transforming the action (3) to a convenient form. After rescaling the fields

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{N\pi/(D-2)}g_{\mu\nu}, \quad \pi \to \pm\sqrt{\frac{2(D-2)}{N(D+N-2)}}\,\pi \qquad (9)$$

and carrying out integrations by parts, we obtain,

$$\mathcal{S} = \int \mathrm{d}^D\mathrm{x}\sqrt{-g^{(D)}}\Big[R^{(D)} - \frac{1}{2}(\partial\pi)^2 + \alpha\,e^{\beta_D\pi}\Big(R_{\mathrm{GB}}$$
$$+ b_1 G_{\mu\nu}\pi^\mu\pi^\nu + b_2(\partial\pi)^2\Box\pi + b_3\left(\partial\pi\right)^4\Big)\Big]$$
$$+ \mathcal{S}_m[e^{-\beta_D\pi}g_{\mu\nu};\psi_m] \qquad (10)$$

where we defined

$$b_1 = 2\frac{D-2}{D+N-2}\left(1 + N\frac{D-4}{(D-2)^2}\right) \qquad (11)$$

$$b_2 = \pm\sqrt{\frac{2}{N}}\left(\frac{D-2}{D+N-2}\right)^{3/2}\left(\frac{N^2}{(D-2)^2} - 1\right) \qquad (12)$$

$$b_3 = -\frac{N^2(D-4) + DN(D-2) - 2(D-2)^2}{4N(D-2)(D+N-2)} \qquad (13)$$

$$\beta_D = \pm\sqrt{\frac{2N}{(D-2)(D+N-2)}} \qquad (14)$$

and $\mathcal{S}_m$ is the matter action with matter fields ψ_m.

We drop in the action the tilde and terms that are total derivatives. We notice that the conformal transformation gives the action an explicit dependance on the dimension D.

In 4-dimensions, the action reduces to

$$\mathcal{S} = \frac{1}{2}\int \mathrm{d}^4\mathrm{x}\sqrt{-g}\Big[R - \frac{1}{2}(\partial\pi)^2 + \alpha e^{\beta\pi}\Big(R_{\mathrm{GB}}$$
$$+ c_1 G_{\mu\nu}\pi^\mu\pi^\nu + c_2(\partial\pi)^2\Box\pi + c_3\left(\partial\pi\right)^4\Big)\Big]$$
$$+ \mathcal{S}_m[e^{-\beta\pi}g_{\mu\nu};\psi_m] \qquad (15)$$

with

$$\beta \equiv \beta_4 = \pm\sqrt{\frac{N}{N+2}}, \qquad (16)$$

$$c_1 = \frac{4}{N+2}, \qquad (17)$$

$$c_2 = \frac{N-2}{N}\beta, \qquad (18)$$

$$c_3 = -\frac{N-1}{N(N+2)}. \qquad (19)$$

In what follows, we shall investigate the dynamics based upon the action (15).

III. FIELD EQUATIONS OF MOTION

The field equations one derives from the action (15) are

$$\Box\pi + \alpha e^{\beta\pi}K = \beta T, \qquad (20)$$

$$G_{\mu\nu} - \frac{1}{2}\left(\pi_\mu\pi_\nu - \frac{1}{2}g_{\mu\nu}(\partial\pi)^2\right) + \alpha e^{\beta\pi}\Sigma_{\mu\nu} = T_{\mu\nu}. \qquad (21)$$

with

$$K = \beta R_{\mathrm{GB}} - c_1 G_{\mu\nu}\left(\beta\pi^\mu\pi^\nu + 2\pi^{\mu\nu}\right) - 4c_3(\partial\pi)^2\Box\pi$$
$$+ 4\left(\beta c_2 - 2c_3\right)\pi^{\mu\nu}\pi_\mu\pi_\nu + \beta\left(\beta c_2 - 3c_3\right)(\partial\pi)^4$$
$$- 2c_2\left[(\Box\pi)^2 - \pi^{\mu\nu}\pi_{\mu\nu} - R_{\mu\nu}\pi^\mu\pi^\nu\right], \qquad (22)$$

$$\Sigma_{\mu\nu} = -2\left(\beta\pi^{\rho\sigma} + (\beta^2 + \frac{c_1}{4})\pi^\rho\pi^\sigma\right)\left[2R_{\rho\mu\nu\sigma} + 2\left(g_{\mu\nu}R_{\rho\sigma}\right.\right.$$
$$\left.\left. + R_{\mu\nu}g_{\rho\sigma} - 2g_{\rho(\mu}R_{\nu)\sigma}\right) + R(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\sigma}g_{\mu\nu})\right]$$
$$+ \frac{1}{2}g_{\mu\nu}\left(\beta c_2 - c_3\right)(\partial\pi)^4 + (2c_3 - \beta c_2)(\partial\pi)^2\pi_\mu\pi_\nu$$
$$+ \left(c_2 - \frac{\beta}{2}c_1\right)\left[\pi_\mu\pi_\nu\Box\pi - 2\pi^\sigma\pi_{\sigma(\mu}\pi_{\nu)} + g_{\mu\nu}\pi^\rho\pi_{\rho\sigma}\pi^\sigma\right]$$
$$+ c_1\left[\pi_{\mu\rho}\pi^\rho_{\ \nu} - \pi_{\mu\nu}\Box\pi + \frac{1}{2}G_{\mu\nu}(\partial\pi)^2\right.$$
$$\left. + \frac{1}{2}g_{\mu\nu}\left((\Box\pi)^2 - \pi_{\rho\sigma}\pi^{\rho\sigma}\right) + \frac{\beta}{2}(\partial\pi)^2\left(g_{\mu\nu}\Box\pi - \pi_{\mu\nu}\right)\right]. \qquad (23)$$

In the analysis to follow, we shall focus on equations of motion (20) to study mass screening induced by non-linear terms in a simple tractable background.

IV. MASS SCREENING – VAINSHTEIN MECHANISM

In order to investigate the effects of the non linear terms in the action, we shall study the model in a flat spherically symmetric background.

In this case, the theory reduces to a special case of KGB [28] coupled to matter,

$$\mathcal{S} = \frac{1}{2}\int \mathrm{d}^4\mathrm{x}\left[K(\pi, X) + G(\pi, X)\Box\pi\right] + \mathcal{S}_m[e^{-\beta\pi}g_{\mu\nu}; \psi_m] \qquad (24)$$

with

$$K(\pi, X) = X - 4\alpha\frac{N-1}{N(N+2)}e^{\beta\pi}X^2 \qquad (25)$$

$$G(\pi, X) = -2\alpha\beta\frac{N-2}{N}e^{\beta\pi}X, \text{ and } X = -\frac{(\partial\pi)^2}{2} \qquad (26)$$

And the special case of $N = 2$ gives rise to K-essence. The equation of motion is

$$\pi'' + 2\frac{\pi'}{r} + \alpha e^{\beta\pi}\frac{\pi'}{N(2+N)r^2}\left[8(N-1)r\pi'^2\right.$$
$$+ (N^2 + N - 3)r^2\pi'(\beta\pi'^2 + 4\pi'')$$
$$\left. - 4\beta(N^2 - 4)(\pi' + 2r\pi'')\right] = -\beta\rho \qquad (27)$$

If we integrate this equation from $r = 0$ to a distance outside the body in the case of $\alpha = 0$, we have $\pi' = -\frac{\beta r_s}{r^2}$ and the fifth force is of the order of the gravitational force,

$$\left|\frac{\mathcal{F}_\pi}{\mathcal{F}_g}\right| = \frac{\beta r^2}{r_s}|\pi'| = \frac{N}{N+2} \simeq 1 \qquad (28)$$

Let us note that we have two different values of β; it is easy to see that if we change $\beta \to -\beta$ the action remains unchanged provided that, $\pi \to -\pi$. Therefore the fifth force is invariant under the change of sign of β.

Unfortunately we can not get analytical solutions in case $\alpha \neq 0$. However, we can have derive asymptotic solutions at large and short distances.

At large distances, we observed that the solution is trivial. This solution should change as we approach the source of matter because of the effect of the non linear terms. Therefore these corrections to the asymptotic solutions become crucial when we enter the Vainshtein radius.

We define this scale as the radius where a perturbation of this trivial solution becomes important. It is easy to show that the Vaishtein radius can be approximated by

$$\text{For } N = 2, \ R_V^4 \simeq \alpha r_s^2 \qquad (29)$$

$$\text{For } \forall \ N \neq 2, \ R_V^3 \simeq \alpha r_s \qquad (30)$$

In case, $\sqrt{\alpha}$ is of the order of the Hubble scale ($\tilde{\alpha} \equiv H_0^2\alpha = 1$), we have for $N = 1$ $R_V \simeq 10^2$pc and $R_V \simeq 2.10^{-2}$pc for $N = 2$, which is in perfect agreement with the Fig.1, where we considered the Sun as the central body.
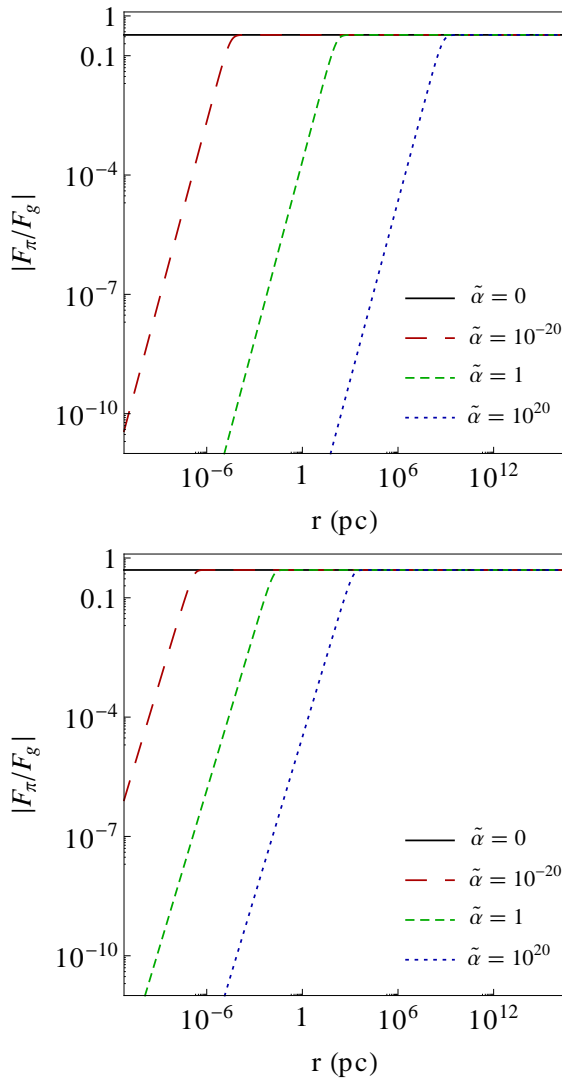
Figure 1: (Top):The ratio of the fifth force and the gravitational force versus the distance from the source in parsecs, for $N = 1$. In the numerics we considered $r_s \equiv r_s(\text{Sun})$ and $\bar{\alpha} \equiv H_0^2 \alpha$. (Bottom):The same evolution for $N = 2$. The fifth force is negligeable at small distances compared to the gravitational force.

In the Fig.1, we show that for $N = 1$ and $N = 2$ there is a screening effect at distances smaller that the Vainshtein radius. For dimensions larger than 2, the evolution of the fifth force versus the radial coordinate is the same as in case of $N = 1$. In fact for $N = 2$ we don't have the G-Essence term[1] which gives an additional effect to the screening mechanism as it can be seen in the Fig.1.

We also find from numerical analysis, that for the solution to be continuous, we need for $N \leq 2$ in case of

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[1] The extra term in the Lagrangian compared to K-Essence: $G(\pi, X)$

$\alpha > 0$ and $\alpha < 0$ for $N > 2$.

V. STABILITY OF SOLUTIONS

We consider the test field approximation where we expand the field $\pi \to \pi + \phi$ and neglecting the back reaction on the metric. The equation for the scalar field (ϕ) in the first order are given by,

$$G_{\text{eff}}^{\mu\nu} \phi_{\mu\nu} + V^\mu \phi_\mu + M\phi = 0 \tag{31}$$

where the induced metric is

$$G_{\text{eff}}^{\mu\nu} = Ag^{\mu\nu} + B\pi^\mu \pi^\nu + C\pi^{\mu\nu} \tag{32}$$

with

$$A = 1 + \frac{4\alpha}{N} e^{\beta\pi} \left(\frac{N-1}{N+2} \pi'^2 - \beta(N-2) \left(\pi'' + \frac{2}{r}\pi' \right) \right)$$

$$B = 4 \frac{N^2 - 2}{N(N+2)} \alpha e^{\beta\pi}$$

$$C = 4\beta \frac{N-2}{N} \alpha e^{\beta\pi} \tag{33}$$

The field equation admits a well-posed initial value-formulation locally if the effective metric $G_{\text{eff}}^{\mu\nu}$ is Lorentzian.

The equation (31) can be expanded as

$$G_{\text{eff}}^{00}\partial_t^2\phi + G_{\text{eff}}^{11}\partial_r^2\phi + G_{\text{eff}}^{22}r^2\partial_\Omega^2\phi + \text{ first derivatives of } \phi + \cdots = 0 \tag{34}$$

where $\partial_\Omega^2\phi$ is the angular part of the Laplacian.

This equation is Lorentzian with a signature $(-,+,+,+)$, if we have $G_{\text{eff}}^{00} < 0$, $G_{\text{eff}}^{11} > 0$ and $G_{\text{eff}}^{22} > 0$.

These conditions are exactly the same as the ghost free condition and the stability of the Laplacian used in the literature.

In fact the equation (34) can be derived from the action

$$\delta^2 S = \int -\frac{1}{2} G_{\text{eff}}^{00} \left[(\partial_t\phi)^2 - c_r^2 (\partial_r\phi)^2 - c_\Omega^2 (\partial_\Omega\phi)^2 \right] \mathrm{d}^4 x \tag{35}$$

where

$$c_r^2 = -\frac{G_{\text{eff}}^{11}}{G_{\text{eff}}^{00}} = 1 + \frac{B}{A}\pi'^2 + \frac{C}{A}\pi'' \tag{36}$$

$$c_\Omega^2 = -r^2 \frac{G_{\text{eff}}^{22}}{G_{\text{eff}}^{00}} = 1 + \frac{C}{rA}\pi' \tag{37}$$

The ghost condition fixes the sign of $G_{\text{eff}}^{00} < 0$ and $G_{\text{eff}}^{11} > 0$ and $G_{\text{eff}}^{22} > 0$ via the positivity of the sound

speed $c_r^2 > 0$, $c_\Omega^2 > 0$(also know as the stability of the Laplacian).

When the non linear terms (α-terms) are dominant, we can reduce the evolution equation for the scalar field to

$$N = 2, \quad 2\pi' + 3r\pi'' = 0 \Rightarrow \pi' \propto r^{-2/3}, \qquad (38)$$

$$N \neq 2, \quad \pi' + 2r\pi'' = 0 \Rightarrow \pi' \propto r^{-1/2}. \qquad (39)$$

Thus it becomes easy to estimate the sound speed at small distances,

$$N = 2, \quad c_r^2 = 3, \quad c_\Omega^2 = 1 \qquad (40)$$

$$N \neq 2, \quad c_r^2 = 4/3, \quad c_\Omega^2 = 1/3. \qquad (41)$$

The propagation of the perturbation of the scalar field is therefore superluminal in the radial direction $c_r^2 > 1$ at small distances. However, we shall demonstrate in the next section for the special case $N = 2$ that the superluminal does not implies a non causal propagation of the perturbations. It is in fact just a redefinition of the maximum sound speed via a larger light cone structure of the space time compared to the Minkowsky space.

VI. SPECIAL CASE OF $N = 2$

In the particular case of $N = 2$ where an emergent geometry is present (see [29] for more details), we have

$$c_r^2 = \frac{1 - 3\alpha X e^{\beta\pi}}{1 - \alpha X e^{\beta\pi}} \qquad (42)$$

Hence as soon as the non linear (α) term becomes dominant, we have $c_r^2 \propto 3$.

The non linear terms which are necessary for local constraints create automatically a superluminal propagation. This behavior is present in models involving Galileons and their extensions [18, 30, 31].

We should, however, emphasize that we do not have this propagation in the Mikowski space-time but in an extended structure of space-time because of the non linear terms.

In fact, in the special case of $N = 2$, the model reduces to a particular K-essence; therefore by performing a conformal transformation we have,

$$\tilde{G}_{\text{eff}}^{\mu\nu} = \frac{1}{K_{,X}^2 c_r} G_{\text{eff}}^{\mu\nu} \qquad (43)$$

We can rewrite the equation for perturbations as [29]

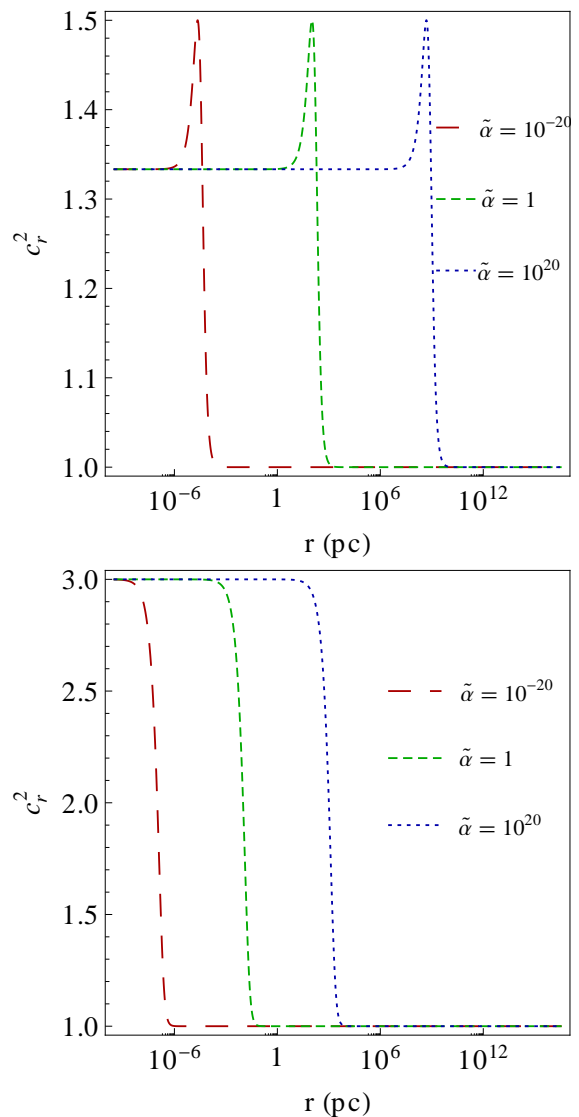$$\tilde{G}_{\text{eff}}^{\mu\nu} D_\mu D_\nu \phi - M_{\text{eff}}^2 \phi = 0 \qquad (44)$$

Figure 2: (Top): The radial sound speed versus the distance in parsecs for $N = 1$, (Bottom): The same figure for $N = 2$

where D_μ is the covariant derivative associated to the effective metric $\tilde{G}_{\text{eff}}^{\mu\nu}$.

and

$$M_{\text{eff}}^2 = -\frac{\alpha}{K_{,X}^2 c_r} \left(\frac{3}{4} X^2 - \beta X \Box\pi + \beta\pi^\mu \pi_{\mu\nu} \pi^\nu \right) e^{\beta\pi} \qquad (45)$$

We notice the difference with [29] where the scalar field was time dependant [2].

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[2] In fact, if $\pi \equiv \pi(t)$, the sound speed in an homogenous and isotropic Universe is $c_s^2 = (1 - 2X \frac{K_{,XX}}{K_{,x}})^{-1}$ but in the case where $\pi \equiv \pi(r)$ the sound speed in the radial direction is $c_r^2 = 1 + 2X \frac{K_{,XX}}{K_{,x}}$.
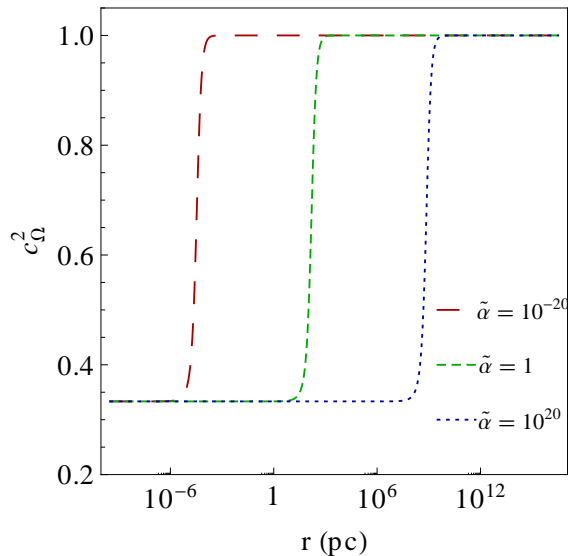
Figure 3: The angular sound speed versus the radial distance in parsecs for $N = 1$

It is also interesting to note that the mass term is null when $\alpha \to \infty$.

Therefore the emergent geometry defined by the metric $\tilde{G}^{\mu\nu}_{\text{eff}}$ defines a new structure of the space-time, the light cone is larger than the standard one if and only if $\frac{K_{,XX}}{K_{,X}} < 0$. In the limit where the non linear terms are dominant, we have $\frac{K_{,XX}}{K_{,X}} = -\frac{2}{\pi'^2} < 0$.

We emphasize also that this new structure of the space-time is stably causal. In fact the Minkowski time could define a future directed timelike vector field,

$$\tilde{G}^{\mu\nu}_{\text{eff}} \partial_\mu t \partial_\nu t = -\frac{1}{K_{,X} c_r} \qquad (46)$$

which is negative as soon as the hyperbolicity conditions are satisfied.
Therefore no CCCs (Causal Closed Curves) can exist.

The superluminal propagation does not create any causal inconsistencies. In fact the perturbations of the scalar field do not propagate in the Minkowski space-time but rather in some form of "aether" because of the presence of the background field $\pi(r)$. The maximum of the speed of the field is just a redefinition of the speed of light in this new space-time. The causal structure is not changed, in the sense that we do not have CCCs in this case.

VII. BACKGROUND COSMOLOGICAL DYNAMICS

We concentrate on spatially flat Friedman-Lemaître-Robertson-Walker (FLRW) universes with scale factor $a(t)$,

$$ds^2 = -dt^2 + a^2 d\mathbf{x}^2 \qquad (47)$$

$$\ddot{\pi} + 3H\dot{\pi} - \alpha e^{\beta\pi} K = \beta\rho, \qquad (48)$$

$$3H^2 - \frac{1}{4}\dot{\pi}^2 - \alpha e^{\beta\pi} \Sigma^0_{\ 0} = \rho, \qquad (49)$$

$$3H^2 + 2\dot{H} + \frac{1}{4}\dot{\pi}^2 - \alpha e^{\beta\pi} \Sigma^1_{\ 1} = 0. \qquad (50)$$

The following equations are obtained

$$\Sigma^0_{\ 0} = -12\beta H^3 \dot{\pi} + \frac{9}{2} c_1 H^2 \dot{\pi}^2 - \frac{c_2}{2} \dot{\pi}^3 (\beta\dot{\pi} - 6H)$$
$$+ \frac{3}{2} c_3 \dot{\pi}^4, \qquad (51)$$

$$\Sigma^1_{\ 1} = -4\left[(\beta\ddot{\pi} + \beta^2\dot{\pi}^2 + 2\beta H\dot{\pi})H^2 + 2\beta H\dot{H}\dot{\pi}\right]$$
$$+ \frac{1}{2} c_1 \dot{\pi}\left[\dot{\pi}(2\dot{H} + 3H^2 + 2\beta H\dot{\pi}) + 4H\ddot{\pi}\right]$$
$$+ \frac{1}{2} c_2 \dot{\pi}^2(2\ddot{\pi} + \beta\dot{\pi}^2) - \frac{1}{2} c_3 \dot{\pi}^4, \qquad (52)$$

$$K = 24\beta H^2(H^2 + \dot{H}) - 3c_1\left[H^2(\beta\dot{\pi}^2 + 2\ddot{\pi} + 6H\dot{\pi})\right.$$
$$\left. + 4H\dot{H}\dot{\pi}\right] + c_2\dot{\pi}\left[\dot{\pi}(4\beta\ddot{\pi} + \beta^2\dot{\pi}^2 - 6\dot{H} - 18H^2)\right.$$
$$\left. - 12H\ddot{\pi}\right] - 3c_3\dot{\pi}^2\left[\beta\dot{\pi}^2 + 4H\dot{\pi} + 4\ddot{\pi}\right]. \qquad (53)$$

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble rate while a dot stands for a derivative with respect to the cosmic time t.

VIII. STABILITY CONDITIONS IN AN FLRW UNIVERSE

In order to derive the stability conditions of the theory in the context of an isotropic and homogeneous Universe, we study linear perturbation in a FLRW background. We consider the following metric

$$ds^2 = -(1 + 2\alpha)\, dt^2 - a\beta_{,i} dt dx^i$$
$$+ a^2 \left[\delta_{ij}(1 + 2\psi) + 2\gamma_{;ij} + 2h_{ij}\right] dx^i dx^j. \qquad (54)$$

where α, β, ψ and γ are scalar metric perturbations, and h_{ij} is the traceless and divergence-free tensor perturbations.

We did not considered vector perturbations in the line element because of the absence of an anisotropic fluid.

A. Scalar perturbations

For scalar perturbations, we can neglect the matter contributions at late times, and it was noticed in that

the calculations simplify if we work in the uniform-field gauge $\delta\pi = 0$ [32].

Therefore we can show that the action at the second order can be written in the following form

$$\delta^2 S = \frac{1}{2}\int \mathrm{dx}^3 \mathrm{dt} a^3 Q^{(s)}\left[\dot\psi^2 - \frac{c_s^{2(s)}}{a^2}(\partial_i\psi)^2\right] \qquad (55)$$

where

$$Q^{(s)} = \frac{\dot\pi^2 + 6\frac{\left(Q_a^{(s)}\right)^2}{2+Q_b^{(s)}} + 2Q_c^{(s)}}{\left(H + \frac{Q_a^{(s)}}{2+Q_b^{(s)}}\right)^2},$$

$$c_s^{2(s)} = 1 + 2\frac{Q_d^{(s)} + \frac{Q_a^{(s)}Q_e^{(s)}}{2+Q_b^{(s)}} - \left(\frac{Q_a^{(s)}}{2+Q_b^{(s)}}\right)^2 Q_f^{(s)}}{\frac{\dot\pi^2}{2} + 3\frac{\left(Q_a^{(s)}\right)^2}{2+Q_b^{(s)}} + Q_c^{(s)}}. \qquad (56)$$

where we defined

$$Q_a^{(s)} = \alpha\left[4\beta H^2 - 2c_1\dot\pi H - c_2\dot\pi^2\right]\dot\pi e^{\beta\pi}, \qquad (57)$$

$$Q_b^{(s)} = \alpha\left[8\beta H - c_1\dot\pi\right]\dot\pi e^{\beta\pi}, \qquad (58)$$

$$Q_c^{(s)} = \alpha\left[3c_1 H^2 + 2c_2\dot\pi\left(3H - \beta\dot\pi\right) + 6c_3\dot\pi^2\right]\dot\pi^2 e^{\beta\pi}, \qquad (59)$$

$$Q_d^{(s)} = \alpha\left[c_1\dot H + c_2\left(\beta\dot\pi^2 + \ddot\pi - \dot\pi H\right) - 2c_3\dot\pi^2\right]\dot\pi^2 e^{\beta\pi}, \qquad (60)$$

$$Q_e^{(s)} = \alpha\left[8\beta\dot H - c_1\left(\beta\dot\pi^2 + 2\ddot\pi - 2\dot\pi H\right) + 2c_2\dot\pi^2\right]\dot\pi e^{\beta\pi}, \qquad (61)$$

$$Q_f^{(s)} = \alpha\left[4\left(\beta\ddot\pi + \beta^2\dot\pi^2 - \beta\dot\pi H\right) + c_1\dot\pi^2\right]e^{\beta\pi}. \qquad (62)$$

We recover the results derived in [33].

The ghost condition fixes the sign of $Q^{(s)} > 0$, and also the positivity condition of the sound speed fixes $c_s^{(s)} > 0$. As we mentioned before that hyperbolicity of equations of motion implies that we have a well posed Cauchy problem (at least locally).

B.   Tensor perturbations

We can also show that for the tensor perturbations we have

$$\frac{1}{a^3 Q^{(t)}}\left(a^3 Q^{(t)}\dot h_j^i\right)^\cdot - c_s^{(t)}\frac{\Delta}{a^2}h_j^i = \frac{1}{Q^{(t)}}\delta T^{(t)i}{}_j, \qquad (63)$$

where

$$Q^{(t)} = 2 + \alpha\left(8\beta H - c_1\dot\pi\right)\dot\pi e^{\beta\pi},$$

$$c_s^{(t)} = \frac{1}{Q^{(t)}}\left[2 + \alpha\left(8\left(\beta\ddot\pi + \beta^2\dot\pi^2\right) + c_1\dot\pi^2\right)e^{\beta\pi}\right]. \qquad (64)$$

Similar to the case of scalar perturbations, we have the two conditions of stability,

$$Q^{(t)} > 0, \quad c_s^{(t)} > 0 \qquad (65)$$

IX.   CONCLUSION AND PERSPECTIVES

In this paper we have studied simple extension of General Relativity in the context of Lovelock theory. We derived the equations of the reduced theory in 4 dimensions. We have shown that locally in a flat spherically symmetric background, the non linear terms coming from the Gauss-Bonnet term in higher dimensions induce Vainshtein mechanism. We found that in $4 + 2$-dimensions, the Vainshtein radius can be approximated by, $R_V^4 \simeq \alpha r_s^2$ whereas in case of $4 + N$-dimensions by $R_V^3 \simeq \alpha r_s$ with $N \neq 2$. We have shown that at distances lower than the Vainshtein radius, the fifth force is negligibly small compared to the gravitational force.

We have investigated the behavior of the scalar field inside the Vainshtein sphere and derived stability conditions of the model. We have reaffirmed that the hyperbolicity of the equations is equivalent to the ghost condition and the stability of the Laplacian. We found that the model has a superluminal propagation of the perturbation of the scalar field in the flat spherically symmetric solution. This faster than light solution appears as soon as the non linear terms of the model become dominant. We have shown, in the special case of $N = 2$ that the causality structure of the space time is well defined even in the presence of the superluminal propagation. In fact, we shown that the field propagates in a space-time which is not anymore the Minkowski one but some kind of "aether" because of the presence of the background field $\pi(r)$. This modification of the structure of the space-time is related to the domination of the non linear terms in the Lagrangian. We observed that the light cone gets wider in the aether as compared to the case of Minkowski space-time provided that the stability conditions hold thereby demonstrating that no CCCs appears even in the presence of the superluminal propagation.

Finally in the context of an isotropic and homogeneous Universe, we derived Friedmann equations for the field and established the stability conditions in the context of the scalar and tensor perturbations.

It will be interesting to investigate the cosmological dynamics and observational constraints on the model under consideration in a separate publication. It is also important to investigate the model with general Lovelock terms in simple and non-trivial topology of extra dimensions. We defer this work to our future investigations.

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