Quantum fluctuations in planar domain-wall space-times: A possible origin of primordial preferred direction

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We study the gravitational effects of a planar domain wall on quantum fluctuations of a massless scalar field during inflation. By obtaining an exact solution of the scalar field equation in de Sitter space, we show that the gravitational effects of the domain wall break the rotational invariance of the primordial power spectrum without affecting translation invariance. The strength of rotational violation is determined by one dimensionless parameter β, which is a function of two physical parameters, the domain wall surface tension σ and cosmological constant Λ. In the limit of small β, the leading effect of rotational violation of the primordial power spectrum is scale-invariant.

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Introduction. Inflationary cosmology was originally proposed to solve the horizon, flatness, and monopole problems [1–3]. The monopole problem, or, more generally, the topological defect problem, arises when an early epoch of symmetry breaking produces defects such as monopoles, cosmic strings or domain walls. These objects redshift more slowly than radiation and would come to dominate the energy density of the universe (or leave a signal in the CMB), in conflict with observation [4]. Because physical distances increase exponentially during inflation, the number density of topological defects is driven to zero. Heavy topological defects (i.e., associated with energy scales larger than that of inflation) then presumably leave almost no detectable evidence, and the only defects we can observe today are those arising from phase transitions which occurred after inflation [5].

In this Letter, we show that gravitational fields of heavy domain walls will affect primordial density fluctuations at the early stage of inflation. If these density fluctuations have re-entered the horizon, they can leave an imprint on CMB anisotropies. We calculate the quantum fluctuations of a massless scalar field in the presence of an infinite planar domain wall and a positive cosmological constant Λ. The physical wavelengths of fluctuation modes increase exponentially during inflation: $\lambda_p \sim \lambda \exp(\sqrt{\Lambda/3}\tau)$. Fluctuations near a domain wall are affected by its gravitational field as they are stretched out beyond the horizon. By obtaining an exact solution of the scalar field equations in this geometry, we show that the gravitational effects of the domain wall on the primordial power spectrum will remain after inflation.

We assume an infinite domain wall because it simplifies our calculations. In a realistic phase transition there are no truly infinite domain walls, only closed walls that divide space into disjoint regions. The typical radius of curvature ξ of closed domain walls is no larger than the horizon size d_H: $\xi \lesssim d_H$, due to causality [6]. However, for short wavelength fluctuation modes ($\lambda << \xi$) in regions near a closed wall, the gravitational effects are similar to those computed from an infinite wall. If our visible universe originated from such a pre-inflationary region, the effect of the domain wall may still be observable and can be calculated using the methods developed here.

Violation of rotational and translational symmetry in the primordial power spectrum has been investigated recently [7, 8], motivated in part by possible large-scale CMB anomalies, where the quadrupole and octopole of the CMB have an apparent alignment with each other [9, 10]. A possible explanation for the anomalies is a preferred direction in the primordial power spectrum [7, 11]. In this Letter, we show that the gravitational field of a planar domain wall will naturally cause the violation of rotational symmetry in the power spectrum without breaking translational invariance.

Planar domain walls in de-Sitter space-time. In the standard cosmological model, the Universe is homogeneous and isotropic with respect to cosmic time evolution. Domain walls, once formed, will evolve to minimize their surface area, subject to interactions with the background enviroment [6]. If the interactions are significant this motion can be overdamped and the wall motion relatively slow. We neglect any motion relative to the thermal rest frame and take the wall to be comoving along the cosmic time direction.

The metric of a planar domain wall in de-Sitter space-time with reflection symmetry has been obtained in [12]:

$$ds^2 = \frac{1}{\alpha^2\,(\eta+\beta|z|)^2}(-d\eta^2 + dz^2 + dx^2 + dy^2), \qquad (1)$$

where the wall is placed at $z = 0$. $\alpha = \sqrt{\Lambda/12\Gamma}(\Gamma+1)$, $\beta = (\Gamma-1)/(\Gamma+1)$, satisfying $-1 < \beta \leqslant 0$, and Γ is a dimensionless parameter

$$\Gamma = 1 + \frac{3\epsilon - \sqrt{48\epsilon + 9\epsilon^2}}{8}, \qquad (2)$$

where $\epsilon = \kappa^2\sigma^2/\Lambda$ and σ is the surface tension of the domain wall. Eq. (2), which gives $0 < \Gamma \leqslant 1$, is only valid for the coordinate ranges $-\infty < \eta + \beta|z| < 0$. When $\sigma = 0$, the metric (1) is simply that of a steady-state Universe in the conformal time [13]. Throughout this Letter we use the units $c = \hbar = 1$ and $\kappa = 8\pi G$.

Domain walls produce repulsive gravitational forces [14]. To understand the gravitational effects of metric (1), we consider observers stationary relative to the wall on the $z > 0$ side, with 4-velocities described by a future-pointing unit time-like vector field $U = -\alpha(\eta + \beta z)\partial_\eta$. Their 4-acceleration, which is defined by $\mathcal{A} \equiv \nabla_U U$, has a constant magnitude $|\mathcal{A}| \equiv \sqrt{g(\mathcal{A}, \mathcal{A})} = |\alpha\beta| = \kappa\sigma/4$ and z-component $A_z \equiv g(\nabla_U U, -\alpha(\eta + \beta z)\partial_z) = -\kappa\sigma/4$, where the minus sign denotes the acceleration toward the wall. This implies that the gravitational field of a planar domain wall produces a constant repulsive force on each observer, independent of their distance from the wall. For this reason, translation invariance is not violated by the gravitational field of the wall. Because of the reflection symmetry, the same is true for the $z < 0$ side.

In the coordinates $\check{\eta} = (\eta + \beta z)/\sqrt{1-\beta^2}$, $\check{z} = (z + \beta\eta)/\sqrt{1-\beta^2}$, $\check{x} = x$ and $\check{y} = y$, the metric (1) becomes (for $z > 0$)

$$ds^2 = \frac{1}{\frac{\Lambda}{3}\check{\eta}^2}(-d\check{\eta}^2 + d\check{z}^2 + d\check{x}^2 + d\check{y}^2), \tag{3}$$

which describes a steady-state Universe in conformal time. However, if one uses the coordinates $(\check{\eta}, \check{z}, \check{x}, \check{y})$ on the $z < 0$ side, the metric becomes $ds^2 = (\frac{\Lambda}{3}(\check{\eta} - \frac{2\beta\check{z}}{1-\beta^2})^2)^{-1}(-d\check{\eta}^2 + d\check{z}^2 + d\check{x}^2 + d\check{y}^2)$. The coordinate transformations between (η, z, x, y) and $(\check{\eta}, \check{z}, \check{x}, \check{y})$ are very similar to Lorentz transformations (boosts) and β is analogous to the relative velocity of two inertial frames. One may notice that the wall is not stationary in $(\check{\eta}, \check{z}, \check{x}, \check{y})$. It is known that the motion of a uniformly accelerated observer O along the x^1 direction in Minkowski space-time with the Minkowski coordinates (x^0, x^1, x^2, x^3) is described by $x^0 = A^{-1}\sinh A\tau, x^1 = A^{-1}\cosh A\tau$ and $(x^2, x^3) = \text{const.}$, where $A = |\mathcal{A}|$ and τ is the proper time of the observer [15]. So O's trajectory is hyperbolic, i.e. $(x^1)^2 - (x^0)^2 = A^{-2}$, in Minkowski space-time. However, the wall's motion in de-Sitter space with the coordinates $(\check{\eta}, \check{z}, \check{x}, \check{y})$ gives $\check{\eta} = -e^{-\alpha\tau}/\alpha\sqrt{1-\beta^2}$ and $\check{z} = -\beta e^{-\alpha\tau}/\alpha\sqrt{1-\beta^2}$, where τ is the wall's proper time. It turns out that the wall's trajectory, which has constant magnitude of acceleration $|\alpha\beta|$, is a straight line $\check{z} = \beta\,\check{\eta}$ in de-Sitter space with the coordinates $(\check{\eta}, \check{z}, \check{x}, \check{y})$. In the coordinates $(\check{\eta}, \check{z}, \check{x}, \check{y})$, stationary observers, who are in relative motion with respect to the wall with 4-velocities $-\sqrt{\Lambda/3}\check{\eta}\,\partial_{\check{\eta}}$, will follow geodesics. We conclude that the stationary observers associated with two different coordinates (η, z, x, y) and $(\check{\eta}, \check{z}, \check{x}, \check{y})$ will correspond to uniformly accelerated observers and geodesic observers, respectively.

Before we discuss quantum fluctuations, it is helpful to describe the metric (1) by introducing proper-time coordinate $\tau = -\frac{1}{\alpha}\ln[-\alpha(\eta \pm \beta z)]$ and $z' = \sqrt{1-\beta^2}z$, so Eq. (1) becomes

$$ds^2 = -d\tau^2 \pm \frac{2\beta\, e^{\alpha\tau}}{\sqrt{1-\beta^2}}d\tau dz' + e^{2\alpha\tau}(dz'^2 + dx^2 + dy^2), \tag{4}$$

where $\pm$ corresponds to $z' > 0$ and $z' < 0$ sides, respectively. It is clear that the metric (4) also has the reflection symmetry about $z' = 0$. Moreover, the stationary observers, whose 4-velocities are ∂_τ, also have constant acceleration $|\mathcal{A}| = |\alpha\beta|$. For $\beta = 0$, i.e. $\sigma = 0$, the metric (4) becomes the metric (3) in $(\check{\tau}, \check{z}, \check{x}, \check{y})$ coordinates, where $\check{\tau} = -\sqrt{3/\Lambda}\ln(-\sqrt{\Lambda/3}\check{\eta})$. Since the metric (1) is z-dependent, one might expect that the primordial density fluctuations will violate translational invariance. On the other hand, as discussed above, the gravitational force due to a planar domain wall is z-independent, so density fluctuations should be translationally invariant. From the metric (4), it becomes clear that the primordial power spectrum will be translation invariant, since the metric (4) only depends on τ. Moreover, the appearance of the cross term $g_{\tau z}$ indicates that the gravitational effects of planar domain walls will break the rotational invariance, i.e. $O(3)$ symmetry, of the power spectrum. In post-Newtonian theory [15], the metric components g_{0i} are associated with the angular momentum of gravitating sources.

Quantum fluctuations in planar domain-wall space-times. Quantum fluctuations in de-Sitter space-time have been widely studied [3, 13, 16]. In particular, it is known that time-like geodesic observers in de-Sitter space-time will detect thermal radiation with temperature $T = \sqrt{\Lambda/12\pi^2}$ [17]. A stationary observer with 4-velocity ∂_τ in de-Sitter space-time, which is described by the metric (4) with vanishing β, will perceive an isotropic thermal bath of radiation [13]. However, in the presence of a planar domain wall the stationary observer with velocity ∂_τ has constant acceleration A_z, so one should expect that, besides the particle production due to the de-Sitter horizon, the constantly accelerating observer should detect extra particles, which are associated with the acceleration A_z. A well-known example is the Unruh effect, which show that a constantly accelerating observer along z-axis in Minkowski space-time will see particles with temperature $T = A_z/2\pi$, though an inertial observer will detect no particles [18].

To understand the gravitational effects of a planar domain wall on primordial density fluctuations, we start from a massless scalar field ϕ satisfying the field equation

$$\mathrm{d} \star \mathrm{d}\phi = 0, \tag{5}$$

where d is the exterior derivative and $\star$ is the Hodge map associated with the metric g. Mode functions $\phi_{\check{\mathbf{k}}}$, which are exact solutions for $z > 0$, are

$$\phi_{\check{\mathbf{k}}}(\check{x}^i) = \check{\eta}^{\frac{3}{2}}\left[c_1(\check{k})H^{(1)}_{3/2}(\check{k}\check{\eta}) + c_2(\check{k})\,H^{(2)}_{3/2}(\check{k}\check{\eta})\right]e^{i\check{\mathbf{k}}\cdot\check{\mathbf{x}}}, \tag{6}$$

where $H^{(i)}_{3/2}$ are Hankel functions, $\check{k} \equiv (\check{k}_z^2 + \check{k}_x^2 + \check{k}_y^2)^{1/2}$ and $\check{x}^i = (\check{\eta}, \check{\mathbf{x}})$. For simplicity, we will only consider the solution $\phi_{\check{\mathbf{k}}}$ for $z > 0$. Using reflection symmetry, the $z < 0$

solution can be obtained. By noting that the metric (1) in the coordinates $(\check{\eta}, \check{z}, \check{x}, \check{y})$ is the metric (3), the normalization of Eq. (6) is straightforward and gives $|c_2(\check{k})|^2 - |c_1(\check{k})|^2 = \pi\Lambda/12$. The choice of $c_1(\check{k})$ and $c_2(\check{k})$ corresponds to the choice of vacuum state [16]. We require that when $\beta = 0$, the vacuum state is identical to the Bunch-Davies vacuum, i.e. $c_1(\check{k}) = 0$ and $c_2(\check{k}) = \sqrt{\pi\Lambda/12}$ [3, 16].

Since $\check{z}$ depends on the variable η, we should rewrite Eq. (6) in the coordinates (η, z, x, y). Furthermore, it is more useful to introduce the proper-time τ, which satisfies $e^{-\alpha\tau} = -\alpha(\eta + \beta z)$, so Eq. (6) becomes

$$\phi_{\mathbf{k}}(x^i) = \sqrt{\frac{\Lambda}{6}}\frac{1}{k\sqrt{k}}\left(\frac{1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}}{\sqrt{1-\beta^2}}\right)^{-3/2}\left(i + \frac{k}{\alpha(1-\beta^2)}e^{-\alpha\tau}(1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})\right)e^{i(k_z+\beta k)z+ik_x x+ik_y y+i\frac{k}{\alpha}e^{-\alpha\tau}}, \tag{7}$$

where $x^i = (\tau, x, y, z)$, $k \equiv (k_z^2 + k_x^2 + k_y^2)^{1/2} = \frac{\check{k}-\beta\check{k}_z}{\sqrt{1-\beta^2}}$, $k_z = \frac{\check{k}_z-\beta\check{k}}{\sqrt{1-\beta^2}}$, $k_x = \check{k}_x$ and $k_y = \check{k}_y$. $\hat{\mathbf{k}}$, $\hat{\mathbf{z}}$ are unit vectors and $\hat{\mathbf{k}}\cdot\hat{\mathbf{z}} = k_z/k$. When β goes to zero, Eq. (7) returns to the well-known solution of a massless scalar field in de-Sitter space-time. Moreover, the $\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}$ terms indicate the existence of a preferred direction in the primordial density spectrum. Eq. (7) also tells us that the rotational violation will appear not only in the low-frequency k modes but also high-frequency modes. It means that the gravitational effects of the constant acceleration will affect all frequency modes.

To quantize the ϕ field, one may expand ϕ in creation and annihilation operators, $a_{\mathbf{k}}\dagger$ and $a_{\mathbf{k}}$, as

$$\phi = \int \frac{d^3k}{(2\pi)^{3/2}}\; a_{\mathbf{k}}\phi_{\mathbf{k}}(x^i) + a^\dagger_{\mathbf{k}}\phi^*_{\mathbf{k}}(x^i), \tag{8}$$

with the vacuum state $|0\rangle$, satisfying $a_{\mathbf{k}}|0\rangle = 0$. The vacuum expectation value of ϕ^2 is $\langle\phi^2(x^i)\rangle = \frac{1}{(2\pi)^3}\int|\phi_{\mathbf{k}}(x^i)|^2 d^3k$. It is convenient to introduce physical momenta $p = ke^{-\alpha\tau}$, which are exponentially decreasing during inflation, to obtain

$$\langle 0|\phi^2(x^i)|0\rangle = \int\frac{d^3p}{(2\pi p)^3}\left[\frac{\Lambda}{6}\left(\frac{1\pm\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}}{\sqrt{1-\beta^2}}\right)^{-3} + \frac{\sqrt{1-\beta^2}\,p^2}{2(1\pm\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})}\right] \tag{9}$$

where $\pm$ denotes $\langle\phi^2(x^i)\rangle$ for $z > 0$ and $z < 0$ sides, respectively. For those p modes with physical wavelengths λ_p well inside the horizon, i.e. $p \gg \sqrt{\Lambda/3}$, the second term of Eq. (9) dominates and $\beta = 0$ simply gives the vacuum fluctuations in Minkowski space-time: $\frac{1}{(2\pi)^3}\int\frac{d^3p}{2p}$. However, when λ_p crosses the horizon, i.e. $p \lesssim \sqrt{\Lambda/3}$, the first term, which is time-independent, becomes dominant and taking $\beta = 0$ yields the well-known scale-invariant Harrison-Zeldovich spectrum.

Eq. (9) only depends on τ, so the density fluctuations will preserve translation invariance. Moreover, we argue that rotational violation will still remain after domain walls are inflated away. If domain walls are present during the early stage of inflation, the density fluctuations of high-frequency k-modes, i.e. $\lambda \ll \xi$, are described by Eq. (9). After domain walls are inflated away, the space-time returns to pure de-Sitter space-time and physical wavelengths continue to grow exponentially, i.e. $\lambda_p = \lambda\exp(\sqrt{\Lambda/3}\tau)$. So short-wavelength modes, which have been affected by the gravitational field of the planar domian wall, will be stretched out beyond the horizon and Eq. (9) indicates that the rotational symmetry violation in primordial density fluctuations will become constant during inflation. Inflation only eliminates the initial inhomogeneities (i.e., by driving the number density of walls to zero) without erasing the violation of rotational symmetry in the quantum fluctuations. Whether these rotationally asymmetric fluctuations have yet returned to our observable Universe depends on the duration (number of e-foldings) of inflation.

It is convenient to express the quantities $\langle\delta(\mathbf{q},t)\delta(\mathbf{q}',t)\rangle$, where $\delta(\mathbf{q},t) = (2\pi)^{-3/2}\int d^3x\;\delta(\mathbf{x},t)e^{-i\mathbf{q}\cdot\mathbf{x}}$ is a Fourier transform of primordial density fluctuations $\delta(\mathbf{x},t)$, in terms of mode functions $\delta_{\mathbf{k}}(\mathbf{x},t)$:

$$\langle\delta(\mathbf{q},t)\delta(\mathbf{q}',t)\rangle = \int d^3k\;\delta_{\mathbf{k}}(\mathbf{q},t)\delta^*_{\mathbf{k}}(\mathbf{q}',t), \tag{10}$$

where $\delta_{\mathbf{k}}(\mathbf{q},t) = (2\pi)^{-3/2}\int d^3x\;\delta_{\mathbf{k}}(\mathbf{x},t)e^{-i\mathbf{q}\cdot\mathbf{x}}$ are Fourier coefficients of $\delta_{\mathbf{k}}(\mathbf{x},t)$. Gaussian distributions give $\delta_{\mathbf{k}}(\mathbf{q},t) = \delta_k(t)\delta^3(\mathbf{k}-\mathbf{q})$ and the power spectrum $P_t(q)$ is defined by $\langle\delta(\mathbf{q},t)\delta(\mathbf{q}',t)\rangle = P_t(q)\delta^3(\mathbf{q}-\mathbf{q}')$, where $P_t(q) = |\delta_q(t)|^2$. It is easy to show that $P_t(q)$ is invariant under rotation and translation [19]. For rotational symmetry violation, we should have $\delta_{\mathbf{k}}(\mathbf{q},t) = \delta_{\mathbf{k}}(t)\delta^3(\mathbf{k}-\mathbf{q})$ and the power spectrum $P_t(\mathbf{q}) = |\delta_{\mathbf{q}}(t)|^2$ [7]. By calculating $\langle\delta^2(\mathbf{x},t)\rangle$, we obtain $\langle\delta^2(\mathbf{x},t)\rangle = \frac{1}{(2\pi)^3}\int d^3q\,|\delta_{\mathbf{q}}(t)|^2$, which is independent of $\mathbf{x}$. So it is clear that $P_t(\mathbf{q})$ is translationally invariant.

We are interested in large-scale modes, we so concentrate on the first term of Eq. (9). The resulting power spectrum $P_t(\mathbf{k}) = |\phi_{\mathbf{k}}(\tau)|^2$ is

$$P_t(\mathbf{k}) = \frac{\Lambda(1-\beta^2)^{\frac{3}{2}}}{12\,k^3}\left[(1+\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^{-3} + (1-\beta\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^{-3}\right], \tag{11}$$

where $P_t(\mathbf{k})$ has been made to satisfy reflection symmetry, i.e. $P_t(\mathbf{k}) = P_t(-\mathbf{k})$. So Eq. (11) is valid for both $z > 0$ and $z < 0$. In the limit of $|\beta| \ll 1$, i.e. the dimensionless parameter $\epsilon \ll 1$, we expand Eq. (11) with respect to β to obtain

$$P_t(\mathbf{k}) = \frac{\Lambda}{6}\frac{(1-\frac{3}{2}\beta^2)}{k^3}\left(1+6\beta^2(\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^2+\cdots\right). \quad (12)$$

This is of the form $P_t(\mathbf{k}) = P_t(k)(1 + g_*(\hat{\mathbf{k}}\cdot\mathbf{n})^2 + \cdots)$ (where g_* is a constant), which was suggested in [7]. Eq. (12) yields a scale-invariant power spectrum with $g_* = 6\beta^2$. In fact, Eq. (11) shows that magnitudes of rotational symmetry violation are solely determined by β, so the resulting power spectrum does not depend on frequency k, and is scale-invariant. Of course, our results only apply to the period during which one can approximate a realistic domain wall (located near the small region from which inflation produced the observable universe) by a planar infinite wall. After sufficient inflation the physical distance between our progenitor region and the domain wall will be large enough that the gravitational effects become sensitive to the curvature (deviation from planarity) of the wall. Thus, at best, we can only apply Eq. (12) over some finite range in k, and outside this range scale invariance will be violated.

Discussion. Because discrete symmetries are common in models of fundamental physics, domain walls are a particularly plausible type of topological defect. However, their effects are so strong that one can largely rule them out in the post-inflationary big bang. (See [20] for discussion of how to detect domain walls using gravitational waves.) In this Letter we found analytical solutions of the scalar field equation in the gravitational background of a planar domain wall and positive cosmological constant. These results allow us to study how domain walls affect primordial density fluctuations in the early stage of the inflation. Interestingly, even as the walls are inflated away they leave a characteristic imprint on the quantum fluctuations of the inflaton, which could lead to observable CMB anisotropies. Under the approximation $|\beta| \ll 1$, our results assume the form suggested in the model-independent analysis of preferred primordial direction [7].

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