Orbital effects of a time-dependent Pioneer-like anomalous acceleration

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ABSTRACT

We work out the impact that the recently determined time-dependent component of the Pioneer Anomaly (PA), interpreted as an additional exotic acceleration of gravitational origin with respect to the well known PA-like constant one, may have on the orbital motions of some planets of the solar system. By assuming that it points towards the Sun, it turns out that both the semi-major axis a and the eccentricity e of the orbit of a test particle would experience secular variations. For Saturn and Uranus, for which modern data records cover at least one full orbital revolution, such predicted anomalies are up to $2-3$ orders of magnitude larger than the present-day accuracies in empirically determining their orbital parameters from the usual orbit determination procedures in which the PA was not modeled. Given the predicted huge sizes of such hypothetical signatures, it is unlikely that their absence from the presently available processed data can be attributable to an "absorption" for them in the estimated parameters caused by the fact that they were not explicitly modeled. The magnitude of a constant PA-type acceleration at 9.5 au cannot be larger than 9×10^{-15} m s^{-2} according to the latest observational results for the perihelion precession of Saturn.

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1. Introduction

According to the latest analysis of extended data records of the Pioneer 10/11 spacecraft (Turyshev et al. 2011), the small frequency drift (blue-shift) (Anderson et al. 1998, 2002) observed analyzing the navigational data of both the spacecraft, known as Pioneer Anomaly (PA), may present a further time-dependent component in addition to the well known constant term. Both linear and exponential models were proposed (Turyshev et al. 2011) for the PA; according to Turyshev et al. (2011), the exponential one is directly connected to non-gravitational effects (Toth and Turyshev 2009) since it takes into account the possible role of the on-board power generators suffering a radioactive decay.

In this letter we work out the orbital effects of such a new term in the hypothesis that the time-dependent PA component is due to some sort of long-range modification of the known laws of gravitation resulting in an additional anomalous acceleration with respect to the nearly sunward constant one, having magnitude (Anderson et al. 2002)

$$|A_{\rm Pio}| = (8.74 \pm 1.33) \times 10^{-10} \text{ m s}^{-2}, \tag{1}$$

in terms of which the constant part of the PA has often been interpreted. Indeed, in this case it should act on the major bodies of the solar system as well, especially those whose orbits lie in the regions in which the PA manifested itself in its presently known form. In this respect, we will not consider the exponential model. Recent studies (Bertolami et al. 2008; Rievers at al. 2009; Bertolami et al. 2010; Rievers et al. 2010a,b; Francisco et al. 2011; Rievers and Lämmerzahl 2011), preceding the one by Turyshev et al. (2011), pointed towards a mundane explanation of the PA in terms of non-gravitational effects pertaining the spacecraft themselves.

2. The orbital effects of the linear model

Since it is (Turyshev et al. 2011)

$$\dot{A}_{\rm Pio} \approx -2 \times 10^{-11}\ {\rm m\ s^{-2}\ yr^{-1}}, \tag{2}$$

then the time-dependent linear component of the postulated PA-type acceleration (Turyshev et al. 2011)

$$A = (t - t_0)\,\dot{A}_{\rm Pio} \tag{3}$$

can be treated as a small perturbation of the dominant Newtonian monopole $A_{\rm N}$ over timescales of the order of an orbital period $P_{\rm b}$ for all the planets of the solar system. Table 1 explicitly shows this fact for Saturn, Uranus, Neptune and Pluto which move just in the spatial regions in which the PA perhaps started to appear (Saturn), or fully manifested itself (Uranus, Neptune, Pluto) in its presently known form. Thus, the Gauss equations for

Table 1: Comparison between the magnitudes of the Newtonian monopole accelerations $A_{\rm N}$ of the outer planets of the solar system and their putative Pioneer-type accelerations A of eq. (3) over timescales comparable to their orbital periods $P_{\rm b}$. Also the values of the planetary semi-major axes a, in au, and eccentricities e are displayed. The gravitational parameter of the Sun is $GM_{\odot} = 1.32712 \times 10^{20}$ m^3 s^{-2}.

Planet	a (au)	e	$\lvert A_{\rm N}\rvert$ (m s^{-2})	$P_{\rm b}$ (yr)	$\lvert A\rvert$ (m s^{-2})
Saturn	9.582	0.0565	6×10^{-5}	29.457	6×10^{-10}
Uranus	19.201	0.0457	2×10^{-5}	84.011	2×10^{-9}
Neptune	30.047	0.0113	6×10^{-6}	164.79	3×10^{-9}
Pluto	39.482	0.2488	4×10^{-6}	247.68	5×10^{-9}

the variation of the osculating Keplerian orbital elements (Brouwer and Clemence 1961), which are valid for any kind of disturbing acceleration $\boldsymbol{A}$, independently of its physical origin, can be safely used for working out the orbital effects of eq. (3). In particular, the Gauss equations for the semi-major axis a and eccentricity e of the orbit of a test particle moving around a central body of mass M are

$$\begin{aligned}\frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}}\left[eA_R\sin f + A_T\left(\frac{p}{r}\right)\right],\\ \frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na}\left\{A_R\sin f + A_T\left[\cos f + \frac{1}{e}\left(1-\frac{r}{a}\right)\right]\right\}:\end{aligned} \tag{4}$$

they allow to work out the rates of changes of a and e averaged over one orbital period $P_{\rm b}$ as

$$\left\langle\frac{d\Psi}{dt}\right\rangle = \left(\frac{1}{P_{\rm b}}\right)\int_0^{P_{\rm b}}\left(\frac{d\Psi}{dt}\right)_{\rm K} dt,\ \Psi = a, e. \tag{5}$$

In eq. (5) $(d\Psi/dt)_{\rm K}$ are the right-hand-sides of eq. (4) evaluated onto the unperturbed Keplerian ellipse. In eq. (4) A_R, A_T are the radial and transverse components of a the generic disturbing acceleration $\boldsymbol{A}$, $p \doteq a(1-e^2)$ is the semilatus rectum, $n \doteq \sqrt{GM/a^3}$ is the unperturbed Keplerian mean motion related to the orbital period by $n = 2\pi/P_{\rm b}$, G is the Newtonian constant of gravitation, and f is the true anomaly. Since the new data analysis (Turyshev et al. 2011) does not rule out the line joining the Sun and the spacecrafts as a direction for the PA, we will assume that eq. (3) is entirely radial, so that $A_R = A, A_T = 0$. Using the eccentric anomaly E as fast variable of integration turns out to be computationally more convenient. To this aim, useful relations are

$$\begin{aligned}dt &= \left(\frac{1-e\cos E}{n}\right)dE,\\ t-t_0 &= \left(\frac{E-e\sin E-E_0+e\sin E_0}{n}\right),\\ \sin f &= \frac{\sqrt{1-e^2}\sin E}{1-e\cos E}.\end{aligned} \tag{6}$$

As a result, a and e experiences non-vanishing secular variations

$$\begin{aligned}\left\langle\frac{da}{dt}\right\rangle &= -\frac{\dot{A}_{\rm P}a^3e(2+e)}{GM},\\ \left\langle\frac{de}{dt}\right\rangle &= -\frac{\dot{A}_{\rm P}a^2e(2+e)\left(1-e^2\right)}{2GM}.\end{aligned} \tag{7}$$

Notice that eq. (7) are exact in the sense that no approximations in e were assumed. Moreover, they do not depend on t_0.

In order to make a meaningful comparison of eq. (7) with the latest empirical results from planetary orbit determinations, we recall that modern data records cover at least one full orbital revolution for all the planets with the exception of Neptune and Pluto. Pitjeva (2007), in producing the EPM2006 ephemerides, made a global fit of a complete suite of standard dynamical force models acting on the solar system's major bodies to more than 400,000 observations of various kinds ranging over $\Delta t = 93$ yr $(1913-2006)$. Among the about 230 estimated parameters, there are the planetary orbital elements as well. According to Table 3 of Pitjeva (2007), the formal, statistical errors in a for Saturn and Uranus are

$$\begin{aligned} \sigma_{a_{♄}}^{(\mathrm{EPM2006})} &= 4,256 \ \mathrm{m} \\ \sigma_{a_{♅}}^{(\mathrm{EPM2006})} &= 40,294 \ \mathrm{m}, \end{aligned} \tag{8}$$

so that

$$\begin{aligned} \sigma_{\dot{a}_{♄}}^{(\mathrm{EPM2006})} &= 46 \ \mathrm{m\ yr}^{-1} \\ \sigma_{\dot{a}_{♅}}^{(\mathrm{EPM2006})} &= 433 \ \mathrm{m\ yr}^{-1}, \end{aligned} \tag{9}$$

can naively be inferred for their rates by simply dividing eq. (8) by Δt. The PA was not modeled in the EPM2006. It is important to remark that the figure for $\sigma_{a_{♄}}$ quoted in eq. (8) was obtained without processing the radiotechnical observations of the Cassini spacecraft. According to eq. (7), the putative PA-induced secular changes of the semi-major axes of Saturn and Uranus are

$$\begin{aligned} \left\langle \dot{a}_{♄}^{(\mathrm{Pio})} \right\rangle &= 42,505 \ \mathrm{m\ yr}^{-1} \\ \left\langle \dot{a}_{♅}^{(\mathrm{Pio})} \right\rangle &= 290,581 \ \mathrm{m\ yr}^{-1}. \end{aligned} \tag{10}$$

They are about 3 orders of magnitude larger than eq. (9): even by re-scaling the formal uncertainties of eq. (9) by a factor of 10, the PA-type anomalous rates of eq. (10) would still be about 2 orders of magnitude too large to have escaped from a detection. Such conclusions are confirmed, and even enforced, by using the latest results published by Pitjeva & Pitjev (2011) who explicitly estimated secular changes of the semi-major axes of the first six planets with the EPM2010 ephemerides based on more than 635,000 observations of different types over $\Delta t = 97$ yr (1913-2010). Pitjeva & Pitjev (2011), whose goal was a different one, did not model the PA. They obtain[1]

$$\dot{a}_{♄}^{(\mathrm{EPM2010})} = 13 \ \mathrm{m\ yr}^{-1}, \tag{11}$$

which is 4 orders of magnitude smaller than the predicted value of eq. (10).

Incidentally, we note that if eq. (3) acted on, say, the Earth, it would cause a variation in its semi-major axis as large as

$$\left\langle \dot{a}_{\oplus}^{(\mathrm{Pio})} \right\rangle = 14.5 \ \mathrm{m\ yr}^{-1} \tag{12}$$

[1]A 35.9% correlation between $\dot{a}_{♄}$ and $a_{♄}$ was reported by Pitjeva & Pitjev (2011).

corresponding to a shift of

$$\Delta a_{\oplus}^{(\mathrm{Pio})} = 1,348.5\ \mathrm{m} \tag{13}$$

over $\Delta t = 93$ yr. The formal, statistical uncertainty in the terrestrial semi-major axis is Pitjeva (2007)

$$\sigma_{a_{\oplus}}^{(\mathrm{EPM2006})} = 0.138\ \mathrm{m}, \tag{14}$$

i.e. about 4 orders of magnitude smaller than eq. (13). Even by re-scaling eq. (14) by a factor of 10, eq. (13) would still be 3 orders of magnitude too large. The EPM2010 ephemerides yield[2] Pitjeva & Pitjev (2011)

$$\dot{a}_{\oplus}^{(\mathrm{EPM2010})} = 2 \times 10^{-5}\ \mathrm{m\ yr}^{-1}, \tag{15}$$

i.e. 6 orders of magnitude smaller than the prediction of eq. (12).

In principle, it may be argued that eq. (3) was not included in the mathematical models fitted to the planetary data in the orbit determination process, so that its signature may have partly or totally been absorbed in the estimation of, say, the planetary state vectors: thus, the entire observational record should be re-processed by using ad-hoc modified dynamical force models explicitly including eq. (3) itself. However, this argument may have a validity especially when the magnitude of a putative anomalous effect of interest is close to the accuracy of the orbit determination process: it is not the case here. Moreover, as far as the constant PA term is concerned, independent analyses (Standish 2008; Fienga et al. 2009; Standish 2010) in which it was explicitly modeled as a radial extra-acceleration acting on the outer planets substantially confirmed its neat incompatibility with the observations, as suggested[3] in earlier studies (Iorio & Giudice 2006; Iorio 2007).

Concerning the possibility that the PA is present at Saturn, Turyshev et al. (2011) yield an equivalent acceleration

$$A_{\mathrm{Pio}}^{(\hbar)} = (4.58 \pm 11.80) \times 10^{-10}\ \mathrm{m\ s}^{-2} \tag{16}$$

for its constant term. It is known that a radial constant extra-acceleration causes a secular precession of the perihelion (Iorio & Giudice 2006; Sanders 2006; Sereno and Jetzer 2006; Adkins and McDonnell 2007) given by

$$\dot{\varpi}^{(\mathrm{Pio})} = \frac{A_{\mathrm{Pio}}\sqrt{1-e^2}}{na}. \tag{17}$$

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[2]A 0.6% correlation between $\dot{a}_{\oplus}$ and $a_{\oplus}$ was reported by Pitjeva & Pitjev (2011).

[3]Concerning the inner planets of the solar system, from an analysis of their determined orbital motions Anderson et al. (2002) pointed out that there was no room for a constant, sunward PA-like exotic acceleration acting on them.

The present-day accuracy in empirically constraining any anomalous precession $\Delta\dot{\varpi}$ of the perihelion of Saturn from the INPOP10a ephemerides, in which the PA was not modeled, is (Fienga et al. 2011)

$$\sigma_{\Delta\dot{\varpi}_{♄}} = 0.65 \text{ milliarcsec cty}^{-1}. \tag{18}$$

Thus, eq. (17) and eq. (18) yield

$$\left|A_{\rm Pio}^{(♄)}\right| \leq \frac{na}{\sqrt{1-e^2}}\sigma_{\Delta\dot{\varpi}_{♄}} = 9\times 10^{-15} \text{ m s}^{-2}, \tag{19}$$

which is up to 5 orders of magnitude smaller than eq. (16).

3. Conclusions

If the time-dependent linear part of the PA was a genuine effect of gravitational origin causing an extra-acceleration, it would affect the orbital motions of the planets with secular variations of their semi-major axis and eccentricity by assuming that it is oriented towards the Sun. The magnitude of such putative effects for Saturn and Uranus is up to $2-3$ orders of magnitude larger than the current accuracy in determining their orbital motions from the observations without modeling the PA itself. A partial removal of a PA-like signature may, in principle, have occurred in the standard parameter estimation procedure, so that the entire planetary data set should be re-processed with ad-hoc modified dynamical models explicitly including a hypothetical time-dependent PA-type acceleration as well. However, supported by the results of analogous tests for the constant form of the PA actually implemented by independent teams of astronomers with different purposely modified dynamical models, we feel that it is unlikely that such a potential removal of a PA-type signature can really justify the absence of such a huge effect in the currently available processed planetary data. Concerning the existence of a constant PA-type acceleration at heliocentric distances of about 9.5 au as large as 1.6×10^{-9} m s^{-2}, latest results about the maximum allowed size of anomalies in the perihelion precession of Saturn yield a figure 5 orders of magnitude smaller.

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