A Model Independent Method to Study Dark Matter induced Leptons and Gamma rays

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Abstract

We propose a novel method to directly determine dark matter induced electron/positron spectrum at the source from experimental measurements at the earth, without reference to specific particle physics models. The dark matter induced gamma rays emitted via inverse Compton scattering are obtained in a model independent way. For illustration, we predict the flux of gamma rays from the Fornax cluster in the decaying dark matter scenario, which turns out to be in disagreement with recent Fermi-LAT measurements. In addition, gamma rays with energy larger than 1 GeV are shown to be almost independent of choices of cosmic ray propagation model and of dark matter density profile.

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As one of the dominant components of the universe, dark matter (DM) has yet to show its existence other than its gravitational effects. The nature of DM can be explored via searches at colliders, as well as in direct and indirect detection experiments. Recently, indirect detection of DM has attracted great attention due to the cosmic ray electron/positron excesses observed by Pamela [1] and Fermi [2, 3]. But the interpretation of these experimental results is subtle. It is not easy to exclude the possibility that these excesses may origin from nearby astrophysical sources. Even assuming the DM annihilation/decay to account for the Pamela and Fermi observations, one has to face particle physics model dependence.

In this paper we will show that, it is possible to tightly constrain the decaying DM interpretation of electron/positron excesses in a particle physics model independent way, by considering the gamma rays from nearby clusters. In other words, constraints can be obtained without any assumptions on details of the DM model.

Experimentally, Pamela and Fermi-LAT measure only the energy spectra of cosmic rays at the earth. To compare with theoretical predictions, one usually starts from a specific DM model to calculate the fluxes at the source and their propagation through the Galaxy. Obviously, it is much desired to extract their fluxes at the source where they are generated, in a model independent way. It this paper, we shall see that the $e^{\pm}$ fluxes at the source can be obtained by solving an integral equation analytically, without introducing a specific DM model. Accordingly, gamma rays emitted by these DM-induced energetic leptons via inverse Compton scattering (ICS), can be predicted independent of any DM model, which can be tested against the recent experimental measurement [4] of gamma rays from nearby clusters by the Fermi-LAT collaboration.[1]

Conventionally, the $e^{\pm}$ propagation in the Galaxy is governed approximately by the diffusion equation

$$K(E)\cdot\nabla^2 f_e^{\rm DM}(E,\vec{r}) + \frac{\partial}{\partial E}\left[B(E) f_e^{\rm DM}(E,\vec{r})\right] + Q_e^{\rm DM}(E,\vec{r}) = 0 \ . \tag{1}$$

Here $f_e^{\rm DM}(E,\vec{r})$ is the DM-induced electron/positron number density per unit energy. $K(E)$ stands for the diffusion coefficient, which can be parameterized as $K(E) = K_0 (E/{\rm GeV})^{\alpha}$, with K_0 and α given in Table I. $B(E)$ describes the energy loss, which is effectively given as $B(E) = E^2/({\rm GeV}\cdot\tau_E)$, with $\tau_E = 10^{16}$ s being a typical time scale in the Galaxy. For

[1] Our method should also be applicable to the annihilating DM scenario, but we will focus on decaying DM scenario in this paper while leave the case of DM annihilation for future work [5].

Model	α	K_0 in kpc^2/Myr	L in kpc
MIN	0.55	0.00595	1
MED	0.70	0.0112	4
MAX	0.46	0.0765	15

TABLE I. Parameters in propagation models. MIN/MED/MAX refer to models which yield minimal/medium/maximal positron flux, respectively [6].

decaying DM, the source term $Q_e^{\rm DM}(E,\vec{r})$ can be expressed as

$$Q_e^{\rm DM}(E,\vec{r}) = \rho^{\rm DM}(\vec{r})\sum_i \frac{\Gamma_i^{\rm DM}}{M^{\rm DM}}\frac{dN_i^{\rm DM}}{dE} = \rho^{\rm DM}(\vec{r})\, X(E)\ . \tag{2}$$

Here $\rho^{\rm DM}(r)$, $\Gamma_i^{\rm DM}$, $M^{\rm DM}$ and $dN_i^{\rm DM}/dE$ are the DM density, the decay width of a particular decay channel, DM particle mass and the $e^\pm$ spectrum per DM decay via a particular channel, respectively. The summation is over all possible decay channels and $X(E)$ contains all the particle physics information.

Given $X(E)$, the DM induced $e^\pm$ at the earth can be determined by solving Eq. (1) in a solid flat cylinder [6–8] as[2]

$$f_e^{\rm DM}(E,\overrightarrow{r_\odot}) = \frac{\tau_E}{E^2}\sum_{m,n=1}^{\infty} B_{mn}\int_E^\infty dE'\exp\left[\lambda_{mn}\left(E^{\alpha-1}-(E')^{\alpha-1}\right)\right]\, X(E')\ , \tag{3}$$

where

$$\begin{aligned} B_{mn} &= \frac{2\sin(m\pi/2)}{J_1^2(\zeta_n)R^2L}J_0\left(\frac{\zeta_n\, r_\odot}{R}\right)\int_0^R dr\, r\int_{-L}^{L}dz\,\rho^{\rm DM}(\sqrt{r^2+z^2})J_0\left(\frac{\zeta_n\, r}{R}\right)\sin\left[\frac{m\pi}{2L}(z+L)\right]\ , \\ \lambda_{mn} &= \left(\frac{\zeta_n^2}{R^2}+\frac{m^2\pi^2}{4L^2}\right)K_0\,\tau_E\,\frac{1}{\alpha-1} \end{aligned} \tag{4}$$

with the cylinder coordinates $z \in [-L,\ L]$ in the z-direction and $r \in [0,\ R]$ ($R = 20$ kpc) in radius. Here J_n is the n-th order Bessel function and ζ_n's are successive zeros of J_0. The solar system is at $r_\odot = 8.5$ kpc.

[2] In practice, one has to truncate the infinite series to a finite sum. When $E' \simeq E$, the series in Eq. (3) converges very slowly since there is no exponential suppression at $E' = E$. In this range the solution is better expressed in an alternative form [8]

$$f_e^{\rm DM}(E,\overrightarrow{r_\odot}) = \frac{\tau_E}{E^2}\int_E^\infty dE'X(E')\exp\left[\frac{K_0\,\tau_E}{1-\alpha}\left(E^{\alpha-1}-(E')^{\alpha-1}\right)\nabla^2\right]\ \rho^{\rm DM}(\vec{r})\,|_{\vec{r}=\overrightarrow{r_\odot}}\ .$$

Taking the MED propagation model and Navarro-Frenk-White (NFW) DM density profile [9] for illustration, and reordering the series in Eq. (3) from small to large $|\lambda_{mn}|$, we shall take the first 1413 terms of the series in Eq. (3). This sum agrees well with the alternative form of $f_e^{\rm DM}$ within 0.1% error in the range $E' \simeq E$.

Surprisingly, the DM-induced $e^{\pm}$ spectrum $X(E)$ at the source can be determined in a DM-model independent way once $f_e^{\rm DM}(E, \overrightarrow{r_\odot})$ is known. Eq. (3) is actually the so-called Volterra integral equation and its inverse solution can be obtained analytically

$$X(E) = \frac{dg(E)}{dE} + (\alpha - 1)E^{\alpha-2} \int_{\infty}^{E} dE' \frac{dg(E')}{dE'} R\left(E^{\alpha-1} - (E')^{\alpha-1}\right) \ , \tag{5}$$

where[3]

$$g(E) = -\frac{E^2}{\tau_E} f_e^{\rm DM}(E, \overrightarrow{r_\odot}) \bigg/ \sum_{m,n=1}^{\infty} B_{mn} \ , \qquad R(x) = \boldsymbol{L}^{-1}\left[\frac{1}{p\widetilde{K}(p)} - 1\right] \ , \tag{6}$$

with

$$\widetilde{K}(p) = \boldsymbol{L}\left[K(x)\right] = \boldsymbol{L}\left[\sum_{m,n=1}^{\infty} B_{mn} \exp[\lambda_{mn} x] \bigg/ \sum_{m,n=1}^{\infty} B_{mn}\right] \ . \tag{7}$$

Here $\boldsymbol{L}$ denotes the Laplace transform and $\boldsymbol{L}^{-1}$ its inverse. The $\boldsymbol{L}$ here can be performed trivially while the Cauchy's residue theorem is needed to perform $\boldsymbol{L}^{-1}$ analytically. The number of singularities in $1/(p\widetilde{K}(p)) - 1$ is the same as the number of terms in the truncated series (as large as 1413 for the case of MED propagation model and NFW DM density profile). In principle, one can find all singularities and their residues. But this demands excessive amount of computer power and it is unnecessary, as we will see presently. Notice that all singularities have negative real parts as $\lambda_{mn} < 0$ and furthermore, there is an exponential suppression factor $\exp(px)$ in the residue. Singularities in the region $-200 < Re(p) < 0$ yield very good approximation, as evidenced by the left part of Fig. 2. They are obtained with the help of the argument principle in complex analysis. This part constitutes one of the major technical hurdles of our analysis and see [5] for further details.

On the other hand, $f_e^{\rm DM}(E, \overrightarrow{r_\odot})$ can be obtained by subtracting off the conventional astrophysical background from the $e^{\pm}$ spectrum measured by Fermi-LAT.[4] We take the pre-Fermi conventional model as the $e^{\pm}$ background. They can be parameterized as [10]

$$\begin{aligned} \Phi_{e^-}^{\rm bkg}(E) &= \frac{82.0\epsilon^{-0.28}}{1 + 0.224\epsilon^{2.93}} \\ \Phi_{e^+}^{\rm bkg}(E) &= \frac{38.4\epsilon^{-4.78}}{1 + 0.0002\epsilon^{5.63}} + 24.0\epsilon^{-3.41} \end{aligned} \tag{8}$$

[3] In practice, the infinite series will be truncated, in the same vein of Eq. (3).

[4] They have reported the electron spectrum in the range from 7 GeV to 1 TeV [2, 3].

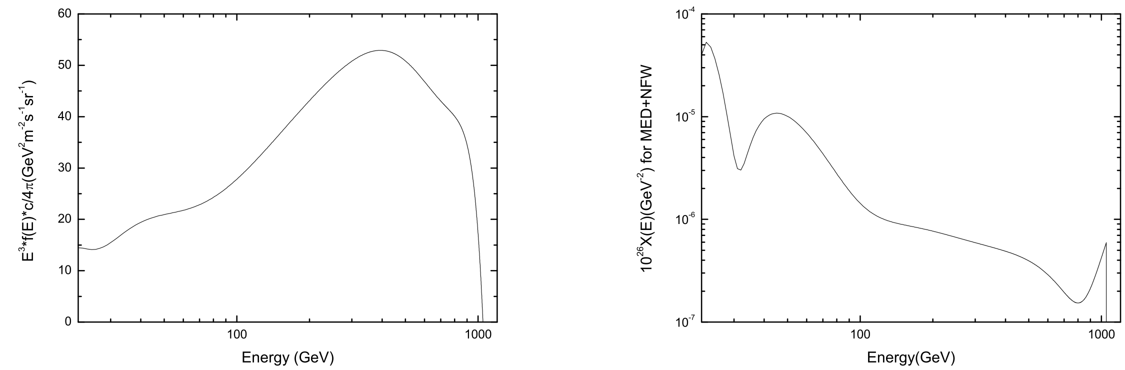

FIG. 1. Left: $f_e^{\rm DM}(E, r_\odot)$ extracted from the Fermi-LAT spectrum of $e^\pm$ by subtracting off the background $\Phi_{e^\pm}^{\rm bkg}(E)$. Right: $X(E)$ determined from $f_e^{\rm DM}(E, r_\odot)$, assuming the MED propagation model and NFW DM density profile.

in units of $\mathrm{GeV}^{-1}\mathrm{m}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$ with $\epsilon = E/1$ GeV. The total $e^\pm$ background flux at the Earth can be expressed as

$$\Phi_{e^\pm}^{\oplus}(E_\oplus) = \frac{E_\oplus^2}{E^2}\left[\Phi_{e^+}^{\rm bkg}(E) + {\rm N} * \Phi_{e^-}^{\rm bkg}(E)\right] \tag{9}$$

with the normalization factor chosen to be $\mathrm{N} = 0.8$. To account for the solar modulation effects, the force field approximation $E_\oplus = E + e\,\phi_F$ with $\phi_F = 0.55$ GV has been taken. With this astrophysical background, the introduction of an additional leptonic component from decaying DM could provide a plausible interpretation of not only Fermi-LAT $e^\pm$ excess but also PAMELA anomaly in the positron fraction (See, e.g., [10–12]).

Shown in the left part of Fig. 1 is a fit function of $f_e^{\rm DM}(E, \overrightarrow{r_\odot})$ obtained by subtracting off $e^\pm$ background from the Fermi-LAT data.[5] Taking this fit function of $f_e^{\rm DM}(E, \overrightarrow{r_\odot})$ as input, one may obtain $X(E)$ via Eq. (5). Shown in the right part of Fig. 1 is the $X(E)$ thus obtained for the MED propagation model and NFW DM density profile with local DM density $\rho_\odot = 0.3$ GeV/cm^3. As discussed above, we have made certain approximations in obtaining $X(E)$. To estimate the theoretical errors, we have taken $X(E)$ as an input in Eq. (3) to get a new $f_e^{\rm DM}(E, \overrightarrow{r_\odot})$. Shown in the left part of Fig. 2 is a comparison of this new $f_e^{\rm DM}(E, \overrightarrow{r_\odot})$ with the original fit function. One sees clearly that the errors are very small, never beyond few percents.

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[5] The fit function of $E^2 * f_e^{\rm DM}(E, \overrightarrow{r_\odot})$ should be a monotonic decreasing function to guarantee the deduced spectrum function $X(E)$ to be non-negative, which represents the DM-induced $e^\pm$ spectrum at the source.

Taking this spectrum function $X(E)$ as an input, the ICS gamma rays can be deduced from the scattering of energetic $e^{\pm}$ on starlight and CMB photons. One can then check these predictions against experimental measurements of gamma rays from inside/outside the Galactic halo. We remind that the constraints obtained in this way does not depend on any details of the DM model. Recently, Fermi-LAT has measured gamma rays from nearby clusters of galaxies with an 18-month data set [4]. These clusters are supposed to be highly DM dominated and isolated at high galactic latitudes. High signal-to-noise ratios are anticipated for gamma-ray observations targeting nearby clusters. Recent model-dependent studies [13, 14] have shown that gamma rays from the Fornax cluster provides the strongest constraint for decaying DM. In the following we will focus on the DM induced gamma rays from the Fornax cluster. Certainly, there may exist other sources in clusters that can emit gamma-rays, besides DM annihilation/decay. Nevertheless the ICS gamma-rays predicted from $X(E)$ can give lower limits on the gamma ray flux. In the Fornax cluster, the ICS gamma rays comes mainly from the scattering of $e^{\pm}$ on the CMB, while the effects of dust and starlight can be neglected [14, 15]. We follow the same method in [14, 16] to calculate the ICS gamma rays semi-analytically, with corresponding viral masses M_{200}, M_{500} adopted from [17]. Shown in the right part of Fig. 2 is the predicted gamma ray spectrum, which seems to disagree with the Fermi-LAT measurement of gamma rays from the Fornax cluster [4] in the range of $1 - 10$ GeV.

We now address the relevant astrophysical uncertainties besides errors from the total DM mass in the Fornax cluster. For the inverse Compton scattering of electron/positron on the CMB, the astrophysical uncertainty must be small as the spectrum of CMB photons is well known. The main uncertainties arise from choices of propagation model and of DM halo profile in determination of $X(E)$ from $f_e^{\mathrm{DM}}(E, \overrightarrow{r_\odot})$. Shown in the left part of Fig. 3 are the $X(E)$'s obtained by using the MED, MIN and MAX propagation models, respectively, with the default NFW DM density profile. Shown in the right part of Fig. 3 are the $X(E)$'s corresponding to the NFW, Einasto [18] and Isothermal [19] density profiles, respectively, with the MED propagation model fixed. One sees that the choice of DM halo profile has almost invisible impact on the determination of $X(E)$. This is because the energetic leptons can not propagate a long distance and different DM profiles have very similar behavior except for the region near the Galaxy center. The choice of propagation models do introduce large uncertainties into the determination of $X(E)$, but only for energies less than about 300 GeV.

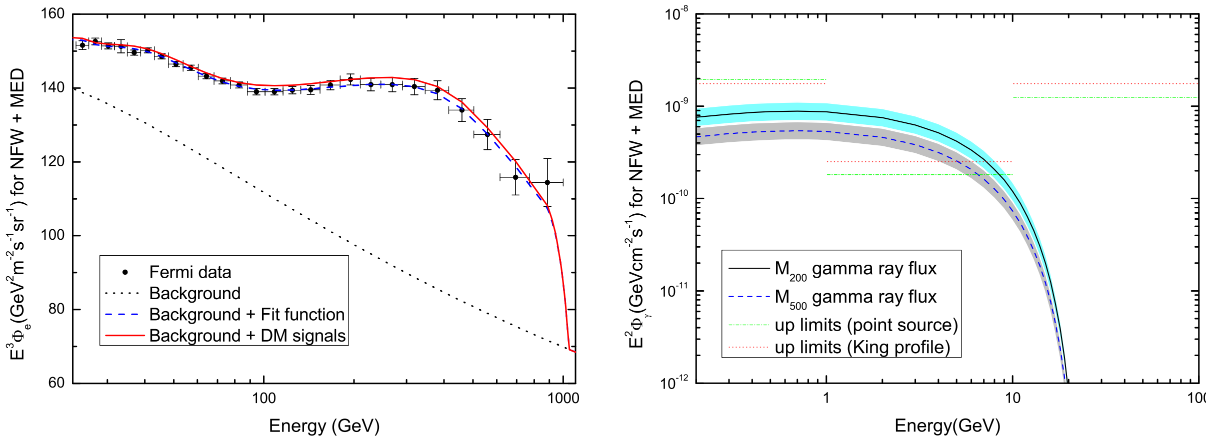

FIG. 2. NFW halo profile and MED propagation model are assumed. Left: The difference between the red solid line and the blue dashed line can be viewed as a demonstration of the theoretical error in determining $X(E)$, as explained in the text. Right: The correspondingly predicted ICS flux of photon is shown in the Fornax cluster. Experimental upper limits are taken from [4].

This is because, very high energy leptons must come from the neighborhood of the solar system. It is reasonable to expect that the propagation effects should not have significant uncertainties in such a small distance. Kinematically, the ICS gamma rays arising from the scattering on the CMB requires $E_e \gtrsim\ m_e\sqrt{E_\gamma/\epsilon}/2$ (ϵ is the energy of CMB photon). This means that the ICS gamma rays with $E_\gamma \gtrsim\ 1$ GeV are produced from the electrons and positrons with $E_e \gtrsim\ 500$ GeV, which has negligible uncertainties due to the choice of propagation model. As a result, the predicted ICS gamma rays in the energy range of $1-10$ GeV have very small theoretical errors.

In summary, we have developed a novel method to determine the DM $e^\pm$ fluxes at the source from the DM $e^\pm$ fluxes measured at the earth, in a DM model independent way by solving the Volterra integral equation. The DM $e^\pm$ fluxes at the earth was obtained by subtracting the conventional astrophysical contribution from the Fermi-LAT measurements of the total flux of electrons and positrons. The DM-induced $e^\pm$ inevitably emit gamma rays via inverse Compton scattering. We have shown that, for decaying dark matter, the predicted gamma rays from the Fornax cluster in the energy range of $1-10$ GeV seems to exceed the upper limits measured by the Fermi-LAT collaboration. In addition, the predicted gamma rays with $E_\gamma \gtrsim\ 1$ GeV are essentially independent of choices of propagation model and of dark matter density profile.

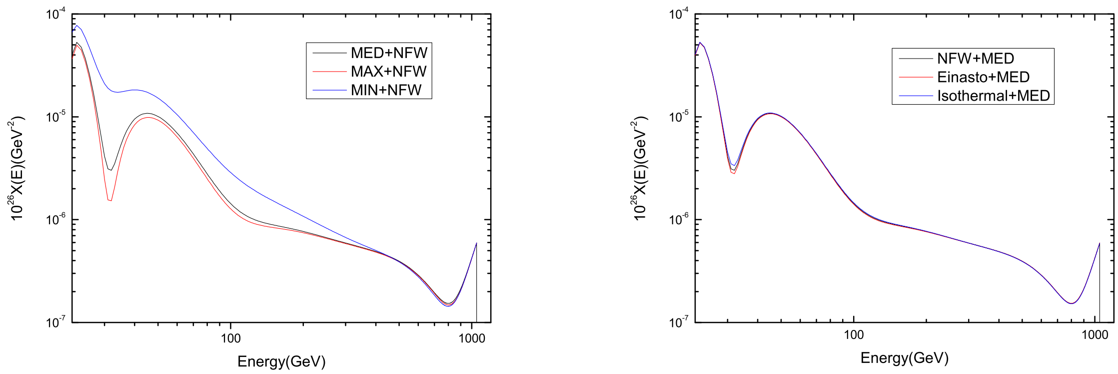

FIG. 3. Astrophysical uncertainties for the determination of $X(E)$ from $f_e^{\rm DM}(E, r_\odot)$. Left: NFW DM density profile is assumed while propagation models are varied. Right: The MED propagation model is assumed while DM density profiles are varied.

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