# NMSGUT-III: Grand Unification upended

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## Abstract

We show that matter yukawa couplings of the New Minimal Supersymmetric (SO(10)) GUT(NMSGUT) are subject to very significant GUT scale threshold corrections. Including these threshold effects relaxes the constraint $y_b - y_\tau \simeq y_s - y_\mu$ operative in $\mathbf{10}-\mathbf{120}$ plet generated tree level MSSM matter fermion yukawas $y_f$. We find accurate fits of the MSSM fermion mass-mixing data in terms of NMSGUT superpotential couplings and 5 independent soft Susy breaking parameters $M_0, M_{1/2}, A_0, M^2_{H,\bar{H}}$ at $M_X$. The fits generally have elevated unification scale $M_X$ near $M_{Planck}$, viable values of $\alpha_3(M_Z)$, and are consistent with current limits on B violation, $b \to s\gamma$, muon magnetic moment anomaly and Standard Model $\rho$ parameter. The associated novel and distinctive soft Susy spectra have light gauginos, a *normal* s-hierarchy and Bino LSP. The Bino LSP is accompanied by second and first generation right chiral sfermions light enough to mediate a consistent WIMP dark matter co-annihilation cosmology and to be discoverable at LHC, while third generation sfermions are in the LHC undiscoverable range of 3-50 TeV. The fits found require $|\mu|, |A_0| \sim 100$ TeV which imply both deep CCB/UFB minimae and stability of the MSSM standard vacuum on cosmological time scales. Our results indicate that a consistent realistic phenomenology may be specifiable in terms of SO(10) (NMS)GUT parameters at $M_X$ alone and that a new viable sector of the soft supersymmetry parameter space may exist if flavour violation constraints can be satisfied in the 43 dimensional parameter space.

## 1 Introduction

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Supersymmetric Grand Unification based on the SO(10) gauge group has received well deserved attention over the last 3 decades. Models proposed fall into two broad classes : those that preserve R-parity down to low energies [1, 2, 6, 9, 12, 5, 7] using as Higgs the special representations ($\overline{\mathbf{126}}$) of SO(10) that contain R-parity even SM singlets and another large class of SO(10) R-parity violating models[8] that attempt to construct viable models using sets of small SO(10) representations even after sacrificing the vital distinction provided by R-parity between matter and Higgs multiplets. Besides structural attractions, such as the automatic inclusion of the conjugate neutrino fields necessary for neutrino

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mass, SO(10) GUTs offer a number of other natural features. Among these are third generation yukawa unification[3, 4], automatic embedding of minimal supersymmetric Left-Right models, natural R-parity preservation[5] down to the weak scale and consequently natural LSP WIMP dark matter, economic and explicitly soluble symmetry breaking at the GUT scale[6], explicitly calculable superheavy spectra[9, 10, 11, 12], interesting gauge unification threshold effects[12, 13, 7, 14, 17] which can lead to a natural elevation of the unification scale to near the Planck scale[14], GUT scale threshold corrections to the QCD coupling $\alpha_3(M_Z)$ of the required[18] sign and size[19] and a deep interplay between the scales of Baryon and Lepton number violation as suggested by the neutrino oscillation measurements and the seesaw formulae connecting neutrino masses to the B-L breaking scale.

The fascination of the MSSM RG flow at large $\tan\beta$ stems from the tendency of third generation yukawa couplings to converge, at the MSSM unification scale [3, 4], in a manner reminiscent of gauge unification in the MSSM RG flow[20, 21]. For suitably large $\tan\beta$ and for close to central input values of SM fermion couplings at the Susy breaking scale $M_S \sim M_Z$, third generation yukawas actually almost coincide at $M_X$. On the other hand, in SO(10) theories with only the simplest possible fermion mass giving Higgs content (a single **10**-plet), when all the complications of threshold effects at $M_X \sim 10^{16} GeV$(not to speak of those at seesaw scales $M_{\bar\nu} \sim 10^7 - 10^{12}\, GeV$) are ignored, one does expect to generate boundary conditions for the gauge and yukawa couplings that are unified gauge group wise and (third generation) flavour wise.

However, fitting the rest of the known fermion data (15 more parameters) definitely requires other Fermion Higgs multiplets (more **10**-plets, $\mathbf{120}, \overline{\mathbf{126}}$ s etc). A principled position (*monoHiggism*?) with regard to the choice of Higgs multiplets responsible for fermion mass (FM Higgs) is to accept *only one* of each irrep present in the conjugate of the direct product of fermion representations. This principle may be motivated by regarding the different Higgs representations as characteristic "FM channels" through which the fermion mass (FM) is transmitted in structurally distinguishable ways. For example the Georgi-Jarlskog mechanism distinguishes the **45** plet Higgs in SU(5) ($\overline{\mathbf{126}}$ in SO(10)) from the $\mathbf{5} + \bar{\mathbf{5}}$ (**10** in SO(10)) due to their ability to explain the quark-lepton mass relations in the second and third generations respectively. Similarly the $\overline{\mathbf{126}}$ in SO(10) is peculiarly suitable for implementing the Type I and Type II seesaw mechanisms for neutrino mass. If one duplicates the Higgs multiplets transforming as the *same* gauge group representation, for example by taking multiple **10**-plets in SO(10), then one abandons the quest for a structural explanation of the pattern of fermion masses in favour of "just so" solutions. Moreover the many Higgs doublets typically present in the theory mix with the **10**-plet derived Higgs doublets to make up the light doublets of the effective MSSM. Both these features dilute the expectation of exact third generation yukawa unification.

The technical difficulties of $b-\tau$ unification are attendant upon the fact that the GUT scale values of the third generation Yukawa couplings, in sharp contrast to the three gauge couplings, are highly infrared sensitive(see [22] for a cogent discussion). The accuracy of unification of third generation Yukawa couplings after the MSSM RG flow up to the 1-loop GUT scale $M_X^0(\sim 10^{16.25}$ GeV) is strongly dependent on the low energy values of the yukawa couplings in the MSSM and thus requires large $\tan\beta$. It is also strongly dependent on the precise values of $y_b^{MSSM}(M_Z)$ and $y_t^{MSSM}(M_Z)$(both of which can receive significant Susy threshold corrections) besides $\alpha_3(M_Z)$ (which needs significant GUT threshold corrections to be consistent with experiment[18]). The large threshold corrections [4, 23] to the relations between Standard Model(SM) and MSSM Yukawa couplings for down type quarks precisely in the large $\tan\beta$ MSSM variants that are in focus in a SO(10) context, combined with infrared sensitivity of yukawa unification casts doubt on the naturalness of exact $\mathbf{t}-\mathbf{b}-\tau$ unification. As these yukawa couplings have been measured more precisely over the same period as the investigation of third generation yukawa unification in Susy SO(10) models, the strain of these models has increased with time, and various just so mechanisms are invoked to preserve the exact third generation unification. Yet it would be strange if Susy SO(10) had no mechanisms to accommodate not only this difficulty but even the full complexity of the fermion spectrum.

Many detailed investigations[3, 4] of the yukawa unification problem since the early days have focussed on models based on small Higgs representations($\mathbf{16}, \mathbf{10}, \mathbf{45}$) and seek to understand only the third generation fermion masses. These studies assume that no Higgs Multiplet other than the **10**-plet could contribute significantly to the third generation masses and use that to justify the assumption that high scale threshold corrections to the yukawa couplings are negligible. In the few studies that attempt to tackle the full fermion spectrum, recourse is had to a subset of the many possible non-renormalizable operators that contribute to fermion yukawa couplings in the effective theory below the unification scale. Since the first two generation yukawas are so small relative to the third generation it is possible to build models using small non-renormalizable contributions that allow one to fit three generation charged fermion data[24, 25] and even keep the operator dimension five contributions to baryon violation within or near to experimental limits[25]. When other aspects, such as GUT scale spontaneous symmetry breaking, are to be addressed multiple Higgs **10,45,16**-plets are often introduced[8]. These then require ad-hoc discrete symmetries to replace the R/matter parity that is broken in such theories. These symmetries also serve to prevent interference between the assigned functions of the multiple copies of low dimensional Higgs representation introduced (along with appropriately limited non-renormalizable interactions that in effect play the role of the higher dimensional Higgs irreps that were disallowed at the outset). With stringent unification constraints at GUT scales and large threshold corrections at the Susy breaking scale only narrow regions of the Susy-GUT parameter space remain viable. Moreover specification of the soft Susy spectrum and couplings to ensure simultaneous cancellations to keep the threshold corrections to $m_b^{MSSM}(M_S)$ small and the $b \to s\gamma$ rate consistent with current limits [26, 27, 22] is needed. In general such investigations have focussed on evaluating various non minimal effects such as non-universal gaugino masses, D-term contributions to scalar non universality etc, operating in specific (tight) corners of the soft susy parameter space, to evade the conflicting demands of infra red sensitivity, large infrared threshold corrections and accurate yukawa unification. A much smaller set of papers [1, 2, 5, 9, 6, 13, 7, 16, 14] focusses on renormalizable models without ad-hoc symmetries and confronts the full complexity of the spontaeous symmetry breaking problem conjointly with fitting the fermion spectra [14, 17].

Renormalizable SO(10) Susy GUTs[1, 2, 6] employ not only the **10**-plet but also the other two SO(10)-allowed fermion mass (FM) Higgs representations i.e the $\overline{\mathbf{126}}$ and **120**. Renormalizability of the GUT model is strictly maintained throughout as a criterion for the structure of the superpotential. As a result these theories have a very parameter-economical structure[1, 2, 6, 14]. In 1992, just as yukawa unification came into focus it was proposed[29] that use of only the $\mathbf{10}-\overline{\mathbf{126}}$ FM Higgs might be sufficient to account for the entire fermion mass spectrum *including* the neutrino masses and mixings in terms of their (15) yukawa coupling parameters. This obviously attractive possibility was investigated as a *generic* possibility in SO(10) (i.e without using the formulae in terms of the fundamental GUT parameters but only having the symmetries and (naive[30])parameter counting implied by their FM Higgs yukawa structure). Although initially unsuccessful, increasingly refined studies based upon the increasingly well defined neutrino mass data ultimately showed[31, 32] that the generic $\mathbf{10}-\overline{\mathbf{126}}$ parametrizations permitted accurate fits of all the known fermion mass data using **10**-plet domination of third generation mass terms combined with Georgi-Jarlskog mechanism for the second generation and either the Type I or Type II seesaw mechanisms or a combination thereof to fit the neutrino data. Perforce these generic models always assumed freedom to dial the relative strength of the two seesaw mechanisms.These accurate fits were found even while assuming that the precisely known SM yukawa coupling uncertainties persevered unchanged over the RG flow up to the GUT scales. In the light of the drastic susy threshold corrections at large $\tan\beta$ this assumption was naive and peculiarly uninformed of the attention long paid[4, 23] to the role of these corrections in the other strand of SO(10) GUTs. The appearance of [33, 34] however reminded us of this essential feature and we have thenceforth adopted the much more realistic estimates of theoretical and experimental uncertainties advocated by [33] in our subsequent work[17].

Just as the intriguing results on the successful implementation of the Babu-Mohapatra proposal in generic SO(10) GUTs emerged it was also shown[13, 7, 16, 30] that : (i) The use of MSGUT specific fermion mass formulae implied that neither seesaw mechanism could account for neutrino masses while also fitting the charged fermion data. (ii) Conclusions derived using generic formulae could not be transferred to concrete and specific GUTs because of intrinsic difficulties in untangling highly nonlinear constraints placed by the underlying fundamental GUT on the generic parameters. (iii) In a specific scenario[7] inclusion of the remaining allowed FM Higgs (i.e the **120**-plet) could permit the generation of large enough Type I seesaw masses due to suppressed $\overline{\mathbf{126}}$ couplings and charged fermion mass fits due to the $\mathbf{10}-\mathbf{120}$ combination.

The $\mathbf{10}-\mathbf{120}$ FM Higgs combination proposed by us to tackle the charged fermion fits (with the $\overline{\mathbf{126}}$ couplings too small to affect any but the first generation masses) was seemingly shown[35] to face a generic difficulty in providing large enough strange and down quark masses. This was only to be expected in any scenario where the Georgi-Jarlskog or other mechanism to distinguish second generation down type yukawas is not implemented. This difficulty appears in a quite different light when one considers[17] that Susy threshold corrections at large $\tan\beta$ tend [4, 23] to drastically modify the effective value of the down and strange quark yukawas. Thus we proposed[17] that the excellent fits obtainable for the other (16) fermion data (besides $y_{d,s}$) justified searching for the susy threshold corrections to implement the required corrections to lower $y_{d,s}$ and this could be achieved not only with free soft susy parameters at $M_S$ (as in version 1 of this paper[17]) but even with just 5 GUT compatible $N=1$ supergravity(SUGRY), Non Universal Higgs Masses(NUHM), type soft parameters $(m_{\frac{1}{2}}, m_0, A_0, m^2_{H,\bar{H}})$ at $m_Z$ specified at $M_X$[17] and two more $(|\mu|, B\sim m_A^2)$ determined by electro-weak spontaneous symmetry breaking and run back up to $M_X$ to provide 7 parameters at $M_X$ coding all the SUGRY-NUHM information.

These developments have put detailed consideration of the role of $\mathbf{120}-\overline{\mathbf{126}}$ FM Higgs representations firmly on the agenda of Susy SO(10) yukawa unification. In this paper we argue that the assumption that third generation fermion yukawas are protected from large GUT scale threshold corrections associated with non-**10** -plet FM Higgs irreps is facile. We show that it is belied in the class of Susy SO(10) theories[1, 2, 6, 7, 14, 17] which actually face up to the task of accounting for *all* the available fermion mass data in a fully specified model without invoking uncontrollable higher dimensional operators or ad-hoc symmetries and rely solely on SO(10) gauge symmetry, supersymmetry, and structural(parameter counting) minimality as guiding principles. In particular the NMSGUT with a large **120**-plet coupling (the **120**-plet yukawa is antisymmetric and hence has two eigenvalues that are equal in magnitude and one that is zero) *requires* evaluation of the GUT scale threshold corrections to the fermion Yukawa couplings. The large number of fields, **120**-plet couplings comparable to the **10**-plet couplings, and the fact that these threshold corrections arise from chiral supermultiplet wave function normalization(so that any field that couples to a matter field or a Higgs doublet can run in the self energy loop) raises the possibility that these corrections may be far from negligible[36]. In this paper we show that this is in fact the case.

The particular importance of the wave function corrections for the fermion data fitting program is that they can relax the stringent constraint $y_b - y_\tau \simeq y_s - y_\mu$ that we found [41, 42] operative at $M_X$ in SO(10) models with a **10-120** FM Higgs system. Thus at least any model that employs the **120**-plet must be sufficiently specified to allow the calculation of these very significant corrections which can drastically change the yukawa couplings at $M_X$. Such a specification involves solution of the spontaneous symmetry breaking, calculation of the GUT spectrum and couplings and a RG analysis of threshold effects based thereon. So far, to our knowledge, such a complete calculation is possible only for the SO(10) NMSGUT [7, 14, 17]for which we give the dominant 1-loop contributions( certain sub-dominant mixing effects are postponed to a future calculation). The calculation of the (wave function) renormalization that gives rise to the threshold effects modifying the matter fermion Yukawa couplings highlights the issue of perturbative consistency. One finds that while the corrections to the matter fermion lines of the mass generating Higgs-Yukawa vertices are no more than 25%, the wave function corrections($\Delta_{H,\bar{H}}$) to the (light) MSSM Higgs lines can be much larger. The most serendipitous scenario would be if searches that restricted these corrections to be less than 1 were able to find fits that are also consistent with Baryon decay limits. This is not what we have observed in our searches so far : accurate fits with $|\Delta_{H,\bar{H}}| < 1$ are obstructed in achieving small values ($<< 10^{-18} GeV^{-1}$) of the Baryon violating dimension 5 operator Wilson coefficients and hence give baryon lifetimes $\sim 10^{27}$ years. On the other hand if one allows large values of $\Delta_{H,\bar{H}}$ the searches achieve lifetimes that are a million times or more larger. Thus although threshold corrections to gauge couplings turned out to be mild due to cancellations [12, 7] in spite of early alarms[36] the wave function renormalization effects at one loop are indeed large. Investigation of higher loop contributions is thus be called for to see if the perturbation theory is sensible. In this paper we take the view that it is necessary to pursue the investigation of the realistic features of the SO(10) while including the large 1-loop threshold effects even though full perturbative convergence may take long to prove( or indeed may never be possible : as for instance in the most precise known theory QED).

In Section **2** we briefly review the structure of the NMSGUT[7, 14, 17] to establish the notation for presentation of our results on threshold effects in Section **3** and Appendix **A**. In Section **4** we present illustrative examples to underline the significance of the GUT scale threshold effects and the need to include them. In Section **5** we present examples of improved fits(specially with respect to acceptable $d = 5$ operator Baryon violation rates). In Section **6** we discuss the broad features of the emerging phenomenology of the NMSGUT. In Section **7** we summarize our conclusions and discuss the various improvements in the fitting, RG flows and searches and that are called for. Appendix **A** contains details of the calculation of threshold effects at $M_X$. Appendix **B** contains a discussion of the two loop RGE flow at large $A_0, |M^2_{H,\bar{H}}|$ values which results in the unconventional susy spectra with normal s-hierarchy and gaugino masses not even close to the $1:2:7$ ratio found at one loop in Susy GUTs with universal gaugino masses. Appendix **C** contains tables of parameter values for additional example fits to provide a wider view of the possibilities and as inputs for phenomenological explorations of the viability of our results. .

# 2 NMSGUT recapitulated

The NMSGUT [14] is a renormalizable globally supersymmetric $SO(10)$ GUT whose Higgs chiral supermultiplets consist of AM(Adjoint Multiplet) type totally antisymmetric tensors: $\mathbf{210}(\Phi_{ijkl})$, $\overline{\mathbf{126}}(\overline{\mathbf{\Sigma}}_{ijklm})$, $\mathbf{126}(\mathbf{\Sigma}_{ijklm})(i,j=1...10)$ which break the $SO(10)$ symmetry to the MSSM, together with Fermion mass (FM) Higgs $\mathbf{10}$ $(\mathbf{H}_i)$ and $\mathbf{120}(O_{ijk})$. The $\overline{\mathbf{126}}$ plays a dual or AM-FM role since it also enables the generation of realistic charged fermion and neutrino masses and mixings (via the Type I and/or Type II Seesaw mechanisms); three **16**-plets $\mathbf{\Psi_A}(A=1,2,3)$ contain the matter including the three conjugate neutrinos $(\bar{\nu}_L^A)$. The superpotential (see[6, 9, 10, 12, 7, 14, 17] for comprehensive details ) contains the mass parameters

$$m:\mathbf{210^2} \quad ; \quad M:\mathbf{126}\cdot\overline{\mathbf{126}}; \qquad M_H:\mathbf{10^2}; \qquad m_O:\mathbf{120^2} \tag{1}$$

and trilinear couplings corresponding to the superfield chiral invariants indicated :

$$\begin{aligned} \lambda &: \mathbf{210^3} \quad ; \quad \eta:\mathbf{210}\cdot\mathbf{126}\cdot\overline{\mathbf{126}}; \qquad \rho:\mathbf{120}\cdot\mathbf{120}\cdot\mathbf{210} \\ k &: \mathbf{10}\cdot\mathbf{120}\cdot\mathbf{210} \quad ;\gamma\oplus\bar{\gamma}:\mathbf{10}\cdot\mathbf{210}\cdot(\mathbf{126}\oplus\overline{\mathbf{126}}) \\ \zeta\oplus\bar{\zeta} &: \mathbf{120}\cdot\mathbf{210}\cdot(\mathbf{126}\oplus\overline{\mathbf{126}}) \end{aligned} \tag{2}$$

In addition one has two symmetric matrices $h_{AB}, f_{AB}$ of Yukawa couplings of the the $\mathbf{10},\overline{\mathbf{126}}$ Higgs multiplets to the $\mathbf{16_A}.\mathbf{16_B}$ matter bilinears and one antisymmetric matrix $g_{AB}$ for the coupling of the **120** to $\mathbf{16_A}.\mathbf{16_B}$. One of the complex symmetric matrices can be made real and diagonal by a choice of SO(10) flavour basis. Thus initially complex FM Yukawas contain 3 real and 9 complex parameters. Five overall phases (one for each Higgs), say those of $m, M, \lambda, \gamma, \bar{\gamma}$, can be set by fixing phase conventions. One(complex parameter) out of the rest of the superpotential parameters i.e $m, M_H, M, m_o, \lambda, \eta, \rho, k, \gamma, \bar{\gamma}, \zeta, \bar{\zeta}$ , say $M_H$, can be fixed by the fine tuning condition to keep two doublets light so that the effective theory is the MSSM. After removing un- physical phases this leaves 23 magnitudes and 15 phases as parameters : still in the lead out of any theories aspiring to do as much[6]. As explained in[6, 9, 12] the fine tuning fixes the Higgs fractions i.e the composition of the massless electroweak doublets in terms of the (6 pairs of suitable) doublet fields in the GUT. A subtle point here is that even if the other parameters are taken real the fine tuned $M_H$ (which does not itself enter into the low energy lagrangian) will be complex. Thus strictly speaking one cannot justify the use of only real superpotential parameters by invoking 'spontaneity' of CP violation and we will not do so.

The GUT scale vevs and therefore the mass spectrum are all expressible[6, 12, 10] in terms of a single complex parameter $x$ which is a solution of the cubic equation

$$8x^3 - 15x^2 + 14x - 3 + \xi(1-x)^2 = 0 \tag{3}$$

where $\xi = \frac{\lambda M}{\eta m}$.

In our programs we find it convenient to scan over the "three for a buck"[15] parameter $x$ and determine $\xi$ therefrom . Then the phase of $\lambda$ is adjusted to be that implied by $x$ and the relation $\xi = \frac{\lambda M}{\eta m}$ and is not itself scanned over independently. It is a convenient fact that the 492 fields in the Higgs sector fall into precisely 26 different types of SM gauge representations which can hence be naturally labelled by the 26 letters of the English alphabet[12]. The decomposition of SO(10) in terms of the labels its "Pati-Salam" maximal subgroup $SU(4) \times SU(2)_R \times SU(2)_L$ provided[9] a translation manual from SO(10) to unitary group labels. The complete GUT scale spectrum and couplings of this theory have been given in [12, 14].

The tree level fermion yukawa couplings and neutrino masses of the effective MSSM arising from this GUT below the GUT scale after fine tuning to keep one pair of Higgs multiplets light are given in [14, 17].

As mentioned, in the NMSGUT the conjugate(i.e "right handed") neutrino Majorana masses are 4 or more orders of magnitude smaller than the GUT scale due to very small $\overline{\mathbf{126}}$ couplings. Therefore for purposes of calculating the threshold corrections to the Yukawa couplings at $M_X$ we can consistently treat the conjugate neutrinos as light particles on the same footing as the other 15 fermions of each SM family. These fermion mass formulae, after correcting for threshold effects, are to be confronted with the RG-extrapolated data (from $Q = M_Z$ to $Q = M_X^0 = 10^{16.25}$ GeV including neutrino masses and mixing angles). The calculation of $\Delta_X$ also fixes the scale $m$ of the high scale symmetry breaking[7, 16, 14]. The stringent simultaneous requirements of of a common unification-seesaw scale, gauge unification (including the right high scale gauge RG threshold corrections to shift the GUT prediction of $\alpha_3(M_Z)$ down to acceptable values[19]), third generation yukawa unifcation, as well as fits to all the other fermion masses and mixing matrices, are effective in singling out characteristic and suggestive GUT parameters (including Susy breaking parameters at $M_X$).

# 3 GUT scale Yukawa threshold corrections

The technique of[37] for calculating high scale threshold corrections to yukawa couplings, generalizes the Weinberg-Hall[38] method for calculating threshold corrections to gauge couplings, and has long been available but has not been exploited much; possibly due to the assumption that such effects are always negligible. In supersymmetric theories the superpotential parameters are renormalized only due to wave function correction and this is easy -if tedious- to calculate for the large number of MSSM submultiplets in the **120**-plet which couple to the light fermions and MSSM Higgs at an SO(10) yukawa vertex. There is also a contribution from heavy gauge field couplings to the light(i.e MSSM) fields at the matter yukawa vertices. The calculation involves going to a basis in which the heavy field supermultiplet mass matrices are diagonal. This basis is easily computable given the complete set of mass matrices and trilinear coupling decompositions given in [9, 12, 14]. For a generic heavy field type $\Phi$ the mass terms in the superpotential diagonalize as :

$$\overline{\Phi} = U^{\Phi}\overline{\Phi'} \quad ; \qquad \Phi = V^{\Phi}\Phi' \quad \Rightarrow \quad \overline{\Phi}^T M\Phi = \overline{\Phi'}^T M_{Diag}\Phi' \tag{4}$$

Threshold correction to a Yukawa coupling matrix (which occurs in the superpotential as $W = \bar{f}^T Y_f f H_{\pm}$) then have the form

$$Y_f = Y_f + \Delta_{\bar{f}}^T \cdot Y_f + \Delta_f \cdot Y_f + \Delta_{H^\pm} Y_f \tag{5}$$

where the $\pm$ refers to the $Y = \pm 1$ Higgs multiplets appropriate to give mass to $T_3 = \pm\frac{1}{2}$ fermions and $\Delta_{f,\bar{f}}, \Delta_{H^\pm}$ are the 1-loop wave function correction factors. For a generic interaction superpotential $W = \frac{1}{6}\sum_{ijk} Y_{ijk}\Phi^i\Phi^j\Phi^k$, the quantities $\Delta$, at the renormalization scale $Q$, have the form ($\Delta = -\frac{1}{2}K$ in the notation of [37])

$$\Delta_i^j(Q) = \frac{1}{32\pi^2}(-2g_{10}^2\sum_{k,A} F_1(m_A, m_k, Q) I_{ik}^A I_{kj}^A + \frac{1}{2}\sum_{kl} Y_{ikl}Y_{jkl}^* F_1(m_k, m_l, Q)) \tag{6}$$

We have used $Q = M_X^0 = 10^{-\Delta_X} M_X$ where $\Delta_X$ is the shift (away from the one loop MSSM unification value $Log_{10}M_X^0 = 16.25$) due to loop and threshold effects. Here $A$ is a generic gauge field(adjoint) index and $i, j, k$ are generic chiral field indices. The gauge couplings and generators $g_{10}, I^A$ of SO(10) are related to the usual ($SU(5)$normalization) gauge coupling and generators $g_5 = g, T^A$ by $g_{10} = g/\sqrt{2}, I^A = \sqrt{2}T^A$. When both the fields running in the loop are heavy fields ($F_1$ is a symmetric Passarino-Veltman function) $F_1$ should be taken to be

$$F_{12}(M_A, M_B, Q) = \frac{1}{(M_A^2 - M_B^2)}(M_A^2 ln\frac{M_A^2}{Q^2} - M_B^2 ln\frac{M_B^2}{Q^2}) - 1 \tag{7}$$

which reduces to just

$$F_{11}(M_A, Q) = F_{12}(M_A, 0, Q) = ln\frac{M_A^2}{Q^2} - 1 \tag{8}$$

when one field is light ($M_B \to 0$). When one of the heavy fields in the loop has MSSM doublet type $G_{321}$ quantum numbers $[1, 2, \pm 1]$ (so that one eigenvalue is light while the other *five*[14] are heavy) care should be taken to avoid summing over light-light loops since that calculation belongs to the MSSM radiative corrections.

In the NMSGUT fitting scenario where $|f_{AB}| < 10^{-5}$ it is an excellent approximation to ignore the contributions of the $\overline{\mathbf{126}}$ to the high scale threshold corrections. Moreover while calculating the wave function renormalization of the Higgs line in the matter fermion-antifermion-MSSM Higgs vertex, we shall assume that it is an good approximation to take the MSSM higgs to be dominantly made up of the **10**-plet (which dominance we know is required in order to account for top-bottom-$\tau$ (near) unification which is an intrinsic part of the large $\tan\beta$ scenario) so that we can ignore the admixture of the other 5 MSSM type Higgs doublets pairs present in the theory. Hence, for the non-dominant i.e non **10**-plet derived light Higgs components, the contributions to the wavefunction renormalization of the light Higgs doublets in the theory will all be suppressed by additional small (in keeping with the **10**-plet dominance of light Higgs composition) factors of $|\alpha_i|^2; i = 2....6$ (for $H_0[1,2,1]$) or $|\bar{\alpha}_i|^2, i = 2....6$ (for $\bar{H}_0[1,2,-1]$). That is to say there will be a suppression of the contribution by $|\alpha_i|^2$ or $|\bar{\alpha}_i|^2$ unless the external doublets come from the **10**-plet in the $k\mathbf{10}.\mathbf{120}.\mathbf{210}$ or $\mathbf{10}.\mathbf{210}.(\gamma\mathbf{126}+\bar{\gamma}\overline{\mathbf{126}})$ terms of the superpotential. We can therefore sidestep–for the moment– the elaborate calculation required for calculating the wave function renormalization of the external light Higgs when all its 6 possible components (from the $\mathbf{10}, \mathbf{210}, \mathbf{126}, \overline{\mathbf{126}}, \mathbf{120}$ (two pairs) ) are corrected by wave function renormalization. This is an good ( but not perfect ) approximation in practice as will be seen from the values of the Higgs fractions[6, 12, 14]. For example in case I-1(Table3 the values are :

$$
\begin{aligned}
|\vec{\alpha}|^2 &= \{0.689, 0.0044, 0.0038, 0.1838, 0.0302, 0.0886\} \\
|\vec{\bar{\alpha}}|^2 &= \{0.792, 0.0068, 0.0037, 0.1020, 0.0052, 0.0903\}
\end{aligned}
\tag{9}
$$

where the numbering of the components is[14] seen from :

$$
[1,2,-1](\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_4, \bar{h}_5, \bar{h}_6) \oplus [1,2,1](h_1, h_2, h_3, h_4, h_5, h_6) \equiv \tag{10}
$$

$$
(H^{\alpha}_{\dot{2}}, \bar{\Sigma}^{(15)\alpha}_{\dot{2}}, \Sigma^{(15)\alpha}_{\dot{2}}, \frac{\Phi^{\dot{2}\alpha}_{44}}{\sqrt{2}}, O^{\alpha}_{\dot{2}}, O^{(15)\alpha}_{\dot{2}}\ ) \oplus (H_{\alpha\dot{1}}, \bar{\Sigma}^{(15)}_{\alpha\dot{1}}, \Sigma^{(15)}_{\alpha\dot{1}}, \frac{\Phi^{44\dot{1}}_{\alpha}}{\sqrt{2}}, O_{\alpha\dot{1}}, O^{(15)}_{\alpha\dot{1}})
$$

The decomposition of SO(10) invariant terms in the superpotential and gauge terms yields[9, 12, 14] a large number($\sim 100$ ) of vertices. It then requires a tedious but straightforward calculation to determine the threshold corrections explicitly. The explicit expressions are given in Appendix **A**.

Heretofore such threshold corrections have mostly been argued to be negligible($< 1\%$) although at least one paper [27] faced with the difficulties of literal third generation yukawa unification has considered the possibility, without any explicit model which permitted calculation, that the third generation yukawa unification relations must necessarily be subject to threshold corrections of up to 50%. In which case it was found that the various stratagems invoked to permit precise 3 generation Yukawa unification could become redundant. We shall see that the calculation of the GUT scale 1-loop Yukawa threshold effects in the NMSGUT can actually change the naive(i.e pure **10**-plet) unification relations $y_t = y_b = y_\tau$ significantly. Furthermore the $\mathbf{10}-\mathbf{120}$ plet fermion fits have been shown ( in the absence of GUT scale threshold effects) to require a close equality $|y_b - y_\tau/(y_s - y_\mu)| \approx 1$ at $M_X$ which is very constricting when searching for fits. The fits we exhibited in [14, 17] were all of this type. However in the present case the fits we obtain can deviate significantly from $\frac{y_b - y_\tau}{y_s - y_\mu} \simeq 1$ which was obeyed by both the fits presented in [17] where no threshold corrections were applied to the yukawas at $M_X$. The large values of $\Delta_{H,\bar{H}}$ can radically reduce the magnitudes of SO(10) yukawas required to reproduce the MSSM couplings at $M_X$, thus threshold effects can help in loosening this constriction of fitting freedom. Moreover the changes are such as to help in finding fits with a slower B violation rate. We note again that one must study the higher loop threshold corrections and the steps necessary to define a consistent perturbative expansion (possibly involving some variants of large N resummation and use of the exact SO(10) Susy gauge beta functions[77]) to see if the 1-loop results we find are stable.

# 4 Numerical fits to the fermion data and threshold effects

To appreciate the significance of the threshold corrections at $M_X$ for the matter fermion yukawas it is sufficient to consider the values of the matrices $\Delta_{f,\bar{f}}$ for the various MSSM fields when the formulae derived in the appendix are evaluated using parameters from the examples of fits (found ignoring GUT scale threshold corrections) given in [14, 17]. The NMSGUT is , to our knowledge, the *only* SO(10) model where the spontaneous symmetry breaking and the consequent spectra and Higgs fractions have been explicitly calculated so that the Yukawa threshold corrections can be evaluated for specific GUT parameter based fits.

The example fits from [17] are of the 18 known fermion data namely the yukawa couplings $y_{t,b,\tau,c,s,\mu,u,d,e}$, the CKM angles and phase $\theta^q_{12,13,23}, \delta^q$, the neutrino mass squared differences $\Delta m^2_{21,32}$ and Leptonic mixing angles $\theta^L_{12,13,23}$ in terms of the NMSGUT hard and soft parameters. We search assuming normal neutrino hierarchy, a very light ($\leq 1\ meV$) electron neutrino, and a small neutrino mixing angle $\theta^L_{13} < 5^\circ$[2]. The fits are found by a random search based on the downhill simplex method which requires the definition of a $\chi^2$ function formed from the difference of the GUT implied and target (i.e RG extrapolated from $M_Z$) values of the fermion parameters normalized by the uncertainties in these parameters[33]. An important point is that heretofore fitting in this class of renormalizable SO(10) models assumed that the uncertainties involved were merely the (very small) error estimates for the SM extrapolated to the GUT scale. The complexities induced by the uncertainties due to the strong threshold effects at the Susy and GUT thresholds were never given any shrift in this context. Recently the papers [33, 34] motivated us to use the threshold effects to evade the difficulty of the failure of the Georgi-Jarlskog mechanism in the NMSGUT by lowering the couplings $y_{d,s}$ via threshold corrections due to Susy partners. Thus in [17] we used the more realistic values of[33] for the error estimates and eschewed the spurious precision of most previous efforts in the context of renormalizable MSGUTs.

The first example of an accurate fit presented in [14] was able to achieve a $\chi_X = 0.0538$ for a fit of the 18 fermion data at the scale $M_X$ (accompanied by $\chi_Z = 0.027$ fit at the scale $M_Z$ for matching the run down charged fermion yukawas to the Standard Model results after inclusion of the Susy threshold corrections). When the same data are used with the threshold corrections switched *on* one gets $\chi_X = 773.6$ and the unification parameters are $\Delta_{X,G,3}$ are unacceptable. Clearly the threshold corrections can make a great deal of difference ! The corresponding changes for the second solution are $\chi_X = 286364.7$ and the unification parameters are $\Delta_{X,G,3}$ are unacceptable. It is also worth noting that even though the fermion yukawas generated from the NMSGUT formulae change radically when one inserts the threshold corrections (using couplings determined by the fits found ignoring them), one finds that the accuracy of satisfaction of $y_b - y_\tau = y_s - y_\mu$ as measured by the ratio $R_{b\tau/s\mu} = (y_b - y_\tau)/y_s - y_\mu$ remains good with the ratios for these two solutions changing as : $(1.084, 0.97) \to (1.16, 0.96)$.

[2] After the calculations for this paper were completed the T2K and MINOS results on the first measurements of $\theta^L_{13}$ using reactor muon neutrinos directed at distant neutrino detectors were announced[39, 40]. These results indicate that the likely range is $\theta^L_{13} \sim 10^\circ \pm 5^\circ$. Since the search found fits with $\theta^L_{13} \sim 5^\circ$ we anticipate no problem if the target is raised by 1-5 degrees.

| | | $NTH-1$ | |
|---|---|---|---|
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.010085 | 0.031557 | 0.032194 |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.013965 | 0.037585 | 0.038028 |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.015536 | 0.057369 | 0.057938 |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.015275 | 0.039068 | 0.040438 |
| $Eigenvalues(\Delta_{Q})$ | 0.007961 | 0.024766 | 0.025598 |
| $Eigenvalues(\Delta_{L})$ | 0.005748 | 0.038287 | 0.039675 |
| $\Delta_{\bar{H}}, \Delta_{H}$ | 3.007406 | 1.661375 | |
| | | $NTH-2$ | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.069710 | 1.007792 | 1.033000 |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.066965 | 1.024073 | 1.053104 |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.078188 | 1.049538 | 1.074279 |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.086420 | 0.999649 | 1.027649 |
| $Eigenvalues(\Delta_{Q})$ | 0.063482 | 1.058937 | 1.085331 |
| $Eigenvalues(\Delta_{L})$ | 0.077448 | 1.075814 | 1.104087 |
| $\Delta_{\bar{H}}, \Delta_{H}$ | 9.309859 | 4.858437 | |

Table 1: NTH12: Eigen values of the wave function correction matrices calculated using coupling values found in [17] *without* incorporating threshold corrections. Third generation values are in the first column. Note the significant threshold effects on the Higgs lines and the twofold degeneracy among the first two generations as well as suppression of the third generation corrections due to cancelation between the large gauge and third generation yukawa contributions. Due to reduction of typical SO(10) **16**-plet yukawas both these features are modified in the fits found after incorporating threshold corrections at $M_X$.

Typical values for the fermion line dressing coefficients due to yukawa couplings of the first two generations are of a few percent or smaller as seen in Table 1 where we exhibit eigenvalues of the fermion/Higgs line dressing matrices calculated for the two fits given in [17] which did *not* incorporate threshold corrections. Note that values for the Higgs lines can add up and achieve large values of up to several 100% ! If we switch off these corrections to $\Delta_{H,\bar{H}}$ then the changes in the fits are much smaller but still by no means negligible. These numbers make it clear that the light fermion lines and specially the light Higgs lines suffer very significant threshold corrections. Thus realistic GUT theories must face up to the task of specifying themselves sufficiently explicitly so that the threshold corrections may be calculated, tree level estimates are likely to be only rough pctures or even completely misleading. It is in this sense that we speak of Grand Unification "upended".

# 5 Realistic fits with threshold corrections included

If we use our search programs to find fits after including the threshold effects we can impose strict perturbativity in the sense that no threshold correction may exceed 1 i.e

$$|\Delta_{f,\bar{f},H,\bar{H}}| < 1 \tag{11}$$

The search programs[17] do find solutions (quite far from the examples of [17] in that some couplings, such as $\eta$ underwent major changes) which satisfied this constraint and still provided accurate unification and accurate fits of the fermion mass data. However when one evaluates the rate of Baryon number violation (in the dominant $B \rightarrow Meson\ +\nu$ channels) one finds (as in the case of the fits in [17]) that typical solutions predict lifetimes of $10^{27}$ years or smaller. This is 6 orders of magnitude below the current experimental limits [47]. This problem is an extension of that exhibited by the solutions found without threshold effects[17].

Given the vast parameter space it is natural to ask whether there exist solutions where there is a suppression of the $d = 5$ operator mediated Baryon decay. Such fits can be found by instructing(via a $\chi^2$ penalty for rapid baryon number violation) the 'amoeba' (i.e is the search engine of the downhill simplex method for nonlinear fitting[52]) to look for fits that have sufficiently low B-decay rates. An exhaustive statistical characterization of the parameter space and its possibilities requires the marshalling of considerable (super)computing resources which we shall eventually accomplish. For the moment we avoid over-determining an already excruciating search for viable fits by not also demanding such strict perturbativity, in the above sense, in addition to all the other demands of a sensible supersymmetric phenomenology that we must anyway impose.

Since the complete programs for calculating B-decay rates (based upon the formulae provided in [25, 53]) are large and time consuming it would have slowed down both our search engine and the present computations too much to interface the complete Baryon violation programs (which include renormalizing some 447 variables from $M_X$ to $M_Z$, as well as a time consuming evaluation of the Baryon decay amplitudes) with our search programs at this still exploratory stage. Therefore we adopted the expedient of computing the the maximal absolute magnitude $Max(O^{(4)})$ of the LLLL and RRRR coefficients in the $d = 5, \Delta B \neq 0$ effective superpotential for the NMSGUT[14, 17]. For the solutions found so far this quantity was found to be typically of order $10^{-18}$ to $10^{-16}\, GeV^{-1}$ corresponding to the fast baryon decay rates $\sim 10^{-27}\, yr^{-1}$ obtained. Thus a quick fix to the problem of limiting the B-decay rate while searching for accurate fermion fits is to limit($\tilde{O}$ is the dimensionless operator in units od $|m/\lambda|$) $Max(\tilde{O}^{(4)}) < 10^{-5} GeV^{-1}$. This produced fits with proton lifetimes above $10^{36}$ yrs so we also relaxed the limit in some searches to just $Max(\tilde{O}^{(4)}) < 10^{-4} GeV^{-1}$. When this condition was imposed simultaneously with the requirement of strict perturbativity (11) above, we were unable to find any accurate fits so far. On the other hand if one removes the condition of strict perturbativity but limits $Max(O^{(4)}) < 10^{-21}\, Gev^{-1}$ or $Max(O^{(4)}) < 10^{-22}\, GeV^{-1}$ then it was possible to find accurate fits that gave lifetimes in excess of $10^{33} yrs$. In addition to the above mentioned penalties we also required the fits to satisfy the following consistency/phenomenological constraints :

- As already explained in detail in [14] the gauge unification RG flow is constrained so that perturbation theory in the gauge coupling at unification remains valid, the unification scale is less than $M_{Planck}$ and the GUT threshold contributions to $\alpha_3(M_Z X)$ are in the right range[18, 19, 14] :

$$\begin{aligned} -22.0 \leq \Delta_G &\equiv \Delta(\alpha_G^{-1}(M_X)) \leq 25 \\ 3.0 \geq \Delta_X &\equiv \Delta(Log_{10} M_X) \geq -0.3 \\ -.017 < \Delta_3 &\equiv \alpha_3(M_Z) < -.004 \end{aligned} \tag{12}$$

- We constrain the $|\mu(M_Z)|, |(A_0)_{ii}| (i = 1, 2, 3)$ parameters to be smaller than 150 TeV. Typically these parameters emerge in the range $\sim 50 - 90$ TeV while the gaugino masses $M_i$ are driven to the lower limits imposed(since it is the ratios $|\mu(M_Z)|/M_i, (A_0)/M_i$ which cointrol the efficacy of the large tan$\beta$ corrections for our purposes. This is the price one must pay to correct the fermion yukawas to achievable values in the NMSGUT. Large values of $A_0$ are well known to lead to deep charge and colour breaking (CCB) minimae[67] or unbounded from below (UFB) potentials[68]. However it is also established[54] that the metastable standard vacua that we are considering (with all mass squared parameters of charged or coloured or sneutrino scalar fields *positive* i.e at a local minimum which preserves colour,charge and R-parity) can well be stable on times scales ($\sim$ 10 Giga-years) of order the age of the universe. Thus individual cases need to be re-examined in detail before dismissing any otherwise viable fit out of hand. We consider the situation further in the next section.

- In accordance with experimental constraints [51] we also constrain lightest chargino (essentially wino $\tilde{W}^{\pm}$ ) masses to be greater than 110 GeV. All the charged sfermions as well as the charged Higgs are constrained to lie above 110 GeV and the uncharged Higgs($h^0$) above 105 GeV.

- Since the susy threshold corrections to $y_{d,s,b}$ *necessary* for the survival of the NMSGUT as a viable theory of fermion masses depend on logarithms of ratios of soft susy breaking parameters, our scenario is obviously incapable of *predicting* the all important susy breaking scale. On the other hand this is also a theory that counts providing exact unbroken R-parity down to the lowest scales (so that the LSP is stable and a good Dark Matter candidate) as one of its main virtues. Thus it behooves us to search for fits constrained by requiring the mass of the LSP(which is purely Bino due to the large value $\mu \sim 50TeV$ that emerges) in various ranges best motivated from a LSP dark matter scenario point of view, such as $> 101\,GeV$(range I), $5 - 50\,GeV$ (Range II) and $50 - 101\,GeV$ (Range III), to get an initial glimpse of whether and how the effective Dark Matter scale is linked to the pattern of superpartner masses. Thus in Tables 2 to 19 we provide examples of fits found with the LSP/Bino mass in each of these ranges. In addition, in the Appendix we provide 8 additional examples of fits in these categories. We emphasize that these examples are only indicative and that a thorough investigation should have a finer grain in the LSP mass ranges chosen and must also incorporate loop corrections, specially for sfermion masses besides various improvements discussed below. It must also consider the statistics of fits found by much larger scale systematic searches than those we have so far been able to mount : due to the scale of computer resources required. We note that the Type II and III fits are variations on those in I found by changing just the constraint on the LSP/Bino mass. One of the cases namely II-2 did not yield a satisfactorily exact fit in spite of extensive running, therefore it is omitted from the Appendix.

In Tables of Type X-a ($X = I, II, III$) we give the complete set of NMSGUT parameters defined at the one loop unification scale $M_X^0$ together with the values of the soft Susy breaking parameters ($\{m_0, m_{1/2}, A_0, B, M^2_{H,\bar{H}}\}$) together with Supersymmetric parameter $\mu$. Thus we have tried out only a N=1 Supergravity GUT motivated scenario with relaxation permitting Non universal Higgs masses(SUGRY-NUHM) (this is justified from a GUT point of view since the light doublets are a mixture of doublets from several sources in different SO(10) representations). The procedure followed for finding the fits, specially the use of Susy threshold correction to correct the yukawas $y_{d,s}$ to values consistent with the **10-120** FM Higgs structure, as well as to raise the effective $y_b$ in the MSSM so as relieve the well known tension requiring this SM value of this parameter to lie several standard deviations above its experimental value in order to achieve $b, \tau$ unification[64], is described in detail in the second paper of this series [17]. Besides these parameter values of the SUGRY-NUHM NMSGUT we also give the mass spectrum of superheavy heavy fields including the right handed neutrinos and the Type I and Type II neutrino seesaw masses as well as the unification parameters $\Delta_{X,G,3}$ described in detail in [14].

In tables of Type X-b we have given the values of the target(i.e two loop RGE extrapoled Susy threshold corrected MSSM yukawas and Susy Weinberg operator coefficients) fermion parameters and their uncertainties (estimated a la [33]) together with the achieved values and pulls. The reader may check that the fits are all excellent with typical fractional errors O(0.1%). We remark that we found it tedious and meaningless to push our program to further narrow the fit in view of the large uncertainties and the numerous corrections still to be incorporated in our calculations (see below). We also give the eigenvalues of the GUT scale yukawa vertex threshold correction operators. We note that Tables X-a show that there is a significant lowering of the size of the SO(10) fermion yukawas so that the universal gauge corrections dominate and make the corrections to all three generations almost equal specially when the lowering is pronounced. We give also the values of the the "Higgs fractions [6, 12, 14] $\alpha_i, \bar{\alpha}_i$ crucial for determining the fermion mass formulae [12, 13, 7, 17]. These parameters are determined as a consequence of the GUT scale symmetry breaking and the fine tuning to preserve a light pair of MSSM Higgs doublets. They distill the influence of the SO(10) GUT (and its spinorial clebsches determined appositely for this purpose in [9]) on the low energy fermion physics. The reader may use them together with the formulae given in [14] to check the fits even without entering into the details of our GUT scale mass spectra. We note that the first components of the $\alpha_i, \bar{\alpha}_i$ were chosen real by convention [14](see Appendix **C**).

In Tables of type X-c values of the SM masses at $M_Z$ are compared with those of masses from the run down yukawas achieved in the NMSGUT both before and after large $\tan\beta$ driven radiative corrections. Note that the central value of $m_b(M_Z) = 2.9$ GeV becomes prima facie acceptable in contrast to small $A_0$ scenarios where the need for $m_b(M_Z) > 3.1$ GeV, ie more than one standard deviation from the experimental central value, has been a principal source of tension and anxiety for small $A_0$ models [64] and should perhaps motivate acceptance of exploration of the large $A_0$ parameter space which seems almost unexplored so far.

In Tables of type X-d values of the soft supersymmetry breaking parameters which are the most crucial and remarkable output of this study since they tie the survival of the NMSGUT to a specific type of soft Susy spectrum with large $\mu, A_0, B$ and third generation sfermion masses generation in the 10-100 TeV range. Remarkably and in sharp contrast to received (small $A_0, M^2_{H,\bar{H}}$) wisdom the third s-generation is much *heavier* than the first two generations, which however are themselves not very light *except* for the *right chiral* smuon/selectron , sstrange/sdown and scharm/sup *right handed* charged sfermions which can actually descend close to their experimental lower limits. In doing so they keep alive the effectiveness of the pure Bino LSP (and pure Wino lightest chargino and next to lightest Neutralinos) as candidate dark matter by providing co-annihilation channels of the sort a light *stau* is often enlisted for in standard Susy GUT scenarios. Also remarkable is the friability of the familiar 1 : 2 : 7 ratio of the gaugino masses $M_1, M_2, M_3$ that we discover repeatedly in our calculations. This ratio is almost fixed in stone by one loop RGE and GUT mandated gaugino mass universality for small $A_0$ (and often provokes baroque elaborations of the gauge invariance principle designed to avoid it). Here however the large influence of the $A_0$ parameters can change the RGE flow to the point where the gluino is often *lighter* than the Winos which are sometimes as much thrice the mass of the Bino. Note also that except for the very heavy third generation the actual sizes of the sfermion trilinear couplings are rather modest since they are the product of the $A_0$ parameters and the Yukawas. For the third sgeneration the trilinears are roughly the same size as the masses themselves thus preserving naturality. We remind the reader that a diagonal two loop RGE flow from $M_X$ to $M_Z$ was used to determine these soft susy parameters via the Susy threshold effects since only the diagonal formulae were easily accessible and seemed justified in view of our limited expectations of overall accuracy of sfermion spectra which we have so far evaluated only at *tree* level. In view of the importance of the susy spectra we discuss in detail in Appendix **B** the features of the two loop RGE flow which result in the unconventional spectra noted above.

Finally Tables of type X-e,f give Susy particle determined using two loop RGEs with and without generation mixing switched on. They are so similar as to justify the use of the diagonal values for estimating the Susy threshold corrections. For the case of the lightest sfermions however the corrections are sometimes as large as 10-30%. This again sounds a note of caution regarding the exact numerical values of the lighter sfermion masses we provide. However even after incorporation of Loop corrections in addition to these effects we certainly expect that the broad division into LHC discoverable particles lighter than say 2.5 TeV (the LHC beam energy available per parton which sets the upper limit of discovery potential at the LHC) and those heavier to have some cogency regarding the limits of the discoverable. Thus we have provided an additional Table 20 that collects all superparticles with masses less than this limit. We also note that the Higgs masses were calculated using the 1-loop corrected electroweak symmetry breaking conditions and 1-loop effective potential using a subroutine[56] based on[71]. The wary reader fearful of loop corrections destroying the whole scenario may be reassured at least on that count.

| Parameter | $Value$ | Field $[SU(3), SU(2), Y]$ | $Masses$ $(Units\ of 10^{16}GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0464 | $A[1,1,4]$ | 645.64 |
| $\chi_Z$ | 0.0148 | $B[6,2,5/3]$ | 0.3633 |
| $h_{11}/10^{-6}$ | 0.0212 | $C[8,2,1]$ | $35.72, 325.28, 339.02$ |
| $h_{22}/10^{-4}$ | 0.0344 | $D[3,2,7/3]$ | $35.03, 349.70, 362.74$ |
| $h_{33}$ | 0.0026 | $E[3,2,1/3]$ | $0.58, 26.33, 26.33$ |
| $f_{11}/10^{-6}$ | $0.0781 - 0.1368i$ | | $28.661, 393.61, 441.49$ |
| $f_{12}/10^{-6}$ | $-1.9955 - 0.0830i$ | $F[1,1,2]$ | $6.15, 6.15$ |
| $f_{13}/10^{-5}$ | $0.0580 + 0.0517i$ | | $25.31, 325.28$ |
| $f_{22}/10^{-5}$ | $6.6036 - 4.9627i$ | $G[1,1,0]$ | $0.091, 0.72, 0.72$ |
| $f_{23}/10^{-4}$ | $2.0080 + 2.3459i$ | | $0.718, 30.69, 30.92$ |
| $f_{33}/10^{-3}$ | $-1.0051 + 0.4427i$ | $h[1,2,1]$ | $1.437, 20.71, 34.27$ |
| $g_{12}/10^{-4}$ | $0.0605 + 0.1232i$ | | $541.46, 563.22$ |
| $g_{13}/10^{-5}$ | $-0.0460 + 1.8407i$ | $I[3,1,10/3]$ | 1.26 |
| $g_{23}/10^{-4}$ | $6.3251 + 5.7460i$ | $J[3,1,4/3]$ | $1.387, 14.31, 14.31$ |
| $\lambda/10^{-2}$ | $-0.8601 - 1.4974i$ | | $44.05, 383.45$ |
| $\eta$ | $-10.3248 + 2.5325i$ | $K[3,1,8/3]$ | $50.91, 468.83$ |
| $\rho$ | $0.7042 - 2.2528i$ | $L[6,1,2/3]$ | $24.18, 752.08$ |
| $k$ | $0.0151 - 0.0805i$ | $M[6,1,8/3]$ | 761.55 |
| $\zeta$ | $1.6200 + 0.5400i$ | $N[6,1,4/3]$ | 757.79 |
| $\bar{\zeta}$ | $1.0084 + 0.4594i$ | $O[1,3,2]$ | 1454.74 |
| $m/10^{16}GeV$ | 0.04 | $P[3,3,2/3]$ | $14.50, 1130.98$ |
| $m_o/10^{16}GeV$ | $-21.047e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 1.041 |
| $\gamma$ | 3.71 | $R[8,1,0]$ | $0.40, 1.55$ |
| $\bar{\gamma}$ | $-2.8691$ | $S[1,3,0]$ | 1.7528 |
| $x$ | $0.9397 + 0.6629i$ | $t[3,1,2/3]$ | $1.15, 19.66, 47.99, 78.65$ |
| $\Delta_X$ | 1.16 | | $252.51, 337.25, 7050.35$ |
| $\Delta_G$ | 4.863 | $U[3,3,4/3]$ | 1.480 |
| $\Delta\alpha_3(M_Z)$ | $-0.013$ | $V[1,2,3]$ | 1.046 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.01, 14.65, 613.85$ | $W[6,3,2/3]$ | 877.20 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $0.4997, 927.38, 38856.88$ | $X[3,2,5/3]$ | $0.353, 28.201, 28.201$ |
| $M_\nu(meV)$ | $2.17, 7.63, 42.34$ | $Y[6,2,1/3]$ | 0.44 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.15, 282.68, 11844.25$ | $Z[8,1,2]$ | 1.54 |
| Soft parameters at $M_X$ | $\mathrm{m}_{\frac{1}{2}} = -88.707$ | $\mathrm{m}_0 = 4198.698$ | $\mathrm{A}_0 = -1.1832 \times 10^5$ |
| | $\mu = 9.4206 \times 10^4$ | $\mathrm{B} = -5.9399 \times 10^9$ | $\tan\beta = 50.0000$ |
| | $\mathrm{M}^2_{\bar{\mathrm{H}}} = -7.1782 \times 10^9$ | $\mathrm{M}^2_{\mathrm{H}} = -6.7789 \times 10^9$ | $\mathrm{R}_{\frac{b\tau}{s\mu}} = 2.6935$ |
| $Max(\|L_{ABCD}\|, \|R_{ABCD}\|)$ | $6.1149 \times 10^{-23}\,\mathrm{GeV}^{-1}$ | | |

Table 2: I-1-a : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. Unificaton parameters and mass spectrum of superheavy and superlight fields are also given. The values of $\mu(M_X), B(M_X)$ are determined by RG evolution from $M_Z$ to $M_X$ of the values determined by the EWRSB conditions.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.098620 | 0.801673 | 2.098242 | −0.000472 |
| $y_c/10^{-3}$ | 1.022984 | 0.168792 | 1.023751 | 0.004546 |
| $y_t$ | 0.383855 | 0.015354 | 0.383875 | 0.001336 |
| $y_d/10^{-5}$ | 6.826388 | 3.979784 | 6.843968 | 0.004417 |
| $y_s/10^{-3}$ | 1.294453 | 0.610982 | 1.301389 | 0.011351 |
| $y_b$ | 0.460136 | 0.238811 | 0.460919 | 0.003278 |
| $y_e/10^{-4}$ | 1.201757 | 0.180264 | 1.202499 | 0.004117 |
| $y_\mu/10^{-2}$ | 2.464838 | 0.369726 | 2.468141 | 0.008935 |
| $y_\tau$ | 0.527874 | 0.100296 | 0.523894 | −0.039688 |
| $\sin\theta_{12}^q$ | 0.2210 | 0.001600 | 0.2210 | −0.0005 |
| $\sin\theta_{13}^q/10^{-4}$ | 29.1027 | 5.000000 | 29.1272 | 0.0049 |
| $\sin\theta_{23}^q/10^{-3}$ | 34.2424 | 1.300000 | 34.2402 | −0.0017 |
| $\delta^q$ | 60.0207 | 14.000000 | 60.0550 | 0.0024 |
| $(m_{12}^2)/10^{-5}(eV)^2$ | 5.3580 | 0.567948 | 5.3569 | −0.0019 |
| $(m_{23}^2)/10^{-3}(eV)^2$ | 1.7330 | 0.346597 | 1.7347 | 0.0050 |
| $\sin^2\theta_{12}^L$ | 0.2887 | 0.057748 | 0.2883 | −0.0080 |
| $\sin^2\theta_{23}^L$ | 0.4620 | 0.138613 | 0.4639 | 0.0132 |
| $\theta_{13}^L(degrees)$ | 3.7 | 3.7 | 4.57 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.048796 | 0.048800 | 0.048805 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.046619 | 0.046623 | 0.046629 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.052548 | 0.052552 | 0.052558 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.059078 | 0.059082 | 0.059087 | |
| $Eigenvalues(\Delta_Q)$ | 0.046281 | 0.046284 | 0.046289 | |
| $Eigenvalues(\Delta_L)$ | 0.054386 | 0.054390 | 0.054394 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 71.935369 | 62.415435 | | |
| $\alpha_1$ | $0.8302 + 0.0000i$ | $\bar{\alpha}_1$ | $0.8899 + 0.0000i$ | |
| $\alpha_2$ | $0.0644 + 0.0157i$ | $\bar{\alpha}_2$ | $0.0516 + 0.0644i$ | |
| $\alpha_3$ | $-0.0425 - 0.0442i$ | $\bar{\alpha}_3$ | $-0.0571 - 0.0204i$ | |
| $\alpha_4$ | $-0.3970 + 0.1618i$ | $\bar{\alpha}_4$ | $0.3186 - 0.0237i$ | |
| $\alpha_5$ | $0.1534 + 0.0818i$ | $\bar{\alpha}_5$ | $0.0709 + 0.0141i$ | |
| $\alpha_6$ | $0.1116 - 0.2759i$ | $\bar{\alpha}_6$ | $0.1187 - 0.2760i$ | |

Table 3: I-1-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0464$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. The eigenvalues of the wavefunction renormalization increment matrices $\Delta_i$ for fermion lines and the factors for Higgs lines are given, assuming the external Higgs is 10-plet dominated. Note the close similarity of the eigenvalues which suggests that the small values of the SO(10) yukwas utilized when threshold corrections are in play lead to gauge dominated corrections which are the same for all three generations. The Higgs fractions $\alpha_i, \bar{\alpha}_i$ which control the MSSM fermion yukawa couplings are also given. Notice the dominance of the first components $\alpha_1, \bar{\alpha}_1$ consistently with the assumption made. Right handed neutrino threshold effects have been ignored. We have truncated numbers for display although all calculations are done at double precision.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.63332 | 2.92341 |
| $m_s/10^{-3}$ | 55.00000 | 12.04268 | 55.60296 |
| $m_b$ | 3.00000 | 3.07778 | 3.00662 |
| $m_e/10^{-3}$ | 0.48657 | 0.47249 | 0.48691 |
| $m_\mu$ | 0.10272 | 0.09694 | 0.10270 |
| $m_\tau$ | 1.74624 | 1.73514 | 1.73675 |
| $m_u/10^{-3}$ | 1.27000 | 1.09930 | 1.27007 |
| $m_c$ | 0.61900 | 0.53635 | 0.61968 |
| $m_t$ | 172.50000 | 149.05874 | 172.47396 |

Table 4: I-1-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} = 0.0148$.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 124.94 | $M_{\tilde{\bar{u}}_1}$ | 5392.74 |
| $M_2$ | 328.39 | $M_{\tilde{\bar{u}}_2}$ | 5392.01 |
| $M_3$ | 569.92 | $M_{\tilde{\bar{u}}_3}$ | 17882.28 |
| $M_{\tilde{\bar{l}}_1}$ | 1101.73 | $A_{11}^{0(l)}$ | $-75168.69$ |
| $M_{\tilde{\bar{l}}_2}$ | 165.22 | $A_{22}^{0(l)}$ | $-75083.12$ |
| $M_{\tilde{\bar{l}}_3}$ | 11100.23 | $A_{33}^{0(l)}$ | $-47868.83$ |
| $M_{\tilde{L}_1}$ | 6400.13 | $A_{11}^{0(u)}$ | $-86227.11$ |
| $M_{\tilde{L}_2}$ | 6353.87 | $A_{22}^{0(u)}$ | $-86226.53$ |
| $M_{\tilde{L}_3}$ | 10210.78 | $A_{33}^{0(u)}$ | $-43721.21$ |
| $M_{\tilde{\bar{d}}_1}$ | 2917.32 | $A_{11}^{0(d)}$ | $-75501.89$ |
| $M_{\tilde{\bar{d}}_2}$ | 2916.57 | $A_{22}^{0(d)}$ | $-75501.25$ |
| $M_{\tilde{\bar{d}}_3}$ | 26552.10 | $A_{33}^{0(d)}$ | $-32031.46$ |
| $M_{\tilde{Q}_1}$ | 4928.60 | $\tan\beta$ | 50.00 |
| $M_{\tilde{Q}_2}$ | 4927.98 | $\mu(M_Z)$ | 76666.76 |
| $M_{\tilde{Q}_3}$ | 22679.30 | $B(M_Z)$ | $9.6672 \times 10^8$ |
| $M_{\bar{H}}^2$ | $-6.0301 \times 10^9$ | $M_H^2$ | $-6.3249 \times 10^9$ |

Table 5: I-1-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). The values of soft Susy parameters at $M_Z$ determine the Susy threshold corrections to the fermion yukawas. The matching of run down fermion yukawas in the MSSM to the SM parameters determines soft SUGRY parameters at $M_X$. Note the heavier third sgeneration. The values of $\mu(M_Z)$ and the corresponding soft susy parameter $B(M_Z) = m_A^2 \sin 2\beta/2$ are determined by imposing electroweak symmetry breaking conditions. $m_A$ is the mass of the CP odd scalar in the in the Doublet Higgs. The sign of $\mu$ is assumed positive.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 569.92 |
| $M_{\chi^{\pm}}$ | $328.38, 76666.85$ |
| $M_{\chi^0}$ | $124.94, 328.38, 76666.81, 76666.82$ |
| $M_{\tilde{\nu}}$ | $6399.784, 6353.520, 10210.565$ |
| $M_{\tilde{e}}$ | $1102.65, 6400.31, 160.56, 6354.33, 10105.12, 11196.70$ |
| $M_{\tilde{u}}$ | $4928.29, 5392.61, 4927.63, 5391.92, 17876.33, 22684.87$ |
| $M_{\tilde{d}}$ | $2917.43, 4928.98, 2916.66, 4928.38, 22663.07, 26566.04$ |
| $M_A$ | 219898.08 |
| $M_{H^{\pm}}$ | 219898.09 |
| $M_{H^0}$ | 219898.06 |
| $M_{h^0}$ | 111.45 |

Table 6: I-1-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. Inclusion of such effects changes the spectra only marginally. Due to the large values of $\mu, B, A_0$. The LSP and light chargino are essentially pure Bino and Wino($\tilde{W}_{\pm}$). The light gauginos and light Higgs $h^0$, are accompanied by a light smuon and sometimes selectron. The rest of the sfermions have multi-TeV masses. The mini-split supersymmetry spectrum and large $\mu, A_0$ parameters help avoid problems with FCNC and CCB/UFB instability[54]. The sfermion masses are ordered by generation not magnitude. This is useful in understanding the spectrum calculated including generation mixing effects. The mass of the Higgs particles($M_A, M_{h^0}, M_{H^{\pm}}$) are calculated by incorporating one loop contributions to the Electroweak symmetry breaking i.e to the effective potential[71, 56, 17].

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 570.03 |
| $M_{\chi^{\pm}}$ | $328.43, 76664.05$ |
| $M_{\chi^0}$ | $124.95, 328.43, 76664.02, 76664.02$ |
| $M_{\tilde{\nu}}$ | $6353.56, 6399.82, 10211.527$ |
| $M_{\tilde{e}}$ | $133.31, 1102.50, 6353.86, 6400.35, 10109.89, 11195.65$ |
| $M_{\tilde{u}}$ | $4926.11, 4928.47, 5392.09, 5392.79, 17873.85, 22683.48$ |
| $M_{\tilde{d}}$ | $2916.90, 2917.69, 4926.85, 4929.16, 22661.71, 26564.36$ |
| $M_A$ | 219973.32 |
| $M_{H^{\pm}}$ | 219973.33 |
| $M_{H^0}$ | 219973.30 |
| $M_{h^0}$ | 111.74 |

Table 7: I-1-f : Spectra of supersymmetric partners calculated including generation mixing effects. Inclusion of such effects changes the spectra only marginally. Due to the large values of $\mu, B, A_0$ the LSP and light charginos are essentially pure Bino and Wino($\tilde{W}_{\pm}$). Note that the ordering of the eigenvalues in this table follows their magnitudes, comparison with the previous table is necessary to identify the sfermions

| Parameter | $Value$ | Field $[SU(3), SU(2), Y]$ | $Masses$ $(Units\ of 10^{16} GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0133 | $A[1,1,4]$ | 622.89 |
| $\chi_Z$ | 0.0386 | $B[6,2,5/3]$ | 0.4399 |
| $h_{11}/10^{-6}$ | 0.0224 | $C[8,2,1]$ | $34.59, 332.64, 344.97$ |
| $h_{22}/10^{-4}$ | 0.0283 | $D[3,2,7/3]$ | $33.75, 322.29, 340.48$ |
| $h_{33}$ | 0.0026 | $E[3,2,1/3]$ | $0.70, 26.06, 26.06$ |
| $f_{11}/10^{-6}$ | $0.0782 - 0.1347i$ | | $27.899, 377.48, 429.89$ |
| $f_{12}/10^{-6}$ | $-2.0477 - 0.0905i$ | $F[1,1,2]$ | $6.08, 6.08$ |
| $f_{13}/10^{-5}$ | $0.0543 + 0.0544i$ | | $24.31, 313.95$ |
| $f_{22}/10^{-5}$ | $6.4657 - 4.7231i$ | $G[1,1,0]$ | $0.110, 0.80, 0.80$ |
| $f_{23}/10^{-4}$ | $1.9840 + 2.3506i$ | | $0.872, 33.23, 33.50$ |
| $f_{33}/10^{-3}$ | $-1.0229 + 0.4338i$ | $h[1,2,1]$ | $1.748, 20.51, 33.32$ |
| $g_{12}/10^{-4}$ | $0.0560 + 0.1257i$ | | $510.60, 538.80$ |
| $g_{13}/10^{-5}$ | $-0.1809 + 1.6222i$ | $I[3,1,10/3]$ | 1.52 |
| $g_{23}/10^{-4}$ | $6.2005 + 5.7438i$ | $J[3,1,4/3]$ | $1.661, 14.22, 14.22$ |
| $\lambda/10^{-2}$ | $-1.2230 - 1.7024i$ | | $42.85, 375.68$ |
| $\eta$ | $-10.0887 + 2.4493i$ | $K[3,1,8/3]$ | $49.33, 459.01$ |
| $\rho$ | $0.6745 - 2.1659i$ | $L[6,1,2/3]$ | $23.85, 735.63$ |
| $k$ | $0.0165 - 0.0849i$ | $M[6,1,8/3]$ | 746.01 |
| $\zeta$ | $1.5173 + 0.4434i$ | $N[6,1,4/3]$ | 739.68 |
| $\bar{\zeta}$ | $1.0209 + 0.4394i$ | $O[1,3,2]$ | 1396.55 |
| $m/10^{16}GeV$ | 0.05 | $P[3,3,2/3]$ | $12.83, 1088.74$ |
| $m_o/10^{16}GeV$ | $-20.758e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 1.262 |
| $\gamma$ | 3.67 | $R[8,1,0]$ | $0.49, 1.85$ |
| $\bar{\gamma}$ | $-2.9668$ | $S[1,3,0]$ | 2.1051 |
| $x$ | $0.9290 + 0.6512i$ | $t[3,1,2/3]$ | $1.39, 19.29, 46.66, 79.08$ |
| $\Delta_X$ | 1.15 | | $249.60, 335.17, 6705.69$ |
| $\Delta_G$ | 5.955 | $U[3,3,4/3]$ | 1.780 |
| $\Delta\alpha_3(M_Z)$ | $-0.009$ | $V[1,2,3]$ | 1.258 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.00, 16.54, 687.11$ | $W[6,3,2/3]$ | 854.17 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $0.2400, 912.69, 37917.07$ | $X[3,2,5/3]$ | $0.424, 27.998, 27.998$ |
| $M_\nu(meV)$ | $3.44, 8.12, 42.61$ | $Y[6,2,1/3]$ | 0.54 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.08, 287.79, 11955.91$ | $Z[8,1,2]$ | 1.84 |
| Soft parameters at $M_X$ | $m_{\frac{1}{2}} = -124.831$ | $m_0 = 2371.139$ | $A_0 = -6.1911 \times 10^4$ |
| | $\mu = 5.8956 \times 10^4$ | $B = -1.9501 \times 10^9$ | $\tan\beta = 50.0000$ |
| | $M^2_{\bar{H}} = -3.0748 \times 10^9$ | $M^2_H = -2.9538 \times 10^9$ | $R_{\frac{b\tau}{s\mu}} = 2.9200$ |
| $Max(\|L_{ABCD}\|, \|R_{ABCD}\|)$ | $6.7561 \times 10^{-23}\ \text{GeV}^{-1}$ | | |

Table 8: II-1-a: Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.141027 | 0.817872 | 2.141050 | 0.000028 |
| $y_c/10^{-3}$ | 1.043728 | 0.172215 | 1.043795 | 0.000391 |
| $y_t$ | 0.386577 | 0.015463 | 0.386580 | 0.000221 |
| $y_d/10^{-5}$ | 6.891765 | 4.017899 | 6.919378 | 0.006873 |
| $y_s/10^{-3}$ | 1.311160 | 0.618867 | 1.311543 | 0.000620 |
| $y_b$ | 0.451549 | 0.234354 | 0.453520 | 0.008411 |
| $y_e/10^{-4}$ | 1.221705 | 0.183256 | 1.221655 | −0.000272 |
| $y_\mu/10^{-2}$ | 2.580846 | 0.387127 | 2.580746 | −0.000257 |
| $y_\tau$ | 0.525812 | 0.099904 | 0.525048 | −0.007648 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | −0.0001 |
| $\sin\theta^q_{13}/10^{-4}$ | 29.1374 | 5.000000 | 29.1407 | 0.0007 |
| $\sin\theta^q_{23}/10^{-3}$ | 34.2832 | 1.300000 | 34.2832 | 0.0000 |
| $\delta^q$ | 60.0207 | 14.000000 | 60.0102 | −0.0008 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.4100 | 0.573459 | 5.4100 | 0.0001 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.7497 | 0.349943 | 1.7497 | −0.0001 |
| $\sin^2\theta^L_{12}$ | 0.2887 | 0.057750 | 0.2887 | 0.0000 |
| $\sin^2\theta^L_{23}$ | 0.4621 | 0.138631 | 0.4621 | −0.0001 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 8.18 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.048603 | 0.048607 | 0.048613 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.046536 | 0.046540 | 0.046546 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.052666 | 0.052670 | 0.052675 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.058866 | 0.058870 | 0.058876 | |
| $Eigenvalues(\Delta_Q)$ | 0.046115 | 0.046119 | 0.046124 | |
| $Eigenvalues(\Delta_L)$ | 0.054312 | 0.054316 | 0.054320 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 70.962744 | 62.481215 | | |
| $\alpha_1$ | $0.8263 + 0.0000i$ | $\bar{\alpha}_1$ | $0.8795 + 0.0000i$ | |
| $\alpha_2$ | $0.0646 + 0.0197i$ | $\bar{\alpha}_2$ | $0.0509 + 0.0709i$ | |
| $\alpha_3$ | $-0.0437 - 0.0505i$ | $\bar{\alpha}_3$ | $-0.0594 - 0.0234i$ | |
| $\alpha_4$ | $-0.3966 + 0.1515i$ | $\bar{\alpha}_4$ | $0.3314 - 0.0234i$ | |
| $\alpha_5$ | $0.1606 + 0.0812i$ | $\bar{\alpha}_5$ | $0.0782 + 0.0121i$ | |
| $\alpha_6$ | $0.1088 - 0.2894i$ | $\bar{\alpha}_6$ | $0.1174 - 0.2904i$ | |

Table 9: II-1-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0133$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m+\Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.64294 | 2.90511 |
| $m_s/10^{-3}$ | 55.00000 | 12.18662 | 54.90038 |
| $m_b$ | 3.00000 | 3.05467 | 3.02134 |
| $m_e/10^{-3}$ | 0.48657 | 0.48198 | 0.48770 |
| $m_\mu$ | 0.10272 | 0.10177 | 0.10289 |
| $m_\tau$ | 1.74624 | 1.74369 | 1.74266 |
| $m_u/10^{-3}$ | 1.27000 | 1.11799 | 1.23689 |
| $m_c$ | 0.61900 | 0.54503 | 0.60300 |
| $m_t$ | 172.50000 | 149.51960 | 172.26015 |

Table 10: II-1-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} =$ 0.0386.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 35.00 | $M_{\tilde{\bar{u}}_1}$ | 2490.81 |
| $M_2$ | 113.01 | $M_{\tilde{\bar{u}}_2}$ | 2490.90 |
| $M_3$ | 83.09 | $M_{\tilde{\bar{u}}_3}$ | 16928.60 |
| $M_{\tilde{\bar{l}}_1}$ | 1045.62 | $A_{11}^{0(l)}$ | $-39522.44$ |
| $M_{\tilde{\bar{l}}_2}$ | 1148.28 | $A_{22}^{0(l)}$ | $-39473.12$ |
| $M_{\tilde{\bar{l}}_3}$ | 14376.75 | $A_{33}^{0(l)}$ | $-25104.91$ |
| $M_{\tilde{L}_1}$ | 3699.46 | $A_{11}^{0(u)}$ | $-44729.39$ |
| $M_{\tilde{L}_2}$ | 3714.57 | $A_{22}^{0(u)}$ | $-44729.08$ |
| $M_{\tilde{L}_3}$ | 10786.67 | $A_{33}^{0(u)}$ | $-22492.46$ |
| $M_{\tilde{\bar{d}}_1}$ | 320.40 | $A_{11}^{0(d)}$ | $-39448.33$ |
| $M_{\tilde{\bar{d}}_2}$ | 323.59 | $A_{22}^{0(d)}$ | $-39447.98$ |
| $M_{\tilde{\bar{d}}_3}$ | 19724.24 | $A_{33}^{0(d)}$ | $-16948.89$ |
| $M_{\tilde{Q}_1}$ | 2422.85 | $\tan\beta$ | 50.00 |
| $M_{\tilde{Q}_2}$ | 2423.11 | $\mu(M_Z)$ | 48104.14 |
| $M_{\tilde{Q}_3}$ | 18383.64 | $B(M_Z)$ | $3.0799 \times 10^8$ |
| $M_{\bar{H}}^2$ | $-2.3660 \times 10^9$ | $M_H^2$ | $-2.5187 \times 10^9$ |

Table 11: II-1-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 83.09 |
| $M_{\chi^{\pm}}$ | 113.01, 48104.28 |
| $M_{\chi^0}$ | 35.00, 113.01, 48104.23, 48104.24 |
| $M_{\tilde{\nu}}$ | 3698.868, 3713.977, 10786.469 |
| $M_{\tilde{e}}$ | 1046.59, 3699.78, 1147.01, 3715.55, 10777.59, 14383.72 |
| $M_{\tilde{u}}$ | 2422.23, 2490.54, 2422.10, 2491.00, 16922.19, 18390.64 |
| $M_{\tilde{d}}$ | 321.46, 2423.62, 324.40, 2423.91, 18355.16, 19750.86 |
| $M_A$ | 124120.40 |
| $M_{H^{\pm}}$ | 124120.43 |
| $M_{H^0}$ | 124120.39 |
| $M_{h^0}$ | 130.18 |

Table 12: II-1-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 83.15 |
| $M_{\chi^{\pm}}$ | 113.03, 48102.54 |
| $M_{\chi^0}$ | 35.01, 113.03, 48102.49, 48102.50 |
| $M_{\tilde{\nu}}$ | 3698.88, 3713.99, 10786.642 |
| $M_{\tilde{e}}$ | 1046.52, 1144.84, 3699.80, 3715.67, 10778.23, 14383.76 |
| $M_{\tilde{u}}$ | 2422.27, 2438.70, 2490.58, 2491.10, 16922.33, 18388.56 |
| $M_{\tilde{d}}$ | 321.58, 324.46, 2423.67, 2440.53, 18353.16, 19750.82 |
| $M_A$ | 124163.88 |
| $M_{H^{\pm}}$ | 124163.90 |
| $M_{H^0}$ | 124163.87 |
| $M_{h^0}$ | 130.20 |

Table 13: II-1-f: Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

| Parameter | $Value$ | Field $[SU(3),SU(2),Y]$ | $Masses$ $(Units\ of 10^{16}GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0038 | $A[1,1,4]$ | 756.12 |
| $\chi_Z$ | 0.0030 | $B[6,2,5/3]$ | 0.4240 |
| $h_{11}/10^{-6}$ | 0.0211 | $C[8,2,1]$ | $39.13, 387.00, 401.03$ |
| $h_{22}/10^{-4}$ | 0.0329 | $D[3,2,7/3]$ | $38.83, 385.83, 402.57$ |
| $h_{33}$ | 0.0026 | $E[3,2,1/3]$ | $0.68, 29.71, 29.71$ |
| $f_{11}/10^{-6}$ | $0.0774 - 0.1363i$ | | $31.591, 443.85, 505.56$ |
| $f_{12}/10^{-6}$ | $-1.9810 - 0.0598i$ | $F[1,1,2]$ | $7.05, 7.05$ |
| $f_{13}/10^{-5}$ | $0.0578 + 0.0556i$ | | $28.49, 380.72$ |
| $f_{22}/10^{-5}$ | $6.4063 - 4.9278i$ | $G[1,1,0]$ | $0.105, 0.82, 0.82$ |
| $f_{23}/10^{-4}$ | $2.0067 + 2.2917i$ | | $0.836, 35.65, 35.90$ |
| $f_{33}/10^{-3}$ | $-0.9930 + 0.4524i$ | $h[1,2,1]$ | $1.695, 23.43, 37.99$ |
| $g_{12}/10^{-4}$ | $0.0610 + 0.1236i$ | | $611.35, 639.58$ |
| $g_{13}/10^{-5}$ | $-0.0066 + 1.7851i$ | $I[3,1,10/3]$ | 1.45 |
| $g_{23}/10^{-4}$ | $6.4307 + 5.7410i$ | $J[3,1,4/3]$ | $1.588, 16.35, 16.35$ |
| $\lambda/10^{-2}$ | $-1.0000 - 1.4449i$ | | $49.47, 451.05$ |
| $\eta$ | $-10.5299 + 2.6810i$ | $K[3,1,8/3]$ | $57.16, 553.55$ |
| $\rho$ | $0.6944 - 2.2125i$ | $L[6,1,2/3]$ | $27.44, 883.40$ |
| $k$ | $0.0166 - 0.0797i$ | $M[6,1,8/3]$ | 896.34 |
| $\zeta$ | $1.5479 + 0.5373i$ | $N[6,1,4/3]$ | 888.19 |
| $\bar{\zeta}$ | $1.0081 + 0.4175i$ | $O[1,3,2]$ | 1667.08 |
| $m/10^{16}GeV$ | 0.04 | $P[3,3,2/3]$ | $15.50, 1292.76$ |
| $m_o/10^{16}GeV$ | $-23.876e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 1.199 |
| $\gamma$ | 3.69 | $R[8,1,0]$ | $0.47, 1.78$ |
| $\bar{\gamma}$ | $-2.8264$ | $S[1,3,0]$ | 2.0164 |
| $x$ | $0.9272 + 0.6601i$ | $t[3,1,2/3]$ | $1.33, 22.24, 53.87, 86.47$ |
| $\Delta_X$ | 1.22 | | $286.53, 380.42, 8307.87$ |
| $\Delta_G$ | 5.014 | $U[3,3,4/3]$ | 1.701 |
| $\Delta\alpha_3(M_Z)$ | $-0.009$ | $V[1,2,3]$ | 1.212 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.01, 16.26, 692.32$ | $W[6,3,2/3]$ | 1005.11 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $0.2849, 723.96, 30818.79$ | $X[3,2,5/3]$ | $0.409, 31.922, 31.922$ |
| $M_\nu(meV)$ | $2.09, 7.64, 42.49$ | $Y[6,2,1/3]$ | 0.52 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.11, 276.17, 11756.40$ | $Z[8,1,2]$ | 1.77 |
| Soft parameters at $M_X$ | $m_{\frac{1}{2}} = -107.371$ <br> $\mu = 8.4934 \times 10^4$ <br> $M^2_{\bar{H}} = -5.8077 \times 10^9$ | $m_0 = 3585.150$ <br> $B = -4.7848 \times 10^9$ <br> $M^2_H = -5.5072 \times 10^9$ | $A_0 = -1.0442 \times 10^5$ <br> $\tan\beta = 50.0000$ <br> $R_{\frac{b\tau}{s\mu}} = 2.7107$ |
| $Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$ | $5.8199 \times 10^{-23}\,\text{GeV}^{-1}$ | | |

Table 14: III-1-a : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 1 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.116326 | 0.808437 | 2.116304 | −0.000027 |
| $y_c/10^{-3}$ | 1.031618 | 0.170217 | 1.031400 | −0.001282 |
| $y_t$ | 0.386527 | 0.015461 | 0.386531 | 0.000285 |
| $y_d/10^{-5}$ | 6.872020 | 4.006388 | 6.880189 | 0.002039 |
| $y_s/10^{-3}$ | 1.303124 | 0.615075 | 1.301971 | −0.001874 |
| $y_b$ | 0.468198 | 0.242995 | 0.468608 | 0.001688 |
| $y_e/10^{-4}$ | 1.209876 | 0.181481 | 1.209817 | −0.000328 |
| $y_\mu/10^{-2}$ | 2.485105 | 0.372766 | 2.485397 | 0.000783 |
| $y_\tau$ | 0.532529 | 0.101181 | 0.532449 | −0.000788 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | −0.0001 |
| $\sin\theta^q_{13}/10^{-4}$ | 29.0367 | 5.000000 | 29.0367 | 0.0000 |
| $\sin\theta^q_{23}/10^{-3}$ | 34.1648 | 1.300000 | 34.1652 | 0.0003 |
| $\delta^q$ | 60.0207 | 14.000000 | 60.0123 | −0.0006 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.3985 | 0.572236 | 5.3985 | 0.0001 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.7468 | 0.349356 | 1.7467 | −0.0001 |
| $\sin^2\theta^L_{12}$ | 0.2886 | 0.057728 | 0.2886 | 0.0001 |
| $\sin^2\theta^L_{23}$ | 0.4617 | 0.138503 | 0.4616 | −0.0006 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 5.38 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.051945 | 0.051949 | 0.051955 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.049907 | 0.049911 | 0.049917 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.056963 | 0.056967 | 0.056973 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.063077 | 0.063081 | 0.063087 | |
| $Eigenvalues(\Delta_Q)$ | 0.048996 | 0.049000 | 0.049006 | |
| $Eigenvalues(\Delta_L)$ | 0.058090 | 0.058094 | 0.058099 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 72.623699 | 62.630532 | | |
| $\alpha_1$ | $0.8274 + 0.0000i$ | $\bar{\alpha}_1$ | $0.8895 + 0.0000i$ | |
| $\alpha_2$ | $0.0650 + 0.0179i$ | $\bar{\alpha}_2$ | $0.0526 + 0.0664i$ | |
| $\alpha_3$ | $-0.0415 - 0.0457i$ | $\bar{\alpha}_3$ | $-0.0554 - 0.0217i$ | |
| $\alpha_4$ | $-0.4028 + 0.1724i$ | $\bar{\alpha}_4$ | $0.3209 - 0.0356i$ | |
| $\alpha_5$ | $0.1511 + 0.0810i$ | $\bar{\alpha}_5$ | $0.0705 + 0.0136i$ | |
| $\alpha_6$ | $0.1099 - 0.2714i$ | $\bar{\alpha}_6$ | $0.1150 - 0.2746i$ | |

Table 15: III-1-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0038$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 2 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM}=(m+\Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.63252 | 2.90621 |
| $m_s/10^{-3}$ | 55.00000 | 11.96937 | 55.00879 |
| $m_b$ | 3.00000 | 3.09207 | 3.00111 |
| $m_e/10^{-3}$ | 0.48657 | 0.47228 | 0.48641 |
| $m_\mu$ | 0.10272 | 0.09698 | 0.10252 |
| $m_\tau$ | 1.74624 | 1.74550 | 1.74567 |
| $m_u/10^{-3}$ | 1.27000 | 1.10652 | 1.27009 |
| $m_c$ | 0.61900 | 0.53927 | 0.61899 |
| $m_t$ | 172.50000 | 149.30372 | 172.48084 |

Table 16: III-1-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z=\sqrt{\sum_{i=1}^{9}(m_i^{MSSM}-m_i^{SM})^2/(m_i^{MSSM})^2}=$ 0.0030.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 100.07 | $M_{\tilde{\bar{u}}_1}$ | 4539.23 |
| $M_2$ | 270.77 | $M_{\tilde{\bar{u}}_2}$ | 4538.61 |
| $M_3$ | 428.43 | $M_{\tilde{\bar{u}}_3}$ | 17182.81 |
| $M_{\tilde{\bar{l}}_1}$ | 906.34 | $A_{11}^{0(l)}$ | $-65947.70$ |
| $M_{\tilde{\bar{l}}_2}$ | 142.19 | $A_{22}^{0(l)}$ | $-65871.94$ |
| $M_{\tilde{\bar{l}}_3}$ | 11976.39 | $A_{33}^{0(l)}$ | $-41513.30$ |
| $M_{\tilde{L}_1}$ | 5564.41 | $A_{11}^{0(u)}$ | $-75901.65$ |
| $M_{\tilde{L}_2}$ | 5528.49 | $A_{22}^{0(u)}$ | $-75901.14$ |
| $M_{\tilde{L}_3}$ | 10194.21 | $A_{33}^{0(u)}$ | $-38216.99$ |
| $M_{\tilde{\bar{d}}_1}$ | 2291.12 | $A_{11}^{0(d)}$ | $-66155.83$ |
| $M_{\tilde{\bar{d}}_2}$ | 2290.56 | $A_{22}^{0(d)}$ | $-66155.26$ |
| $M_{\tilde{\bar{d}}_3}$ | 24659.75 | $A_{33}^{0(d)}$ | $-27514.18$ |
| $M_{\tilde{Q}_1}$ | 4205.32 | $\tan\beta$ | 50.00 |
| $M_{\tilde{Q}_2}$ | 4204.83 | $\mu(M_Z)$ | 68576.15 |
| $M_{\tilde{Q}_3}$ | 21289.22 | $B(M_Z)$ | $7.4868\times10^{8}$ |
| $M_{\bar{H}}^2$ | $-4.7966\times10^{9}$ | $M_H^2$ | $-5.0786\times10^{9}$ |

Table 17: III-1-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 4 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 428.43 |
| $M_{\chi^{\pm}}$ | $270.76, 68576.25$ |
| $M_{\chi^0}$ | $100.07, 270.76, 68576.21, 68576.22$ |
| $M_{\tilde{\nu}}$ | $5564.015, 5528.094, 10193.999$ |
| $M_{\tilde{e}}$ | $907.46, 5564.62, 136.00, 5529.05, 10149.71, 12014.32$ |
| $M_{\tilde{u}}$ | $4204.96, 4539.08, 4204.40, 4538.53, 17176.99, 21294.85$ |
| $M_{\tilde{d}}$ | $2291.27, 4205.76, 2290.68, 4205.29, 21272.06, 24674.64$ |
| $M_A$ | 193516.73 |
| $M_{H^{\pm}}$ | 193516.75 |
| $M_{H^0}$ | 193516.72 |
| $M_{h^0}$ | 117.18 |

Table 18: III-1-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 428.53 |
| $M_{\chi^{\pm}}$ | $270.80, 68573.70$ |
| $M_{\chi^0}$ | $100.09, 270.80, 68573.66, 68573.67$ |
| $M_{\tilde{\nu}}$ | $5528.13, 5564.05, 10194.739$ |
| $M_{\tilde{e}}$ | $105.36, 907.31, 5528.77, 5564.65, 10152.37, 12014.38$ |
| $M_{\tilde{u}}$ | $4205.11, 4205.54, 4538.67, 4539.24, 17175.20, 21293.36$ |
| $M_{\tilde{d}}$ | $2290.89, 2291.51, 4205.91, 4206.41, 21270.60, 24673.37$ |
| $M_A$ | 193585.06 |
| $M_{H^{\pm}}$ | 193585.07 |
| $M_{H^0}$ | 193585.04 |
| $M_{h^0}$ | 117.39 |

Table 19: III-1-f : Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

# 6 Structural features and Phenomenology

As mentioned the threshold corrections to the Higgs lines in a light fermion- light higgs vertex can obtain a thick wave function dressing from GUT scale particles leading to an amplification of the effective SO(10) yukawa couplings and making much weaker SO(10) couplings capable of fitting the fermion data. Moreover the threshold corrections of the light fermion yukawa couplings are highly nonlinear, and as such there is no reason to expect that the constraint $y_b - y_\tau \simeq y_s - y_\mu$ found on the basis of the tree level coupling of the **10,120**-plets to the **16**-plets of fermions[41, 42]continues to be effective. The influence of these large corrections can be seen in the value of $R_{b\tau/s\mu} = \frac{y_b - y_\tau}{y_s - y_\mu}$ given in the tables. We see that only in cases $I-2, III-2, III-4$ is $R_{b\tau/s\mu}$ still approximately unity while in the rest it is typically $\sim 3$: which is the magnitude of the ratio when all threshold corrections to SM values are ignored and the couplings are run up using MSSM RGEs. This shows that the tree level constraint can be evaded; and with profit since the magnitudes are no longer mismatched. Note that the cases with $R_{b\tau/s\mu} \sim 1$ typically have $\Delta_{H,\overline{H}}$ smaller by a factor of about 5 than the cases where $R_{b\tau/s\mu} \sim 3$.

The ultra small values of the $\overline{\mathbf{126}}$ couplings ensure that they make little difference to the 2nd and 3rd generation charged fermion yukawas but ensure that $M_{\nu^c}$ are light and thus Type I neutrino masses are viable[7]. Note that for the same reason in all fits the Type II neutrino masses are completely negligible. The other superpotential couplings are unremarkable except that $\eta$ is somewhat large. However one should recall that it occurs in the superpotential divided by $4! = 24$. Actual coefficients of the radiative corrections will typically be powers of ($\sim \frac{|\eta|}{4\pi}$).

As explained in Section **1,2** the unification scale $M_X = M_X^0 10^{\Delta_X}$, (typically $\sim 10^{17.5}$ GeV) is identified with the mass of the $X[3, 2, \pm\frac{5}{3}]$ gauge sub-multiplet ( exchange of which gives rise to d=6 operators for B decay ) and determines the scale parameter $m$ of the superpotential via

$$m = \frac{|\lambda| 10^{16.25+\Delta_X}}{\tilde{M}_X} GeV \tag{13}$$

where

$$\tilde{M}_X = g_r \sqrt{4|\tilde{a} + \tilde{\omega}|^2 + 2|\tilde{p} + \tilde{\omega}|^2}$$

and

$$g_r = \sqrt{2\pi(25.6 + \Delta_G)^{-1}}$$

is the corrected SO(10) gauge coupling. We see that the unification scale is generally elevated( $10^{16.7} - 10^{19}$ GeV) but the SO(10) coupling at unification is still perturbative though sometimes only marginally so.

The right handed neutrino masses are important for Leptogenesis and for lepton RGE flows at intermediate scales. We find $M_{\nu^c}$ is generically in the range $10^9 - 10^{13}$ GeV (with normal hierarchy); which is also the preferred range for Leptogenesis. It is determined by the necessarily (for viable neutrino spectra) ultra small $\overline{\mathbf{126}}$ couplings $f \sim 10^{-8} - 10^{-3}$. It is interesting to note that the threshold corrections may also weaken the influence of the right handed neutrino thresholds on yukawa unification. The interplay of the GUT scale corrections and the $M_{\nu^c}$ thresholds will be interesting to evaluate, specially since the latter are known[65] to lead to tension for yukawa unification. It may be that as in the case of the tension regarding the value of $m_b(M_Z)$ acceptable for yukawa unification[64], which is relieved in our model by the threshold corrections at $M_S = M_Z$, so also GUT scale threshold corrections may help with relieving the effect of right handed neutrino thresholds.

The super heavy masses lie in the range $10^{14} - 10^{20}$ GeV with a few multiplets sometimes having an uncomfortably large mass even greater than the Planck mass. Overall the unification parameter and the spectra of the Fits with the large $\Delta_{H,\bar{H}} \sim 10^2$ and $R_{b\tau/s\mu} \sim 3$ seem more palatable than those for Case I-2 with $R_{b\tau/s\mu} \sim 1$, $\Delta_{H,\bar{H}} \sim 10$.

Let us turn next to the conjectured Soft Spectra determined by requiring EWRSB at $\tan\beta \sim 50$ and large threshold corrections to lower $y_{d,s}$ by a factor of 5 or so. The required values of the $|\mu|, |A_0|$ parameters turn out to be so huge ($\sim 10^2$ TeV) that they will incite controversy driven by concerns regarding deep CCB minima. For the moment we take the pragmatic attitude that we have checked the local stability by ensuing the positivity of all scalar mass squared parameters. The stability against tunneling to CCB minima on cosmological time scales calls for further investigation after loop corrections to scalar masses have been included. However the literature[54, 69] supports the pragmatic attitude we adopt regarding the viability of metastable minima. The seminal and clear investigations of[54] regarding meta-stability in the parameter region of ultra large $\mu, A_0$ are so encouraging that we cannot resist quoting them verbatim. Firstly they note that (our interpolations in square brackets)" *the height of the barrier separating the [metastable] minimum from the CCB minimum is roughly proportional to $1/y_{min}^2$, where $y_{min}$ is the smallest Yukawa coupling associated with the fields that acquire non-zero vev in the CCB minimum. The corresponding tunnelling rates are greatly suppressed for small $y$.* " Thus since it is only the third generation of matter sfermion fields that have appreciable yukawa couplings the violation of the CCB and UFB bounds[67, 68] is not likely to be a matter of concern for the first two generations. Moreover they note "*the most stringent constraints come from the small* $\tan\beta$ *region, where the top Yukawa coupling is larger*". Whereas we are in the large $\tan\beta \sim 50$ regime.

The investigations of [54] focussed on the region $|\mu|, |A_0| < 4$ TeV. Their findings confirmed that in this region "*the larger the trilinear coupling the more dangerous is the corresponding CCB minimum*". However they had the prescience to realize that understanding the behaviour in the regions with much larger $|\mu|, |A_0|$ would illuminate the dynamics of CCB and tunnelling and clarify the operation of decoupling arguments which seem violated by the above tradecraft maxim but are always crucial to establish an intuitive grip on field theoretic dynamics. Thus they note "*it is instructive to examine what happens to the tunnelling probability in the limit of very large $\mu$ and $A_t$ (and large enough squark mass terms to ensure the existence of the [metastable] minimum). In that limit, as the CCB minimum moves away from the SML [standard] minimum, the barrier separating the two becomes thicker, and the false vacuum should become more stable. This is, in fact, what happens.... As expected, the tunnelling probability diminishes for very large values of $A_t$ and $\mu$, and $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$. To summarize, if the global CCB minimum is nearly degenerate with the local SML minimum (thin-wall limit), then the tunnelling probability is extremely small. As the trilinear couplings increase, the false vacuum decay rate increases because the escape point of the bounce moves out of the flat vicinity of the global minimum into the region in which the gradient of the potential is significant.* ***However, a further increase in the size of the trilinear couplings, as well as the consequent increase in the squark mass terms, makes the barrier thicker and pushes the escape point away from the [metastable] minimum. This eventually causes a decrease in***

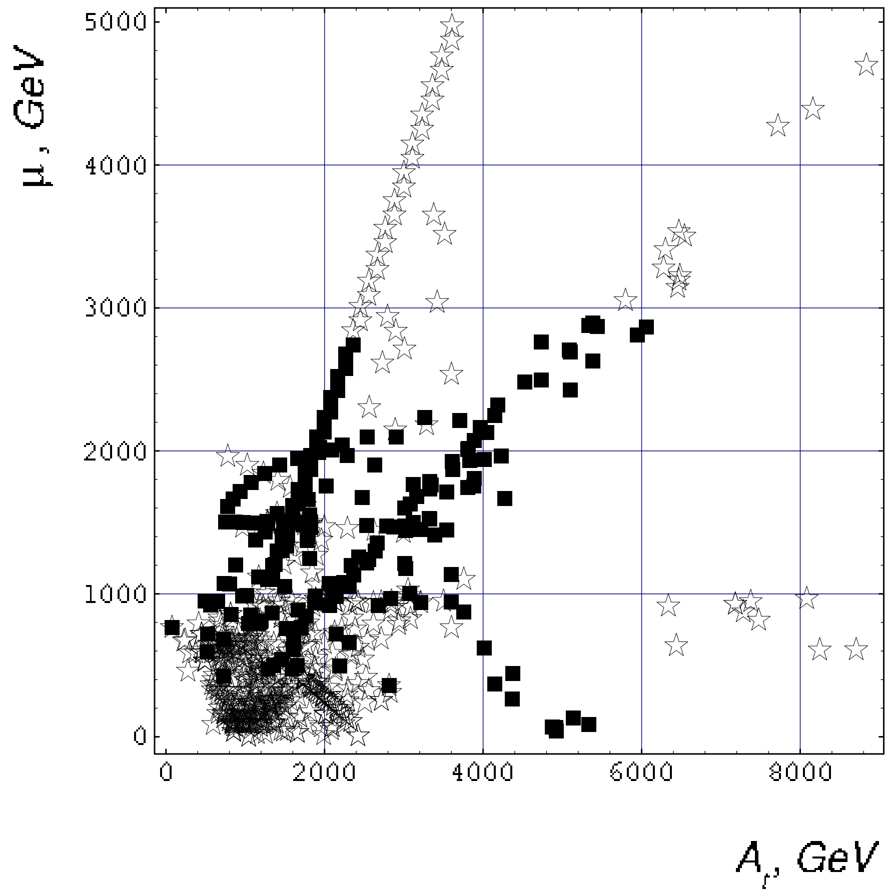


Figure 1: "*Tunneling probability for unphysically large values of $A_t$ and $\mu$. As the CCB minimum moves farther away, it becomes "less dangerous". As before, the stars mark the points with $S > 400$, while the boxes depict those with $S < 400$.*". The points marked with stars correspond to MSSM "standard/realistic vacua" that are long lived on the scale of the age of the universe. From [54] with permission.

***the tunneling rate. In accordance with one's intuition, the low-energy physics is unaffected by the physics at the very high energy scales.*** " These effects are clearly visible in the Fig. 1 and indicate that $|\mu|, |A_0| > 10$ TeV should be utterly safe as regards CCB/UFB issues.

The above arguments justify our retention of the parameter sets which we have found and which prima facie suffer from CCB problems. Our very large values of $\tan\beta, |\mu|, |A_0|$ and $B$ militate for metastability on cosmic time scales. However for the dubious we note two further points : we have not used the full freedom in to choose trilinears at $M_X$, contenting ourselves with a single $A_0(M_X)$ parameter. The requirement of viable metastability could itself be incorporated as a search criteria in future versions of our search routines. In sum it would be premature to dismiss the novel scenario emerging from an apparently well motivated fundamental theory on the basis of technical grounds of CCB instability whose very efficacy in choosing vacua has long been in doubt. We note that recently arguments have been advanced that cast doubt on even the very possibility of tunnelling from a Poincare invariant vacuum to a lower energy state[55]. Should it be the case that a phenomenology package assumes some version of CCB constraints without calculating

| Particle ⇒ | $LSP$ | $Winos$ | $Gluino$ | $Higgs$ | $Sfermions$ |
|---|---|---|---|---|---|
| Case ⇓ | $(\tilde{B})$ | $(\tilde{W}^{\pm,0})$ | $\tilde{g}$ | $h^0$ | $\tilde{\bar{f}}, \tilde{F}$ |
| $I-1$ | 0.125 | 0.328 | 0.570 | 0.111 | $1.1(\tilde{\bar{e}}), 0.16(\tilde{\bar{\mu}})$ |
| $I-2$ | 0.105 | 0.354 | 0.269 | 0.113 | $1.89(\tilde{\bar{e}}), 0.3(\tilde{\bar{\mu}}), 0.82(\tilde{\bar{d}}), 0.83(\tilde{\bar{s}})$ |
| $I-3$ | 0.147 | 0.406 | 0.599 | 0.115 | $1.63(\tilde{\bar{e}}), 0.39(\tilde{\bar{\mu}})$ |
| $I-4$ | 0.104 | 0.302 | 0.351 | 0.128 | $0.12(\tilde{\bar{e}}), 0.89(\tilde{\bar{\mu}}), 1.7(\tilde{\bar{d}}), 1.7(\tilde{\bar{s}})$ |
| $II-1$ | 0.035 | 0.113 | 0.83 | 0.130 | $1.05(\tilde{\bar{e}}), 1.14(\tilde{\bar{\mu}}), 0.32(\tilde{\bar{d}}), 0.32(\tilde{\bar{s}}), 2.49(\tilde{\bar{u}}), 2.49(\tilde{\bar{c}}), 2.49(\tilde{Q}_{1,2})$ |
| $II-3$ | .044 | .144 | 0.09 | .124 | $2.15(\tilde{\bar{e}}), 2.07(\tilde{\bar{\mu}}), 0.46(\tilde{\bar{d}}), 0.46(\tilde{\bar{s}}), 0.20(\tilde{\bar{u}}), 0.20(\tilde{\bar{c}}), 2.45(\tilde{Q}_{1,2})$ |
| $II-4$ | 0.035 | 0.110 | 0.082 | 0.127 | $1.67(\tilde{\bar{e}}), 1.68(\tilde{\bar{\mu}}), 0.364(\tilde{\bar{d}}), 0.36(\tilde{\bar{s}}), 1.36(\tilde{\bar{u}}), 1.36(\tilde{\bar{c}}), 2.11(\tilde{Q}_{1,2})$ |
| $III-1$ | 0.100 | 0.271 | 0.429 | 0.117 | $0.105(\tilde{\bar{e}}), 0.91(\tilde{\bar{\mu}}), 2.29(\tilde{\bar{d}}), 2.29(\tilde{\bar{s}})$ |
| $III-2$ | 0.99 | 0.342 | 0.232 | 0.113 | $1.87(\tilde{\bar{e}}), 0.34(\tilde{\bar{\mu}}), 0.72(\tilde{\bar{d}}), 0.71(\tilde{\bar{s}})$ |
| $III-3$ | 0.98 | 0.273 | 0.387 | 0.118 | $1.06(\tilde{\bar{e}}), 0.14(\tilde{\bar{\mu}}), 2.15(\tilde{\bar{d}}), 2.15(\tilde{\bar{s}})$ |
| $III-4$ | 0.94 | 0.284 | 0.269 | 0.128 | $.09(\tilde{\bar{e}}), 0.11(\tilde{\bar{\mu}}), 1.17(\tilde{\bar{d}}), 1.17(\tilde{\bar{s}})$ |

Table 20: Table of nominally discoverable particles. Mass values (in TeV) below 2.5 TeV, rounded off to two decimal places, calculated at tree level(except the Higgs which include one loop corrections) using two loop RGE equations *including* generation mixing. The principal component of the mass eigenstate is indicated in brackets after the mass value. The eigenstates are quite pure.

metastability then it is the package that must be improved not necessarily our proposed parameters that need to be considered as discredited.

The mass spectra obey a normal s-hierarchy (third sgeneration heavier than degenerate first two generations) coupled with a mini split supersymmetry ($m_{\tilde{f}} \gg M_i$) with pure Bino LSP, Wino($\tilde{W}_{\pm}$) light charginos, and next to lightest neutralino($\tilde{W}_3$), and Higgsino heavy neutralinos and chargino. What is remarkable and interesting from the point of view of the Dark Matter Cosmology is that the quasi inert Bino LSP, which could serve as an ideal form of Cold Dark Matter, is generally accompanied by a light Right sfermion of the first or second generation (often a smuon). This is in sharp contrast to most Susy GUT spectra which predict the stau, stop and sbottom as the lightest sfermions because their masses are driven to lower values by the effect of the their large yukawas. Here however the additional presence of large negative Higgs mass terms drives the the third sgeneration to large masses (see Appendix **B**). Thus our model is marked out from other GUT models by a distinctive low energy spectrum that puts it in a different and novel universality class of models. The LHC at 14 TeV will provide about 1-2 TeV per colliding parton and so, as a rule of thumb, particles lighter than 2.5 TeV and with reasonably large couplings to SM particles may be detectable. Besides the light Bino which is very weakly coupled and the light Higgs $h^0$, the other MSSM gauginos $\hat{W}_a, \tilde{g}$, and from the sfermions some from among $\tilde{\bar{\mu}}, \tilde{\bar{e}}, \tilde{\bar{d}}, \tilde{\bar{s}}, \tilde{\bar{u}}, \tilde{\bar{c}}$ are lighter than 2 TeV and should be discoverable at LHC. In Table 20 we give the "detectable spectra" for each case.

The cosmology of the Bino LSP Dark matter would be determined by the co-annihilation through the sfermion channels (the pseudo-scalar Higgs is unavailable for the purpose being very heavy). The combination of a Bino LSP and very light sfermion is thus ideal for Bino-

LSP WIMP DM and it is interesting to note that this feature emerged naturally from completely independent considerations. Note that-as is often the case - the smuon is the lightest sfermion then such annihilation could lead to a excess of charged leptons relative to nucleons from DM decay. Such signals have indeed been reported recently by a number of experiments[60, 61, 62, 63]. We will return to detailed examination of the rates of the co-annihilation and charged particle production therefrom elsewhere.

In our fits typical gluino masses lie in the $200-600$ GeV range. It is interesting that the ratio of gaugino masses can diverge significantly from the $M_1 : M_2 : M_3 :: 1 : 2 : 7$ ratio dictated by the 1-loop RG invariance of $M_i/g_i^2$ if one begins from universal gaugino masses at $M_X$ : as is simple and plausible in a GUT context. However we find that the gluino can be quite light ( as in Fit I-2 where it is lighter than even the Wino). Further phenomenological analysis to revise the gluino mass bounds in the special soft Susy parameter region of high $\tan\beta$, and multi TeV $\mu, B, A_0$, which has emerged from this analysis as a distinct, novel possibly viable region of soft Susy parameter space, is required before the viability of such fits can be decided. This unusual feature like many others in this scenario, is also due to the effects of the large values of $A_0$ that we have been required to consider by the down and strange quark fitting requirements of the NMSGUT. The large values of $M^2_{H,\bar{H}}, |A_0|$ required by the fermion fitting lead to correspondingly large values of required $|\mu(M_Z)|, B(M_Z)$ (and therefore also of the additional pseudoscalar mass $M_A$ which emerges much to heavy to play any role in Dark Matter cosmology) through the EWRSB conditions that tie them together. The large(negative) values of $M^2_{H,\bar{H}}$ have a dramatic consequence : the one loop RG equations for the evolution of sfermion masses contain terms proportional to $M_H, M^2_{\bar{H}}$ times the yukawa couplings squared $Y_f^\dagger Y_f$. These terms dominate the RGEs for the third generation sfermions and drive their masses far above the masses of the first two s-generations independently of the value of $A_0$. These important features of the RGE are shown graphically in Appendix **B**.

Baryon decay via $d = 5$ operators is, as usual[25, 53], dominated by the chargino mediated channels. The heavy sfermions help with suppressing B-decay. The dominant channels are $Baryon \to Meson + neutrino$. We emphasize that the flavour violation required by $d = 5$ B violation is supplied entirely by the rundown values of the (off diagonal) Super-CKM values determined by the fitting of the fermion yukawas at $M_X$ by the SO(10) light fermion yukawa formulae[9, 12, 6, 7, 14]. Using the formulae given in [14] and adapting the formalism of[25, 53], the proton decay decay rates in the six dominant channels for the 11 Fits we present in this paper are given in Table 21. These lifetimes are enhanced by up to 8 orders of magnitude relative to those calculated for generic fits where[17] no attempt was made to suppress the coefficient of the Baryon violating $d = 5$ operators or to take account of threshold corrections to the yukawa couplings. If one tries to trace how this has been accomplished one sees that the minimum value of the masses of standard B violating triplets $[3, 1, \pm 2/3](t \oplus \bar{t})$ (which anyway dominate -being lighter- the novel $[3, 3, \pm 2/3](P \oplus \bar{P}), [3, 1, \pm 8/3](K \oplus \bar{K})$ triplets) is raised by some two orders of magnitude and in addition the SO(10) yukawa couplings are also reduced significantly relative to those that were necessary to reproduce the MSSM fermion yukawas in the theory without threshold corrections. A point that requires further investigation is the effect of wave function renormalizations on the Baryon decay operators. As matters stand we have not applied such corrections since the external lines are all light fermion lines and the corrections on these lines are typically smaller than the systematic errors in the B- violation calcula-

| Case | $\tau_p(M^+\nu)$ | $\Gamma(p\to\pi^+\nu)$ | $BR(p\to\pi^+\nu_{e,\mu,\tau})$ | $\Gamma(p\to K^+\nu)$ | $BR(p\to K^+\nu_{e,\mu,\tau})$ |
|---|---|---|---|---|---|
| $I-1$ | $2.4\times10^{36}$ | $6.2\times10^{-38}$ | $\{3.6\times10^{-7},0.082,0.918\}$ | $3.5\times10^{-37}$ | $\{2.8\times10^{-5},0.119,0.881\}$ |
| $I-2$ | $6.7\times10^{34}$ | $2.5\times10^{-36}$ | $\{3.8\times10^{-5},0.06,0.94\}$ | $1.3\times10^{-35}$ | $\{1.1\times10^{-4},0.111,0.889\}$ |
| $I-3$ | $5.7\times10^{36}$ | $2.4\times10^{-38}$ | $\{3.2\times10^{-7},0.100,0.900\}$ | $1.5\times10^{-37}$ | $\{2.3\times10^{-5},0.139,0.861\}$ |
| $I-4$ | $5.7\times10^{34}$ | $1.7\times10^{-36}$ | $\{7.4\times10^{-5},0.052,0.948\}$ | $1.6\times10^{-35}$ | $\{9.1\times10^{-5},0.046,0.954\}$ |
| $II-1$ | $1.5\times10^{36}$ | $9.7\times10^{-38}$ | $\{1.8\times10^{-6},0.114,0.886\}$ | $5.9\times10^{-37}$ | $\{3.6\times10^{-5},0.170,0.830\}$ |
| $II-3$ | $1.7\times10^{36}$ | $7.4\times10^{-38}$ | $\{1.9\times10^{-6},0.153,0.847\}$ | $5.1\times10^{-37}$ | $\{2.7\times10^{-5},0.205,0.795\}$ |
| $II-4$ | $6.2\times10^{33}$ | $1.6\times10^{-35}$ | $\{5.8\times10^{-5},0.071,0.929\}$ | $1.5\times10^{-34}$ | $\{8.0\times10^{-5},0.069,0.931\}$ |
| $III-1$ | $2.3\times10^{36}$ | $6.5\times10^{-38}$ | $\{5.7\times10^{-7},0.088,0.912\}$ | $3.7\times10^{-37}$ | $\{2.5\times10^{-5},0.128,0.872\}$ |
| $III-2$ | $5.0\times10^{34}$ | $3.3\times10^{-36}$ | $\{3.3\times10^{-5},0.050,0.950\}$ | $1.7\times10^{-35}$ | $\{9.2\times10^{-5},0.093,0.907\}$ |
| $III-3$ | $2.2\times10^{36}$ | $6.2\times10^{-38}$ | $\{5.9\times10^{-7},0.103,0.897\}$ | $4.0\times10^{-37}$ | $\{2.0\times10^{-5},0.141,0.859\}$ |
| $III-4$ | $6.2\times10^{33}$ | $1.6\times10^{-35}$ | $\{5.8\times10^{-5},0.071,0.929\}$ | $1.5\times10^{-34}$ | $\{8.0\times10^{-5},0.069,0.931\}$ |

Table 21: of $d=5$ operator mediated proton lifetimes $\tau_p$(yrs), decay rates $\Gamma(yr^{-1})$ and Branching ratios in the dominant Meson$^+$ + $\nu$ channels.

tion(see tables of type X-b). It is another matter that dimension six operators containing external light Higgs vevs may be enhanced by wave function renormalization. However, the additional dimensional suppression may well keep these operators subdominant.

We see that we have been able to suppress the B decay rates to lie comfortably within the current limits. Thus the search criteria may even be loosened without conflict with experiment. We note that our programs can already calculate the rates in other channels driven by Gluino, Neutralino, Higgsino etc exchange. However we defer a presentation of the results for the subdominant channels till the various corrections and improvements still needed (see below) needed have been implemented. Our aim was to show that the NMSGUT is quite compatible with the stability of the proton to the degree it has been tested, and even beyond. Firm predictions will ensue only once the susy spectrum is anchored in reality by a discovery of a supersymmetric particle.

The very heavy third sgeneration masses indicate that the rate $\Gamma(b\to s\gamma)$, is likely to be acceptable and uniform among the fits. The Susy contribution to muon (g-2) $\Delta a_\mu = \Delta(g-2)_\mu/2$ may vary considerably since the mass of the smuon in the loop within which the photon couples is quite variable and generally quite low compared to other sfermions. Finally change in the $\rho$ parameter $\Delta\rho$ could also in principle be appreciable due to the 6-8 light superparticle present in most cases. We plugged our susy spectra into the ( tree level) Spheno[56] routines to obtain the contributions shown in Table 22

The $b\to s\gamma$ branching ratio values are right in the centre of the region $(3-4\times10^{-4})\pm15\%$ determined by measurements at CLEO, BaBar and Belle[51, 48, 49, 50]. The current difference between experiment and theory for the muon magnetic moment anomaly is is $\Delta a_\mu = 255(63)(49)\times10^{-11}$[51]. The results in Table 22 are thus certainly in the right ball park and we may well begin to use the value of $\Delta a_\mu$ to discriminate between different models provided one is confident that all instabilities in the parameter determination process have been controlled by adequate attention to loop and threshold effects. At the moment however we simply note that there is no gross conflict. The predicted change in the $\rho$ parameter is so small as to be insignificant compared with the experimental uncertainties $\sim .001$[51].

| Case | $B.R(b \to s\gamma)$ | $\Delta a_\mu$ | $\Delta\rho$ |
|---|---|---|---|
| $I-1$ | $3.294 \times 10^{-4}$ | $5.796 \times 10^{-9}$ | $5.985 \times 10^{-6}$ |
| $I-2$ | $3.293 \times 10^{-4}$ | $5.471 \times 10^{-9}$ | $2.397 \times 10^{-5}$ |
| $I-3$ | $3.294 \times 10^{-4}$ | $2.300 \times 10^{-9}$ | $2.825 \times 10^{-6}$ |
| $I-4$ | $3.293 \times 10^{-4}$ | $7.238 \times 10^{-9}$ | $6.064 \times 10^{-7}$ |
| $II-1$ | $3.290 \times 10^{-4}$ | $1.360 \times 10^{-10}$ | $2.503 \times 10^{-6}$ |
| $II-3$ | $3.287 \times 10^{-4}$ | $1.035 \times 10^{-10}$ | $3.385 \times 10^{-6}$ |
| $II-4$ | $3.278 \times 10^{-4}$ | $1.043 \times 10^{-10}$ | $3.612 \times 10^{-6}$ |
| $III-1$ | $3.293 \times 10^{-4}$ | $8.058 \times 10^{-9}$ | $3.718 \times 10^{-6}$ |
| $III-2$ | $3.293 \times 10^{-4}$ | $6.824 \times 10^{-9}$ | $2.105 \times 10^{-5}$ |
| $III-3$ | $3.295 \times 10^{-4}$ | $8.689 \times 10^{-9}$ | $3.743 \times 10^{-6}$ |
| $III-4$ | $3.294 \times 10^{-4}$ | $7.452 \times 10^{-9}$ | $5.989 \times 10^{-7}$ |

Table 22: Table Low energy constraints from the limits on the branching ratio for $b \to s\gamma, \Delta a_\mu$ and $\Delta\rho$.

The unification scale tends to be raised above $M_X^0$ in the NMSGUT i.e. $\Delta_X > 0$ . This is especially true once we demand that $d = 5$ operators mediating proton decay be suppressed. In fact of the Fits we have exhibited here and in [14, 17] the values of $\Delta_X$ we encounter are $-0.29, 1.82$ for the solutions without GUT scale threshold corrections (to fermion Yukawas) while with threshold corrections one gets $\{1.16, 2.82, 1.28, 0.46\}$ for Cases I-1 to I-4, $\{1.15, 1.36, 0.39\}$ for Cases II-1,3,4. $\Delta_G$, and $\{1.22, 2.82, 1.21, 0.43\}$ for Cases III-1 to I-4. Thus we see that the unification scale-defined as the mass of the B-violating gauginos of type $X[3, 2, \pm\frac{5}{3}]$ is typically raised by one oorder of magintude or more. On the other hand the correction to the inverse value of the fine structure constant ($\Delta_G$) at the unification scale varies over a wide range from -20.0 to 8.1 so that the value of the unification coupling may as well be weak as not. However it remains true that above the new unification scale once we begin to use the SO(10) RGE beta functions the gauge coupling will still explode[57, 58] over an energy scale range of only about 5-10. Smaller $\alpha_G$ can only postpone this a little. An ideal scenario is then that the theory is still weakly coupled at the threshold corrected unification scale $M_X > 10^{17.5}$ GeV but that thereafter the Susy GUT becomes strongly coupled simultaneously with gravity. In that case the Planck scale may be identified as a physical cutoff for the Susy NMSGUT where it condenses as strongly coupled Supersymmetric gauge theory described by an appropriate sigma model. We envisage[58] the possibility that gravity arises dynamically as an induced effect of the quantum fluctuations of the Susy GUT calculated in a coordinate independent framework. This may be realized as a path integral over a background metric that begins to propagate only at low energies leading to the near canonical N=1 Supergravity perturbative NMSGUT as the effective theory below $M_{Planck}$ that we assume in our work.

Besides the high(GUT)and low (Susy/Electroweak) scale thresholds SO(10) theories are also typically subject to threshold corrections and RG flows associated with the couplings of the righthanded neutrinos present in the theory. In fact the NMSGUT scenario makes essential use of intermediate scale heavy neutrinos($10^8 - 10^{13}$ GeV). Such neutrinos are however not inert: particularly as far as their effects on RG evolution of the yukawa couplings of the light leptons above the mass threshold associated with the right handed neutrino masses are concerned. The techniques for inclusion of these RG flows and threshold effects are by now standard[45, 46] and in subsequent full analysis we will include also these effects.

The NMSGUT also provides corrections to the QCD coupling at $M_Z$ that are in the right range($-.017 < \Delta\alpha_3(M_Z) < -.004$ to lower it as required[18, 19].

As a direct consequence of the dominance of the Type I seesaw mechanism, due to the low value of the $\overline{\mathbf{126}}$ coupling, the right handed neutrinos emerge in just the range $10^8 - 10^{14}$ GeV required to implement leptogenesis[59]. The influence of these threshold on the RG evolution has not yet been factored in by us yet and may have important implications for the Yukawa unification. This is straightforward to implement and of high priority for the next round of improvement of the calculation.

The large values of the crucial parameters $|\mu|, |A_0|, M_A, |M^2_{H,\bar{H}}| \sim 10^2\, TeV$ (where $M_A$ is the mass of the pseudo scalar Higgs remnant) play a crucial role in structuring the low energy phenomenology. Due to the large value of $|\mu|$ the LSP is essentially a pure Bino and the lighter chargino is a Wino($\tilde{W}_\pm$) while the heavier one $\tilde{H}^\pm$ is a pure Higgsino, with mass set by the large $\mu$ parameter ( which also sets the scale for the two heavy neutralinos) while the next to lightest neutralino is also an essentially pure Wino($\tilde{W}_3$). Since the required threshold corrections depend on ratios of scalar masses to gaugino masses there is actually a preference for light gaugino masses to enhance the ratios with the heavy masses. Running counter to this is only the constraint that the experimental lower mass limit on charged gauginos and exotic charged and coloured scalars generally is around 100 GeV. Thus we imposed a floor of 110 GeV for all such exotics. It is this chargino limit and the linked behaviour of the Bino and Wino masses that prevents the Bino mass from running to very low values( in our example fits the lowest Bino mass is 35 GeV). Also due to the link between the gaugino masses have We have light gluinos below 500 GeV for the cases I-II where we allowed LSP/Binos in the range $5 - 150$ GeV.

The parameters $\mu, m_A$ at $M_Z$ are determined in terms of the run down Higgs mass parameters $M^2_{H,\bar{H}}$ by the Electroweak symmetry breaking conditions which we implemented at the one-loop level by including Higgs tadpoles calculated using a subroutine from [56] corresponding to the formulae given by [44]. $M_A$ sets the scale of the mass of the scalar Higgs apart from $h^0$ i.e $H^0, H^\pm, A$ all have masses close to $M_A$. The light scalar Higgs $h^0$ typically has a mass in the range of $110 - 125$ GeV. Once the values of $\mu$, $m_A$ ( or $B = m_A^2 Sin2\beta/2$ ) at $M_Z$ are known we run them back up to $M_X$ since they do not interfere with the running of rest of parameters due to the modular structure of the RGE. Thus we give the 7 parameter set $(m_{\frac{1}{2}}, \tilde{m}_0, A_0, M^2_{H,\bar{H}}, |\mu|, B)$ which together with the NMSGUT parameters completely specifies the theory at all scales and yields a distinctive scenario for the low energy supersymmetric accelerator phenomenology as well as for LSP Dark Matter cosmology.

The large but negative values of the Higgs mass-squared parameters found by our search programs have a dramatic consequence that sets the type of Susy spectra obtained in the NMSGUT in marked contradiction with the generally accepted and used patterns of Sfermion masses derived from GUT boundary conditions at high scales. As shown in Appendix **B** these large negative mass-squared parameters drive the third generation sfermions *to be much heavier than the first two generations* even though we assume a common mass squared for the sfermions in all three 16 plets at $M_X^0$. Thus third generation sfermion generally lie in the range 5-50 TeV and are effectively decoupled from electroweak scale physics. However the NMSGUT susy spectra are *not* of the split supersymmetry type since the sfermions of the first two generations typically populate the mass band between the light gauginos and the superheavy third sgeneration. Most of the sparticles emerge above the direct discovery limits but a couple of right chiral sfermions emerge( see Table 20) in the discoverable set; of which squarks are generally relatively heavy but right sleptons and specially the smuon can descend even to the Electroweak breaking scale. It remains to be seen if leptonic flavour violation constraints in will actually permit such light sleptons. However the point to emphasize at this stage is that searches based on the assumption of a light LSP inevitably lead to a characteristic Susy spectrum :

$$M_{\tilde{B}} < \{M_{\tilde{W}}, M_{\tilde{g}}, M_{\tilde{\bar{f}}_{1,2}}\} << M_{\tilde{F}_{1,2}} << M_{\tilde{\bar{f}}_3,\tilde{F}_3} << |\mu|, |A_0|, M_A, |M^2_{H,\bar{H}}| \tag{14}$$

It may be necessary to further constrain the search (and include 1-loop corrections to sfermion masses) to obtain consistency with flavour violation processes involving the first two generations (as can be seen from the Table 22 the generally stringent limits due to $b \to s\gamma$ (specially at large $\tan\beta$ ) when the third sgeneration is lightest are ineffective in our case). Thus whereas it may be premature to point to any one sfermion mass as a *prediction* of the NMSGUT that can be verified at the LHC it seems safe to assert that that the NMSGUT does predict a light Bino and Chargino and more distinctively that the first sfermions discovered, at LHC or later, *must* belong to the first two generations with some preference being exhibited for the smuon and or selectron (provided those cases are consistent with flavour violation constraints).

# 7 Conclusions and Outlook

This paper is the third of a series [14, 17] developing the NMSO(10)GUT as a possibly viable and complete theory of particle physics. The emphasis is to develop the theory on the same lines and as explicitly as the the Standard Model. We have shown that the theory is sufficiently simple as to allow explicit calculation of the spontaneous symmetry breaking, mass spectra and eigenstates and allows a computation of the RG flow in terms of the fundamental GUT parameters to the point where one can attempt to actually fit the low energy data, i.e the SM parameters together with the neutrino mixing data, in its entirety. In carrying out this project the model meets a major obstacle in its inability to fit the unmodified down and strange quark yukawas in the MSSM (renormalized up to $M_X$), which it overcomes by staking its viability on the operation of large $\tan\beta$ driven threshold corrections -providentially known to be operative and important in this context- which lower these yukawas from their SM values by a factor of about 5. As a result the pattern- although unfortunately not the scale- of the Susy breaking parameters tends to become fixed by the need to preserve, or indeed somewhat raise, the b quark yukawa while lowering the $d, s$ quark yukawas when crossing the SM-MSSM threshold(s) . Thus the major unknown for the model, indeed for the whole field of Susy, remains the mass of at least one of the light susy particles. Due to the tight interrelationship of the susy spectra which have been generated from just 5 soft susy parameters at $M_X$ and subjected to multiple stringent demands such as EW symmetry breaking, yukawa modification, Baryon decay suppression, LSP consistency and so on, it seems very plausible that input of any one of the susy particle masses would generate a prediction of all the susy masses in the context of a fit that was pinned to yield that particular mass value along with satisfying the various requirements we have mentioned. In the absence of Susy discovery data and the difficulty of extending the mass ranges that can actually be claimed to have been excluded without egregious assumptions, we can still call upon the the model to stand by it's claim to be a viable theory of Dark matter and yield its stable Bino LSP in the $5-150$ GeV mass range preferred by Cold Dark matter WIMP scenarios. This requirement is sufficient to pin the prediction for the Susy spectrum to a specific and completely novel and distinctive *type*, if not yet to specific masses for specific particles. The model thus predicts light gauginos and discoverable first or second generation right chiral sfermions below 2 TeV and invisible third generation sfermions. Thus we arrive at the attractively dangerous conclusion that : *If as all other GUT scenarios derive, the third sgeneration emerges lightest the NMSGUT may be taken to be falsified.*

This remarkable model has thus added yet more feathers to the already long list of its attractions. Besides providing a natural and minimal context for the supersymmetric seesaw mechanism and the implementation of R-parity as a part of the gauge structure of unification, the theory successfully accounts for the entire available fermion mass data in terms of its own parameters in way consistent with all known phenomenology. It also yields insight into a necessary structure of it's susy breaking parameters and yields a viable and completely natural candidate for the LSP combined with a surprising and novel candidate for the scalar NLSP which can lead to an effective DM scenario. Furthermore the theory naturally pushes the unification scale towards the Planck scale and allows suppression of the dangerous $d = 5$ B violation operators. The conflation of the Planck scale and the unification scale goes a long way towards alleviating a perennial problem of renormalizable SO(10) models [57, 58] namely divergent couplings in the ultraviolet. The unification scale or rather the Landau pole above it becomes a physical cutoff beyond which the theory enters a strongly coupled phase together with gravity.

Till a susy breaking soft mass is pinned by experiment the development of the NMSGUT will continue by facing up to technical challenges that we have postponed in the first phase of the definition of the model as reported in a series of papers([9, 12, 6, 13, 7, 19, 41, 42] and the papers of the current triplet: [14, 17] and this paper). We conclude by itemizing the important issues which may materially alter the numbers obtained by our fitting program so far.

- We have, following[44], chosen $M_Z$ as the scale at which we match the SM to the MSSM. On the one hand this is well motivated since, as we have seen, the weak gauginos, i.e the Bino and Wino tend to emerge as light as we allow them : which is as light as experiment permits i.e $\sim M_Z$ for the Chargino but much lower for the Bino( the connection between the two masses and the limit $M_{\tilde{W}^\pm}$ however does not let the Bino mass descend below about 25 GeV). On the other hand we cannot deny that the model has its own little hierarchy problem with the $\mu, A_0, M_H^2$ parameters all lying in the range of 10-100 TeV. Clearly the threshold effects due to the large differences of the Susy particle spectra from $M_Z$ will cause appreciable threshold corrections to the naive RG running which takes a single undifferentiated Susy breaking scale and identifies it arbitrarily with $M_Z$. Some development of the techniques for incorporating multiple thresholds between the SM and MSSM has already taken place[73] and should prove useful.

- We have calculated only tree tree masses for the susy particles in the theory (the Electro weak symmetry breaking and thus the Higgs masses were however calculated using 1-loop effective potential[44, 56]. This is not a very good approximation in the MSSM with small $A_0$ and it may be worse in scenarios with large $A_0$ parameters. The large $A_0$ parameters lead to large trilinear couplings ($A = A_0 Y_f$) only for the third sgeneration since the Yukawas $Y_f$ for the other generations are so small and may significantly modify the third generation sfermion masses. The complete formulae for calculating these modifications-at least in the flavour diagonal case - have long been known[44]. They will be incorporated in the next version of our search codes.

- We have not yet incorporated the three thresholds associated with the heavy right handed neutrino masses in the theory. Since one progressively introduces the neutrino Dirac couplings as one passes these thresholds going higher in energy, significant effects on the yukawa unification may be anticipated. These thresholds will also be significant when calculating the amount of lepton flavour violation introduced when integrating down from the high scale (driven by the non diagonality of the SO(10) yukawas required to account for the observed quark flavour mixing). It is for this reason, and because the mass insertion formalism is ill adapted to securely evaluate novel scenarios, that we have not generated tables of Lepton flavour violating mass insertions for comparison with the existing analyses on lepton flavour constraints[76]. We note however that we have calculated some of the common Susy sensitive quantities and found them to be compatible with existing limits. Nevertheless the incorporation of constraints on our searches based upon electric dipole moments and the strength of flavour violation operators involving the sfermions of the first two sgenerations are a priority issue for our program.

- It will be clear to the reader familiar with the nuances of the Susy Flavour problem that in terms of a Bottom-up approach our results suggest that a radical extension of the consistent and viable susy parameter space at large $\tan\beta, A_0, |mu|, |M^2_{,\bar{H}}|$ and with $M_{\tilde{f}_3} >> M_{\tilde{f}_3}$ may be possible and the characteristics of such an extension as derived from the NMSO(10)GUT approach could be diametrically opposed to those from all previous GUTs. However that reader will also realize that the inversion of the standard inverted hierarchy of sfermion masses to a normal hierarchy may have drastic implications for the consistency of the theory with flavour violation constraints and EDM constraints for the first two generations (the heavy sthird is likely to pass such constraints -as already seen in the case of the $b \to s\gamma$ constraint : which is commonly a stringent one when the third sgeneration is light but is here totally insensitive see Table 22). Prominent among these well known constraints are those on the electric dipole moments of the electron, muon, neutron etc, the severely constrained Branching rations of the $\Delta F = 1$ decays of Mesons such as $B_d \to \mu\mu$, and the limits on the supersymmetric contributions to $\Delta F = 2$ quantities like $\epsilon_K, \Delta K$ etc. These contributions can now be calculated using codes[78] that do not make any simplifying approximations as done in the mass insertion method[79] which allow only order of magnitude estimates of individual terms but cannot be used to evaluate total contributions including the effect of cancellations, in unconventional scenarios like ours. It may well be that the parameter sets we have presented will fail these tests when they put to them. However we emphasize that, from the point of view of the this series of papers which seeks to pin down the viable points of the 45 dimensional parameter space SO(10) NMSGUT, such a challenge would be no different than the one that was posed by the onus to show compatibility with B violation rates: which the theory overcame by novel use of expedients available to it in a way that pointed out fresh approaches to hoary questions. In the same way it may be that when the requirement of the consistency of the *total* value of such parameters predicted by the NMSGUT is taken into consideration then again new viable parameter sets may emerge. In view of the length of the present paper as well as the considerable further effort required to answer definitively to these important challenges the flavour violation constraints will be taken up in the sequels. We urge the tolerant reader not to prejudge this vital issue but join us in reflecting on the behaviour of the novel type of Susy parameter sets suggested by us.

- In this calculation we found parameter sets leading to quasi-stable Baryons by a shot-gun carrying brute search. However the characterization of the possible cancellations and the least constraining versions compatible with experimental limits remains to be done. The search for fits with suppressed Baryon Number violation was carried out by simply limiting the size of the maximal element of the LLLL and RRRR operator coefficients with no heed paid to the detailed effects of that coefficient on the decay rate in specific channels. A more sophisticated (but hugely more computer intensive) way of doing this would be to calculate the Baryon violation rates in each channel at every iteration and limit the total lifetime. In principle one could hope to implement this given enough (super)computing power.

- Due to the large amount of running time required to find an acceptable solution, we have only scratched the surface of the enormous parameter space and can by no means pronounce on the general structure of the solution space. It will take long runs on a super computing cluster to develop a statistical picture of where the solutions tend to lie. We are now preparing for both the improvements mentioned above and the harnessing of a cluster for the task.

- We have run down only the diagonalized yukawa couplings from $M_X^0$ to $M_Z$ along with the susy breaking parameters which are optimized to fit the eigenvalues of the SM yukawa couplings to the run down diagonal NMSGUT matter yukawas. A complete treatment would run down the full set of coupling matrices obtained from the GUT and apply the large $\tan\beta$ driven Susy threshold corrections to the off diagonal couplings before matching the two sets : or at least the "physical" parameters (eigenvalues, mixing angles and phases) coded in the two pairs. The formalism for applying off diagonal corrections is still somewhat murky so some theoretical progress towards setting out clear algorithms for including all off diagonal 1-loop effects and renormalizations is called for.

- We argued that the CCB and UFB minimae(that certainly exist at the large values of $|\mu|, |A_0|$ central to the needs of the NMSGUT) need not destabilize the metastable standard vacua we have found and cited previous investigations of this issue[54] which are specially encouraging in this regard. The fact remains, however, that these decay rates must actually be estimated for each set of parameters found. On the other hand once the requisite subroutines for calculating the vacuum tunneling rate are in hand one can add them to the search routines to filter the parameter sets.

- Threshold corrections play a central role in our calculation and the large wavefunction dressing of the Higgs doublet lines that we find at one loop demand an investigation of the two loop effects to determine whether they are yet larger still. If so our model, and by implication realistic GUTs generally, may survive only after the Higgs wavefunction dressing has somehow been re-summed to all orders. However we note that this growth of wave function dressing leads to reduction in the actual size of the SO(10) yukawas actually needed to fit the low energy fermion data by a factor of 10-100. Thus the contributions to the fermion lines are effectively degenerate and it is only the large number of heavy fields running in dressing of the light Higgs boson lines that leads to such a large wave function renormalization of the Higgs fields. It is possible that in case of further growth at the multiloop level the theory is being driven to a quantum fixed point dominated by the gauge coupling and therefore re-summable using the the exact beta functions available for supersymmetric gauge theories[77]. In any event we consider that our calculations show that GUTs aiming to be realistic have been "up-ended": in the sense that any Grand unified model with pretensions to realism must define itself explicitly enough to permit calculation of threshold corrections using calculated superheavy field spectra or else risk being under suspicion of being an qualitative scenario that may be destroyed as soon as quantum effects are included.

- As Karl Popper emphasized so insightfully, the virtue of a comprehensive scientific theory that accounts for all known data is that it accepts the challenge and charm of living dangerously and in constant confrontation with experimental data that may falsify it. It must face every new measurement that it claims to be able to account for with its fate hanging in the balance. This indeed is what gives properly scientific models their peculiar power and utility: that speculations living at a safe distance from the cutting edge of Occam's razor rarely possess. The (N)MSO(10)GUT has braved multiple iterations of challenges over the three decades since it was proposed[1, 2] and, so far, has emerged stronger from every challenge, defining the possible in its realm ever more clearly and distinctly. If it dies it will not have lived in vain.

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## Appendix A: Gut Scale threshold corrections to matter yukawas

We give below our results for the threshold corrections to the Yukawa couplings of the matter fields due to heavy fields running in a self energy loop on a line leading into the Yukawa vertex(here we assume the Higgs line is predominantly the **10**-plet derived doublet).

Let us denote by $U^A(V^A)$ the matrices that diagonalize the mass terms for fields of the alphabetized label type A :

$$\overline{\Phi}^T M\Phi = \overline{\Phi'}^T M_{Diag}\Phi' \quad \Leftarrow \quad \overline{\Phi} = U^{\Phi}\overline{\Phi'} \quad ; \qquad \Phi = V^{\Phi}\Phi' \qquad (\mathbf{A1})$$

generically by $W^A$ and the corresponding index ranging over the multiplicity of that field type also by the corresponding lower case roman letter (a). For example for the 6-fold set $[\bar{3},2,-\frac{1}{3}](\bar{E}_1,\bar{E}_2,\bar{E}_3,\bar{E}_4,\bar{E}_5,\bar{E}_6)\oplus[3,2,\frac{1}{3}](E_1,E_2,E_3,E_4,E_5,E_6)$ the generic rotation matrix is denoted $W^E$( i.e $U^E or V^E$ ) carrying indices $e,e'=1,2,3,4,5,6$. Then it is useful to define

$$\begin{aligned}
\mathcal{F}_1(W^A,a) &= \sum_{a'=1}^{a'=dim(W^A)} |W^A_{a,a'}|^2 F_{11}(m_{a'},Q)\\
\mathcal{F}_1'(W^H,h) &= \sum_{h'=2}^{h'=6} |W^A_{h,h'}|^2 F_{11}(m_{h'},Q)\\
\mathcal{F}_1^u(W^A,a,m^{(u)}) &= \sum_{a'=1}^{a'=dim(W^A)} |W^A_{a,a'}|^2 F_{12}(m^{(u)},m_{a'},Q)\\
\mathcal{F}_1^{u'}(W^H,h,m^{(u)}) &= \sum_{h'=2}^{h'=dim(W^H)} |W^H_{h,h'}|^2 F_{12}(m^{(u)},m_{h'},Q) + |W^H_{h,1}|^2 F_{11}(m^{(u)},Q)\\
\mathcal{F}_2(W^A,W^B,a,b) &= \sum_{b'=1}^{b'=dim(W^B)} \sum_{a'=1}^{a'=dim(W^A)} |W^A_{a,a'}|^2 |W^B_{b,b'}|^2 F_{12}(m_{a'},m_{b'},Q)\\
\mathcal{F}_2'(W^A,W^H,a,h) &= \sum_{h'=2}^{h'=6} \sum_{a'=1}^{a'=dim(W^A)} |W^A_{a,a'}|^2 |W^H_{h,h'}|^2 F_{12}(m_{a'},m_{h'},Q)\\
&+ \sum_{a'=1}^{a'=dim(W^A)} |W^A_{a,a'}|^2 |W^H_{h,1}|^2 F_{11}(m_{a'},Q)\\
\mathcal{C}_1(W^A,a,a') &= \sum_{a''=1}^{a''=dim(W^A)} (W^A_{a,a''})^* W^A_{a',a''} F_{11}(m_{a''},Q) \qquad (\mathbf{A2})\\
\mathcal{C}_1'(W^H,h,h') &= \sum_{h''=2}^{a''=dim(W^H)} (W^H_{h,h''})^* W^A_{h',h''} F_{11}(m_{h''},Q)
\end{aligned}$$

$$\mathcal{C}_2(W^A,W^B,a,b,b') = \sum_{b''=1}^{b''=dim(W^B)}\sum_{a'=1}^{a'=dim(W^A)} |W^A_{a,a'}|^2(W^B_{b,b''})^* W^B_{b',b''}F_{12}(m_a,m_{b''},Q)$$

$$\mathcal{C}_2'(W^H,W^B,a,h,h') = \sum_{h''=2}^{h''=6}\sum_{a'=1}^{a'=dim(W^A)} |W^A_{a,a'}|^2(W^H_{h,h''})^* W^H_{h',h''}F_{12}(m_a,m_{h''},Q) + \sum_{a'=1}^{a'=dim(W^A)} |W^A_{a,a'}|^2(W^H_{h,1})^* W^H_{h',1}F_{11}(m_a,Q)$$

The primes on the function names and summations instruct an omission of any light fields (in practice the light Higgs $[1,2,\pm1]$ doublets only) from the sum over the heavy fields of the given type. If the field is one of the unmixed types (i.e $u$ = A, B, I, M, N, O, S, T, U, V, W, Y, Z) then the function carries a superscript (u) thus e.g $\mathcal{F}_1^{(u)}(V_F,1,M_V)$ arises from a coupling between $F_1[1,1,2]$ and $V[1,2,-3]$. Such functions arise in the dressing of the Higgs lines. The calculation is quite tedious but we applied various consistency checks to ensure that we had included contributions from all members of multiplets. Naturally we await the contributions of those patient and interested enough to check our results.

$$\begin{aligned}
(32\pi^2)\Delta_{\bar u} &= 2\bar h^*\bar h\mathcal{F}_1(U_T,1) - 4\bar g^*\bar g\mathcal{F}_1(U_T,7) - 2i\sqrt{2}\bar h^*\bar g\mathcal{C}_1(U_T,1,7) - 2i\sqrt{2}\bar g^*\bar h\mathcal{C}_1(U_T,7,1)\\
&+ \bar h^*\bar h\mathcal{F}_1(V_T,1) - 2\bar g^*\bar g\mathcal{F}_1(V_T,7) - 2\bar g^*\bar g\mathcal{F}_1(V_T,6)\\
&- i(\sqrt{2}\bar h^*\bar g\mathcal{C}_1(V_T,1,7)) - i(\sqrt{2}\bar g^*\bar h\mathcal{C}_1(V_T,7,1)) - \sqrt{2}\bar h^*\bar g\mathcal{C}_1(V_T,1,6)\\
&+ \sqrt{2}\bar g^*\bar h\mathcal{C}_1(V_T,6,1) - 2i(\bar g^*\bar g\mathcal{C}_1(V_T,6,7)) + 2i(\bar g^*\bar g\mathcal{C}_1(V_T,7,6))\\
&+ 2(\bar h^*\bar h\mathcal{F}_1'(V_H,1) - (1/3)\bar g^*\bar g\mathcal{F}_1'(V_H,6) + (\frac{i}{\sqrt{3}})\bar h^*\bar g\mathcal{C}_1'(V_H,1,6))\\
&+ ((\frac{i}{\sqrt{3}})\bar g^*\bar h\mathcal{C}_1'(V_H,6,1)) - \bar g^*\bar g\mathcal{F}_1'(V_H,5) + \bar h^*\bar g\mathcal{C}_1'(V_H,1,5)\\
&- \bar g^*\bar h\mathcal{C}_1'(V_H,5,1) - ((\frac{i}{\sqrt{3}})\bar g^*\bar g\mathcal{C}_1'(V_H,5,6)) + ((\frac{i}{\sqrt{3}})\bar g^*\bar g\mathcal{C}_1'(V_H,6,5)))\\
&- \bar g^*\bar g((64/3)\mathcal{F}_1(V_C,3) + 8\mathcal{F}_1(V_D,3)\\
&+ 8\mathcal{F}_1(U_K,2) + 4\mathcal{F}_1(V_J,5) + 8\mathcal{F}_1(V_L,2))) - (2g^2)(25\mathcal{F}_1(V_G,6)\\
&+ 0.5\mathcal{F}_1(V_J,4) + 0.5\mathcal{F}_1(V_F,3) + 2\mathcal{F}_1(V_X,3) + \mathcal{F}_1(V_E,5)) \qquad \textbf{(A3)}
\end{aligned}$$

$$\begin{aligned}
(32\pi^2)\Delta_{\bar d} &= (2\bar h^*\bar h\mathcal{F}_1(U_T,1) - 4\bar g^*\bar g\mathcal{F}_1(U_T,7) + 2i(\sqrt{2}\bar h^*\bar g\mathcal{C}_1(U_T,1,7))\\
&+ 2\sqrt{2}\bar g^*\bar h\mathcal{C}_1(U_T,7,1)) + \bar h^*\bar h\mathcal{F}_1(V_T,1) - 2\bar g^*\bar g\mathcal{F}_1(V_T,7)\\
&- 2\bar g^*\bar g\mathcal{F}_1(V_T,6) + i(\sqrt{2}\bar h^*\bar g\mathcal{C}_1(V_T,1,7)) + i(\sqrt{2}\bar g^*\bar h\mathcal{C}_1(V_T,7,1))\\
&- \sqrt{2}\bar h^*\bar g\mathcal{C}_1(V_T,1,6) + \sqrt{2}\bar g^*\bar h\mathcal{C}_1(V_T,6,1) + 2i(\bar g^*\bar g\mathcal{C}_1(V_T,6,7))\\
&- 2i(\bar g^*\bar g\mathcal{C}_1(V_T,7,6)) + 2(\bar h^*\bar h\mathcal{F}_1'(U_H,1) - (1/3)\bar g^*\bar g\mathcal{F}_1'(U_H,6)\\
&- \bar g^*\bar g\mathcal{F}_1'(U_H,5) + i((\frac{1}{\sqrt{3}})\bar h^*\bar g\mathcal{C}_1'(U_H,1,6)) + i((\frac{1}{\sqrt{3}})\bar g^*\bar h\mathcal{C}_1'(U_H,6,1))\\
&+ \bar h^*\bar g\mathcal{C}_1'(U_H,1,5) - \bar g^*\bar h\mathcal{C}_1'(U_H,5,1) - i((\frac{1}{\sqrt{3}})\bar g^*\bar g\mathcal{C}_1'(U_H,5,6))
\end{aligned}$$

$$
\begin{aligned}
&+ \quad i((\frac{1}{\sqrt{3}})\bar{g}^*\bar{g}\mathcal{C}_1'(U_H,6,5))) - \bar{g}^*\bar{g}((64/3)\mathcal{F}_1(U_C,3) + 8\mathcal{F}_1(V_E,6) + 4\mathcal{F}_1(V_K,2) \\
&+ \quad 8\mathcal{F}_1(U_J,5) + 8\mathcal{F}_1(V_L,2)) - (2g^2)(0.225\mathcal{F}_1(V_G,6) \\
&+ \quad 0.5\mathcal{F}_1(V_J,4) + 0.5\mathcal{F}_1(V_F,3) + \mathcal{F}_1(V_X,3) + 2\mathcal{F}_1(V_E,5))) \qquad (\mathbf{A4})
\end{aligned}
$$

$$
\begin{aligned}
(32\pi^2)\Delta_{\bar{\nu}} \;=\; & (2\bar{h}^*\bar{h}\mathcal{F}_1'(V_H,1) - 2\bar{g}^*\bar{g}\mathcal{F}_1'(V_H,5) - 6\bar{g}^*\bar{g}\mathcal{F}_1'(V_H,6) \\
& +2\bar{h}^*\bar{g}\mathcal{C}_1'(V_H,1,5) - i2\sqrt{3}\bar{h}^*\bar{g}\mathcal{C}_1'(V_H,1,6) + i2\sqrt{3}\bar{g}^*\bar{g}\mathcal{C}_1'(V_H,5,6) \\
& +(2\bar{h}^*\bar{g}\mathcal{C}_1'(V_H,1,5) - i2\sqrt{3}\bar{h}^*\bar{g}\mathcal{C}_1'(V_H,1,6) + i2\sqrt{3}\bar{g}^*\bar{g}\mathcal{C}_1'(V_H,5,6))^\dagger \\
& +3\bar{h}^*\bar{h}\mathcal{F}_1(V_T,1) - 6\bar{g}^*\bar{g}\mathcal{F}_1(V_T,7) - 6\bar{g}^*\bar{g}\mathcal{F}_1(V_T,6) \\
& +3\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(V_T,1,6) - i3(\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(V_T,1,7)) + 6i(\bar{g}^*\bar{g}\mathcal{C}_1(V_T,6,7)) \\
& +(3\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(V_T,1,6) - i3(\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(V_T,1,7)) + 6i(\bar{g}^*\bar{g}\mathcal{C}_1(V_T,6,7)))^\dagger \\
& -\bar{g}^*\bar{g}(4\mathcal{F}_1(U_F,4) + 24\mathcal{F}_1(U_E,6) + 12\mathcal{F}_1(V_J,5)) \\
& -(2g^2)(0.625\mathcal{F}_1(V_G,6) + 1.5\mathcal{F}_1(V_J,4) + 0.5\mathcal{F}_1(V_F,3) + 3\mathcal{F}_1(V_E,5))) \qquad (\mathbf{A5})
\end{aligned}
$$

$$
\begin{aligned}
(32\pi^2)\Delta_{\bar{e}} \;=\; & (2\bar{h}^*\bar{h}\mathcal{F}_1'(U_H,1) - 2\bar{g}^*\bar{g}\mathcal{F}_1'(U_H,5) - 6\bar{g}^*\bar{g}\mathcal{F}_1'(U_H,6) \\
& +2\bar{h}^*\bar{g}\mathcal{C}_1'(U_H,1,5) - i2\sqrt{3}\bar{h}^*\bar{g}\mathcal{C}_1'(U_H,1,6) + i2\sqrt{3}\bar{g}^*\bar{g}\mathcal{C}_1'(U_H,5,6) \\
& +(2\bar{h}^*\bar{g}\mathcal{C}_1'(U_H,1,5) - i2\sqrt{3}\bar{h}^*\bar{g}\mathcal{C}_1'(U_H,1,6) + i2\sqrt{3}\bar{g}^*\bar{g}\mathcal{C}_1'(U_H,5,6))^\dagger \\
& +3\bar{h}^*\bar{h}\mathcal{F}_1(V_T,1) - 6\bar{g}^*\bar{g}\mathcal{F}_1(V_T,7) - 6\bar{g}^*\bar{g}\mathcal{F}_1(V_T,6) \\
& +3\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(V_T,1,6) + i3(\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(V_T,1,7)) - 2i3(\bar{g}^*\bar{g}\mathcal{C}_1(V_T,6,7)) \\
& +(3\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(V_T,1,6) + i3(\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(V_T,1,7)) - 2i3(\bar{g}^*\bar{g}\mathcal{C}_1(V_T,6,7)))^\dagger \\
& -\bar{g}^*\bar{g}(4\mathcal{F}_1(U_F,4) + 24\mathcal{F}_1(U_D,3) + 12\mathcal{F}_1(V_K,2)) \\
& -(2g^2)(025\mathcal{F}_1(V_G,6) + 1.5\mathcal{F}_1(V_J,4) + 0.5\mathcal{F}_1(V_F,3) + 3\mathcal{F}_1(V_X,3))) \qquad (\mathbf{A6})
\end{aligned}
$$

$$
\begin{aligned}
(32\pi^2)\Delta_u \;=\; & (\bar{h}^*\bar{h}\mathcal{F}_1(U_T,1) - 2\bar{g}^*\bar{g}\mathcal{F}_1(U_T,6) - \sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(U_T,1,6) \\
& +\sqrt{2}\bar{g}^*\bar{h}\mathcal{C}_1(U_T,6,1) + 2\bar{h}^*\bar{h}\mathcal{F}_1(V_T,1) + \bar{h}^*\bar{h}\mathcal{F}_1'(U_H,1) \\
& -\bar{g}^*\bar{g}\mathcal{F}_1'(U_H,5) - (1/3)\bar{g}^*\bar{g}\mathcal{F}_1'(U_H,6) - \bar{h}^*\bar{g}\mathcal{C}_1'(U_H,1,5) \\
& -(i/\sqrt{3})\bar{h}^*\bar{g}\mathcal{C}_1'(U_H,1,6) - (i/\sqrt{3})\bar{g}^*\bar{g}\mathcal{C}_1'(U_H,5,6) + (-\bar{h}^*\bar{g}\mathcal{C}_1'(U_H,1,5) \\
& -(i/\sqrt{3})\bar{h}^*\bar{g}\mathcal{C}_1'(U_H,1,6) - (i/\sqrt{3})\bar{g}^*\bar{g}\mathcal{C}_1'(U_H,5,6))^\dagger + \bar{h}^*\bar{h}\mathcal{F}_1'(V_H,1) \\
& -\bar{g}^*\bar{g}\mathcal{F}_1'(V_H,5) - (1/3)\bar{g}^*\bar{g}\mathcal{F}_1'(V_H,6) - \bar{h}^*\bar{g}\mathcal{C}_1'(V_H,1,5) \\
& -(i/\sqrt{3})\bar{h}^*\bar{g}\mathcal{C}_1'(V_H,1,6) - (i/\sqrt{3})\bar{g}^*\bar{g}\mathcal{C}_1'(V_H,5,6) + (-\bar{h}^*\bar{g}\mathcal{C}_1'(V_H,1,5) \\
& -(i/\sqrt{3})\bar{h}^*\bar{g}\mathcal{C}_1'(V_H,1,6) - (i/\sqrt{3})\bar{g}^*\bar{g}\mathcal{C}_1'(V_H,5,6))^\dagger + \bar{g}^*\bar{g}(-(32/3)\mathcal{F}_1(U_C,3) \\
& -(32/3)\mathcal{F}_1(V_C,3) - 4\mathcal{F}_1(U_D,3) - 4\mathcal{F}_1(U_E,6) \\
& -4\mathcal{F}_1(V_P,2) - 8\mathcal{F}_1(V_P,2) - 6\mathcal{F}_1(U_P,2) \\
& -8\mathcal{F}_1(U_L,2)) - (2g^2)(25\mathcal{F}_1(V_G,6) + 0.5\mathcal{F}_1(V_J,4) \\
& +1.5\mathcal{F}_1(V_X,3) + 1.5\mathcal{F}_1(V_E,5))) = (32\pi^2)\Delta_d \qquad (\mathbf{A7})
\end{aligned}
$$

$$
\begin{aligned}
(32\pi^2)\Delta_e \;=\;& (3\bar{h}^*\bar{h}\mathcal{F}_1(U_T,1) - 6\bar{g}^*\bar{g}\mathcal{F}_1(U_T,6) + 3\sqrt{2}\bar{h}^*\bar{g}\mathcal{C}_1(U_T,1,6)\\
&-3\sqrt{2}\bar{g}.\bar{h}\mathcal{C}_1(U_T,6,1) + \bar{h}^*\bar{h}\mathcal{F}'_1(V_H,1) - \bar{g}^*\bar{g}\mathcal{F}'_1(V_H,5)\\
&-3\bar{g}^*\bar{g}\mathcal{F}'_1(V_H,6) + \bar{h}^*\bar{g}\mathcal{C}'_1(V_H,1,5) - (i\sqrt{3})\bar{h}^*\bar{g}\mathcal{C}'_1(V_H,1,6)\\
&+(i\sqrt{3})\bar{g}^*\bar{g}\mathcal{C}'_1(V_H,5,6) + (\bar{h}^*\bar{g}\mathcal{C}'_1(V_H,1,5) - (i\sqrt{3})\bar{h}^*\bar{g}\mathcal{C}'_1(V_H,1,6)\\
&+(i\sqrt{3})\bar{g}^*\bar{g}\mathcal{C}'_1(V_H,5,6))^\dagger + \bar{h}^*\bar{h}\mathcal{F}'_1(U_H,1) - \bar{g}^*\bar{g}\mathcal{F}'_1(U_H,5)\\
&-3\bar{g}^*\bar{g}\mathcal{F}'_1(U_H,6) - \bar{h}^*\bar{g}\mathcal{C}'_1(U_H,1,5) + (i\sqrt{3})\bar{h}^*\bar{g}\mathcal{C}'_1(U_H,1,6)\\
&+(i\sqrt{3})\bar{g}^*\bar{g}\mathcal{C}'_1(U_H,5,6) + (-\bar{h}^*\bar{g}\mathcal{C}'_1(U_H,1,5) + (i\sqrt{3})\bar{h}^*\bar{g}\mathcal{C}'_1(U_H,1,6)\\
&+(i\sqrt{3})\bar{g}^*\bar{g}\mathcal{C}'_1(U_H,5,6))^\dagger + \bar{g}^*\bar{g}(-4\mathcal{F}_1(V_F,4) - 12\mathcal{F}_1(V_E,6)\\
&-12\mathcal{F}_1(V_D,3) - 18\mathcal{F}_1(U_P,2)) - (2g^2)((9/40)\mathcal{F}_1(V_G,6)\\
&+1.5\mathcal{F}_1(V_J,4) + 1.5\mathcal{F}_1(U_X,3) + 1.5\mathcal{F}_1(U_E,5)) = (32\pi^2)\Delta_\nu \qquad (\mathbf{A8})
\end{aligned}
$$

$$
\begin{aligned}
\frac{(32\pi^2)\Delta_{H^0}}{|V^H_{11}|^2} \;=\;& \bar{\gamma}^2\mathcal{F}'_2(V_G,U_H,2,2) + \gamma^2\mathcal{F}'_2(V_G,U_H,2,3) + \bar{\gamma}\gamma\mathcal{C}'_2(V_G,U_H,2,2,3)\\
&+\bar{\gamma}\gamma(\mathcal{C}'_2(V_G,U_H,2,2,3))^8 + 8(\bar{\gamma}^2\mathcal{F}_2(V_R,U_C,1,1) + \gamma^2\mathcal{F}_2(V_R,U_C,1,2)\\
&+\bar{\gamma}\gamma\mathcal{C}_2(V_R,U_C,1,1,2) + \bar{\gamma}\gamma(\mathcal{C}_2(V_R,U_C,1,1,2))^*) + 3(\bar{\gamma}^2\mathcal{F}_2(V_J,U_D,2,2)\\
&+(\gamma)^2\mathcal{F}_2(V_J,U_D,2,1) + \bar{\gamma}\gamma\mathcal{C}_2(V_J,U_D,2,2,1) + \bar{\gamma}\gamma(\mathcal{C}_2(V_J,U_D,2,2,1))^*)\\
&+3((\bar{\gamma})^2\mathcal{F}_2(U_J,V_E,2,1) + (\gamma)^2\mathcal{F}_2(U_J,V_E,2,2) + \bar{\gamma}\gamma\mathcal{C}_2(U_J,V_E,2,1,2)\\
&+\bar{\gamma}\gamma(\mathcal{C}_2(U_J,V_E,2,1,2))^*) + 3(\bar{\gamma}^2\mathcal{F}_2(U_E,V_T,4,2) + \gamma^2\mathcal{F}_2(U_E,V_T,4,3)\\
&+\bar{\gamma}\gamma\mathcal{C}_2(U_E,V_T,4,3,2) + \bar{\gamma}\gamma(\mathcal{C}_2(U_E,V_T,4,3,2))^*) + 3(\bar{\gamma}^2\mathcal{F}_2(V_X,U_T,2,2)\\
&+\gamma^2\mathcal{F}_2(V_X,U_T,2,3) + \bar{\gamma}\gamma\mathcal{C}_2(V_X,U_T,2,3,2) + \bar{\gamma}\gamma(\mathcal{C}_2(V_X,U_T,2,3,2))^*)\\
&+6\gamma^2\mathcal{F}^u_1(V_L,1,M_Y) + 12\gamma^2\mathcal{F}_1(M_B,M_M) + 3\gamma^2\mathcal{F}_2(V_X,U_T,1,4)\\
&+6\gamma^2\mathcal{F}_2(V_E,U_J,3,1) + 18\bar{\gamma}^2F_1(M_Y,M_W) + 9\bar{\gamma}^2\mathcal{F}_2(V_X,U_P,1,1)\\
&+\gamma^2\mathcal{F}^u_1(V_F,1,M_V) + 3\gamma^2\mathcal{F}'^u_1(V_H,4,M_O) + 2\gamma^2\mathcal{F}'_2(V_G,U_H,4,4)\\
&+18\gamma^2F_1(M_B,M_W) + 9\gamma^2\mathcal{F}_2(V_P,U_E,1,3) + 6\bar{\gamma}^2\mathcal{F}^u_1(V_L,1,M_B)\\
&+3\bar{\gamma}^2\mathcal{F}_2(V_T,U_E,4,3) + 12\bar{\gamma}^2F_1(M_Y,M_N) + 3\bar{\gamma}^2F_1(M_V,M_O)\\
&+2\bar{\gamma}^2F_1(M_V,M_A) + \bar{\gamma}^2\mathcal{F}'_2(U_F,V_H,1,4) + 6\bar{\gamma}^2\mathcal{F}_2(V_K,U_X,1,1)\\
&+4\bar{\gamma}^2\mathcal{F}_2(V_R,U_C,2,1) + 4\gamma^2\mathcal{F}_2(V_R,U_C,2,2) + 0.5\bar{\gamma}^2\mathcal{F}'_2(V_G,U_H,3,2)\\
&+0.5\gamma^2\mathcal{F}'_2(V_G,U_H,3,3) + 1.5\gamma^2\mathcal{F}_2(V_J,U_D,3,1) + 1.5\bar{\gamma}^2\mathcal{F}_2(V_J,U_D,3,2)\\
&+1.5\gamma^2\mathcal{F}_2(V_J,U_E,3,2) + 1.5\bar{\gamma}^2\mathcal{F}_2(V_J,U_E,3,1) + 8\gamma^2\mathcal{F}^u_1(V_C,1,M_Z)\\
&+8\bar{\gamma}^2\mathcal{F}^u_1(V_C,2,M_Z) + \gamma^2\mathcal{F}'_2(U_F,V_H,2,3) + \bar{\gamma}^2\mathcal{F}'_2(U_F,V_H,2,2)\\
&+3\gamma^2\mathcal{F}_2(V_T,U_E,5,1) + 3\bar{\gamma}^2\mathcal{F}_2(V_T,U_E,5,2) + 3\bar{\gamma}^2\mathcal{F}^u_1(V_D,1,M_I)\\
&+3\gamma^2\mathcal{F}^u_1(V_D,2,M_I) + 12\gamma^2\mathcal{F}^u_1(U_C,2,M_Q) + 12\bar{\gamma}^2\mathcal{F}^u_1(U_C,1,M_Q)\\
&+1.5\gamma^2\mathcal{F}'^u_1(U_H,3,M_S) + 1.5\bar{\gamma}^2\mathcal{F}'^u_1(U_H,2,M_S) + 4.5\bar{\gamma}^2\mathcal{F}^u_1(U_D,1,M_U)\\
&+4.5\gamma^2\mathcal{F}^u_1(U_D,2,M_U) + 4.5\bar{\gamma}^2\mathcal{F}^u_1(V_E,1,M_U) + 4.5\gamma^2\mathcal{F}^u_1(V_E,2,M_U)\\
&+|k|^2(\mathcal{F}'_2(V_G,U_H,1,5) + 6\mathcal{F}^u_1(V_L,2,M_B) + 3\mathcal{F}_2(V_T,U_E,6,3)\\
&+6\mathcal{F}^u_1(U_L,2,M_Y) + 3\mathcal{F}_2(V_X,U_T,1,6) + \mathcal{F}'_2(U_F,V_H,4,4)\\
&+\mathcal{F}^u_1(V_F,4,M_V) + 4\mathcal{F}_2(V_R,U_C,2,3) + 8\mathcal{F}^u_1(V_C,3,M_Z)
\end{aligned}
$$

$$
\begin{aligned}
&+0.5\mathcal{F}_2'(V_G,U_H,2,6)+\mathcal{F}_2'(U_F,V_H,2,6)+1.5\mathcal{F}_2(V_E,U_J,6,3)\\
&+3\mathcal{F}_1^u(V_D,3,M_I)+1.5\mathcal{F}_2(V_J,U_D,3,3)+3\mathcal{F}_2(V_T,U_E,5,6)\\
&+1.5\mathcal{F'}_1^u(V_H,6,M_S)+12\mathcal{F}_1^u(V_C,3,M_Q)+4.5\mathcal{F}_1^u(V_D,3,M_U)\\
&+4.5\mathcal{F}_1^u(U_E,6,M_U)+1.5\mathcal{F}_2(V_T,U_E,7,4)+3\mathcal{F}_2(V_K,U_X,2,2)\\
&+1.5\mathcal{F}_2(V_X,U_T,2,7)+3\mathcal{F}_2(V_E,U_J,4,5)+4.5\mathcal{F}_2(V_P,U_E,2,4)\\
&+4.5\mathcal{F}_2(V_X,U_P,2,2))-(2g^2)(+1.5\mathcal{F}_2(V_T,U_X,1,3)\\
&+1.5\mathcal{F}_2(V_E,U_T,5,1)+0.1\mathcal{F}_2'(V_G,V_H,6,1)+0.5\mathcal{F}_2'(V_F,U_H,3,1))) \qquad (\mathbf{A9})
\end{aligned}
$$

$$
\begin{aligned}
\frac{(32\pi^2)\Delta_{\bar{H}^0}}{|U_{11}^H|^2} = {} & (8(\bar{\gamma}^2\mathcal{F}_2(U_R,V_C,1,2)+(\gamma)^2\mathcal{F}_2(U_R,V_C,1,1)+\bar{\gamma}\gamma\mathcal{C}_2(U_R,V_C,1,2,1)\\
&+\bar{\gamma}\gamma(\mathcal{C}_2(U_R,V_C,1,2,1))^*)+\bar{\gamma}^2\mathcal{F}_2'(U_G,V_H,2,2)+(\gamma)^2\mathcal{F}_2'(U_G,V_H,2,3)\\
&+\bar{\gamma}\gamma\mathcal{C}_2'(U_G,V_H,2,2,3)+\bar{\gamma}\gamma(\mathcal{C}_2'(U_G,V_H,2,2,3))^*+3(\bar{\gamma}^2\mathcal{F}_2(U_J,V_D,2,1)\\
&+(\gamma)^2\mathcal{F}_2(U_J,V_D,2,2)+\bar{\gamma}\gamma\mathcal{C}_2(U_J,V_D,2,1,2)+\bar{\gamma}\gamma(\mathcal{C}_2(U_J,V_D,2,1,2))^*)\\
&+3(\bar{\gamma}^2\mathcal{F}_2(V_J,U_E,2,2)+(\gamma)^2\mathcal{F}_2(V_J,U_E,2,1)+\bar{\gamma}\gamma\mathcal{C}_2(V_J,U_E,2,2,1)\\
&+\bar{\gamma}\gamma(\mathcal{C}_2(V_J,U_E,2,2,1))^*)+3(\bar{\gamma}^2\mathcal{F}_2(U_X,V_T,2,2)\\
&+(\gamma)^2\mathcal{F}_2(U_X,V_T,2,3)+\bar{\gamma}\gamma\mathcal{C}_2(U_X,V_T,2,2,3)+\bar{\gamma}\gamma(\mathcal{C}_2(U_X,V_T,2,2,3))^*)\\
&+3(\bar{\gamma}^2\mathcal{F}_2(V_E,U_T,4,2)+(\gamma^2\mathcal{F}_2(V_E,U_T,4,3)+\bar{\gamma}\gamma\mathcal{C}_2(V_E,U_T,4,3,2)\\
&+\bar{\gamma}\gamma(\mathcal{C}_2(V_E,U_T,4,3,2))^*)+12\gamma^2F_1(M_Y,M_N)+6\gamma^2\mathcal{F}_1^u(U_L,1,M_B)\\
&+3\gamma^2\mathcal{F}_2(U_T,V_E,4,3)+6\gamma^2\mathcal{F}_2(U_K,V_X,1,1)+18\bar{\gamma}^2F_1(M_B,M_W)\\
&+9\bar{\gamma}^2\mathcal{F}_2(V_E,U_P,3,1)+3\gamma^2F_1(M_V,M_O)+2\gamma^2F_1(M_V,M_A)+\gamma^2\mathcal{F}_2'(V_F,U_H,1,4)\\
&+18\gamma^2F_1(M_Y,M_W)+\bar{\gamma}^2\mathcal{F}_1^u(U_F,1,M_V)+3\bar{\gamma}^2F_1(M_V,M_O)\\
&+2\bar{\gamma}^2\mathcal{F}_2'(V_G,V_H,4,4)+9\gamma^2\mathcal{F}_2(V_P,U_X,1,1)+3\bar{\gamma}^2\mathcal{F}_2(V_T,U_X,4,1)\\
&+6\bar{\gamma}^2\mathcal{F}_2(V_J,U_E,1,4)+6\bar{\gamma}^2\mathcal{F}_1^u(V_L,1,M_Y)+12\bar{\gamma}^2F_1(M_B,M_M)\\
&+4\bar{\gamma}^2\mathcal{F}_2(V_R,V_C,2,2)+4\gamma^2\mathcal{F}_2(V_R,V_C,2,1)+0.5\bar{\gamma}^2\mathcal{F}_2'(V_G,U_H,3,2)\\
&+0.5\gamma^2\mathcal{F}_2'(V_H,U_G,3,3)+1.5\gamma^2\mathcal{F}_2(V_D,U_J,2,3)+1.5\bar{\gamma}^2\mathcal{F}_2(V_D,U_J,1,3)\\
&+1.5\gamma^2\mathcal{F}_2(V_J,U_E,3,1)+1.5\bar{\gamma}^2\mathcal{F}_2(V_J,U_E,3,2)+8\gamma^2\mathcal{F}_1^u(U_C,2,M_Z)\\
&+8\bar{\gamma}^2\mathcal{F}_1^u(U_C,1,M_Z)+\gamma^2\mathcal{F}_2'(V_F,U_H,2,3)\\
&+\bar{\gamma}^2\mathcal{F}_2'(V_F,U_H,2,2)+3\gamma^2\mathcal{F}_2(U_T,V_E,5,2)+3\bar{\gamma}^2\mathcal{F}_2(U_T,V_E,5,1)\\
&+3\bar{\gamma}^2\mathcal{F}_1^u(U_D,2,M_I)+3\gamma^2\mathcal{F}_1^u(U_D,1,M_I)+12\gamma^2\mathcal{F}_1^u(V_C,1,M_Q)\\
&+12\bar{\gamma}^2\mathcal{F}_1^u(V_C,2,M_Q)+1.5\gamma^2\mathcal{F'}_1^u(V_H,3,M_S)+1.5\bar{\gamma}^2\mathcal{F'}_1^u(V_H,2,M_S,M_H,6)\\
&+4.5\bar{\gamma}^2\mathcal{F}_1^u(U_E,2,M_U+4.5\gamma^2\mathcal{F}_1^u(U_E,1,M_U+4.5\bar{\gamma}^2\mathcal{F}_1^u(V_D,1,M_U)\\
&+4.5\gamma^2\mathcal{F}_1^u(V_D,2,M_U)+|k|^2(\mathcal{F}_2'(V_G,V_H,1,5)+6\mathcal{F}_1^u(V_L,2,M_Y)\\
&+3\mathcal{F}_2(U_X,V_T,1,6)+\mathcal{F}_2'(V_F,U_H,4,4)+\mathcal{F}_1^u(U_F,4,M_V)\\
&+6\mathcal{F}_1^u(U_L,2,M_B)+3\mathcal{F}_2(U_T,V_E,6,3)+8\mathcal{F}_1^u(U_C,3,M_Z)\\
&+\mathcal{F}_2'(V_F,U_H,2,6)+4\mathcal{F}_2(V_R,V_C,2,3)+0.5\mathcal{F}_2'(V_G,V_H,3,6)\\
&+1.5\mathcal{F}_2(U_E,V_J,6,3)+3\mathcal{F}_1^u(U_D,3,M_I)+1.5\mathcal{F}_2(V_D,U_J,3,3)\\
&+3\mathcal{F}_2(V_E,U_T,6,5)+1.5\mathcal{F'}_1^u(U_H,6,M_S)+12\mathcal{F}_1^u(U_C,3,M_Q)\\
&+4.5\mathcal{F}_1^u(U_D,3,M_U)+4.5\mathcal{F}_1^u(V_E,6,M_U)+1.5\mathcal{F}_2(V_E,U_T,4,7)
\end{aligned}
$$

$$+3\mathcal{F}_2(U_K,V_X,2,2)+1.5\mathcal{F}_2(U_X,V_T,2,7)+3\mathcal{F}_2(U_E,V_J,4,5)$$
$$+4.5\mathcal{F}_2(U_P,V_E,2,4)+4.5\mathcal{F}_2(U_X,V_P,2,2))-(2g^2)(+1.5\mathcal{F}_2(U_T,V_X,1,3) \quad (15)$$
$$+1.5\mathcal{F}_2(U_E,T,5,1)+0.1\mathcal{F}_2'(G,U_H,6,1)+0.5\mathcal{F}_2'(U_F,H,3,1))) \quad (\mathbf{A10})$$

# Appendix B : Discussion of RGE features

We used the two loop Renormalization group evolution equations for the softly broken MSSM given in [75] to evolve randomly chosen SUGRY-NUHM parameters $\{m_{\tilde{f}}, m_{1/2}, A_0, B, M^2_{H,\bar{H}}\}$ together with the $\mu$ parameter and the rest of the superpotential parameters down from $M_X^0 = 10^{16.25}$ GeV to $M_Z$. It is notable that the universal gaugino mass $M_{1/2}$ at $M_X$ is found to be negative in all our fits. Furthermore the Higgs mass parameters $M^2_{H,\bar{H}}$ are very large and negative being typically $\sim -10^4$ TeV$^2$. The sfermion beta functions at one loop contain terms proportional to the Higgs mass parameters squared [74, 75]. For example :

$$\beta^{(1)}_{\mathbf{m}^2_Q} = (\mathbf{m}^2_Q + 2M^2_H)\mathbf{Y}^\dagger_u\mathbf{Y}_u + (\mathbf{m}^2_Q + 2M^2_{\bar{H}})\mathbf{Y}^\dagger_d\mathbf{Y}_d + ........ \quad (16)$$

which dominate the RGE for the third generation sfermions and drive their masses far above the those of the first two generations as one flows from $M_X$ to $M_Z$. This behaviour is only slightly modulated by the contributions of $A_0$ and is a one-loop feature immune to significant modification by the two loop contributions. The presence of terms $A^\dagger A$ added to the Higgs contributions has a countervailing effect on the scalar mass evolution since it tends to decrease the mass squared in the infrared. However the huge Higgs masses always prevail resulting in the third sgeneration always being heavier than the first two. This is an invariant feature of our spectra. Heavy third sgeneration is a distinctive feature of our fits and counterposes them to all(to our knowledge) previous GUT based predictions which have a third sgeneration *lighter* than the first two.

These tendencies can be clearly seen in the plots of the RG evolution given in Figures 2-5 which refer to the actual two loop RG evolution (Fig. 2) of $M^2_{\tilde{d}}$ , and the next three figures refer to hypothetical cases with $\{A_0(M_X) = 0\}$(Fig.3), $\{M^2_H(M_X) = 0 = M^2_{\bar{H}}(M_X)\}$(Fig.4),$\{M^2_H(M_X) = 0 = M^2_{\bar{H}}(M_X) = A_0(M_X)\}$(Fig.5).

Similarly we can understand why the one loop prediction ($M_1 : M_2 : M_3$ as $g_1^2 : g_2^2 : g_3^2$ which is $\simeq 1 : 2 : 7$ at $M_Z$.) for the ratio of gaugino masses $M_i$ which follows from the 1-loop RG invariance of $M_i/g_i^2, i = 1,2,3$ is badly violated at two loops when $A_0$ is large. Although the $1 : 2 : 7$ seems set in stone by the known gauge couplings at $M_Z$ if GUT mandated equality of gaugino masses at $M_X$ is accepted, the influence of the additional terms[74, 75] at two loop in the gaugino mass RGE :

$$\frac{d}{dt}M_a = \frac{2g_a^2}{16\pi^2}B_a^{(1)}M_a + \frac{2g_a^2}{(16\pi^2)^2}\Big[\sum_{b=1}^{3}B^{(2)}_{ab}g_b^2(M_a+M_b) \quad (17)$$
$$+\sum_{x=u,d,e}C_a^x\left(\mathrm{Tr}[Y_x^\dagger A_x] - M_a\mathrm{Tr}[Y_x^\dagger Y_x]\right)\Big] \quad (18)$$

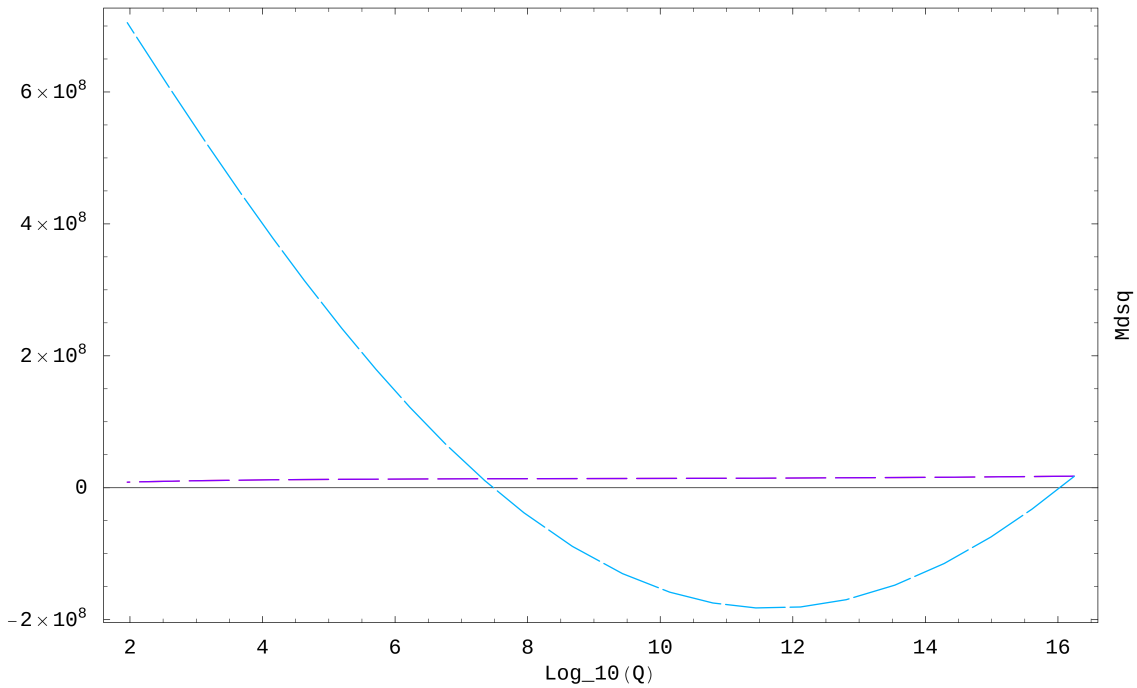


Figure 2: Two loop RG evolution of $M^2_{\tilde{d}}$ from $M^0_X$ to $M_Z$ for Case I-1. Red: $M^2_{\tilde{d}}$, Blue: $M^2_{\tilde{s}}$, Green: $M^2_{\tilde{t}}$. Note the strong growth in the the third sgeneration mass at low energies. The same behaviour is exhibited by all sfermions.

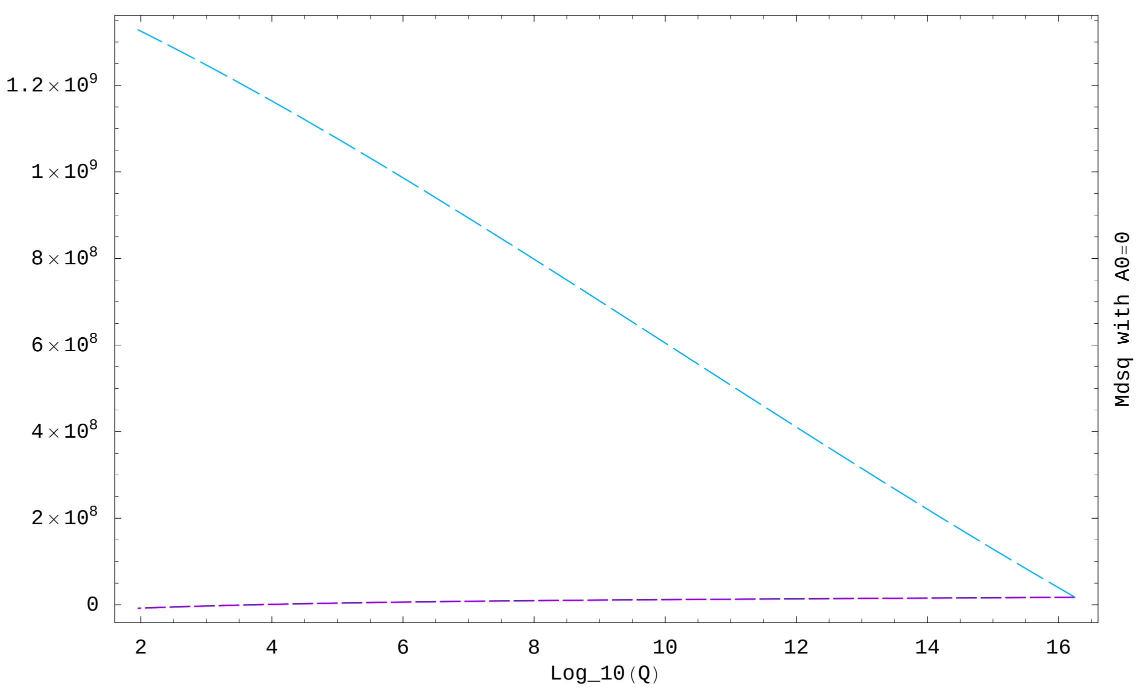


Figure 3: Little effect of $A_0$ : Hypothetical Two loop RG evolution of $M^2_{\tilde{d}}$ from $M^0_X$ to $M_Z$ with $A_0(M_X) = 0$ for Case I-1. Red: $M^2_{\tilde{d}}$, Blue: $M^2_{\tilde{s}}$, Green: $M^2_{\tilde{t}}$. Note the strong growth in the the third sgeneration mass at low energies. The same behaviour is exhibited by all sfermions.

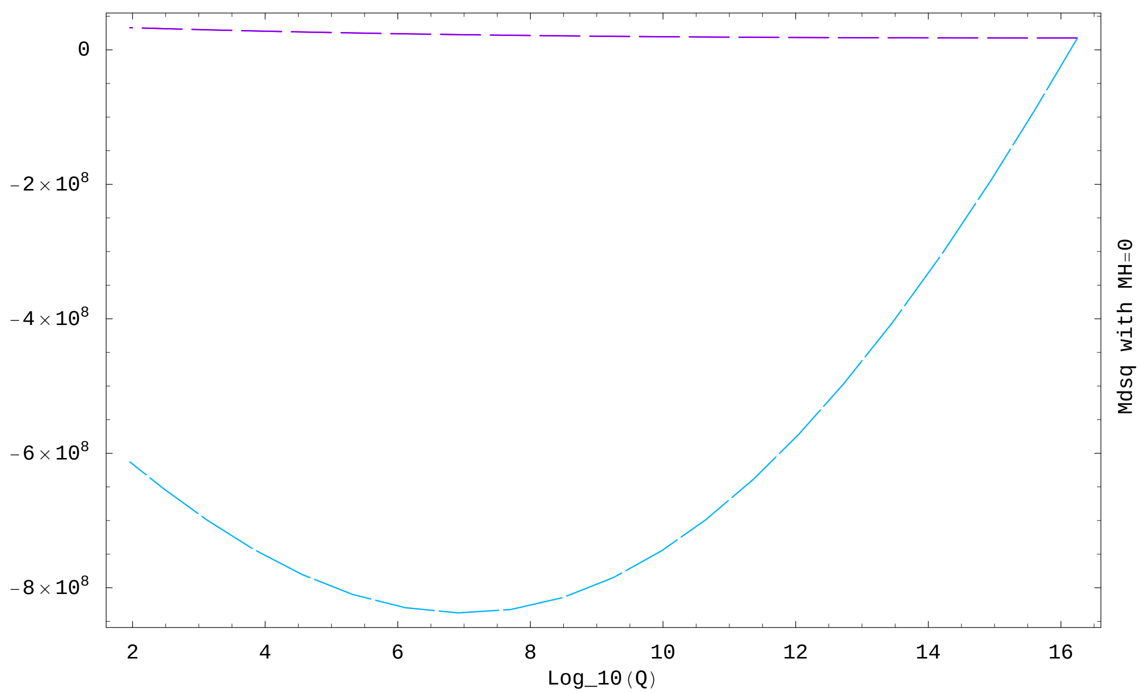


Figure 4: Effect of large $M^2_{H,\bar{H}}$ : Hypothetical Two loop RG evolution of $M^2_{\tilde{d}}$ from $M^0_X$ to $M_Z$ with $M^2_H(M_X) = M^2_{\bar{H}}(M_X) = 0$ for Case I-1. Red: $M^2_{\tilde{\bar{d}}}$, Blue: $M^2_{\tilde{\bar{s}}}$, Green: $M^2_{\tilde{\bar{t}}}$. Note the strong *decrease* in the the third sgeneration mass at low energies while the first two generations are unaffected. Putting $A_0 = 0$ has essentially no effect except that the increase becomes linear. The same behaviour is exhibited by all sfermions.

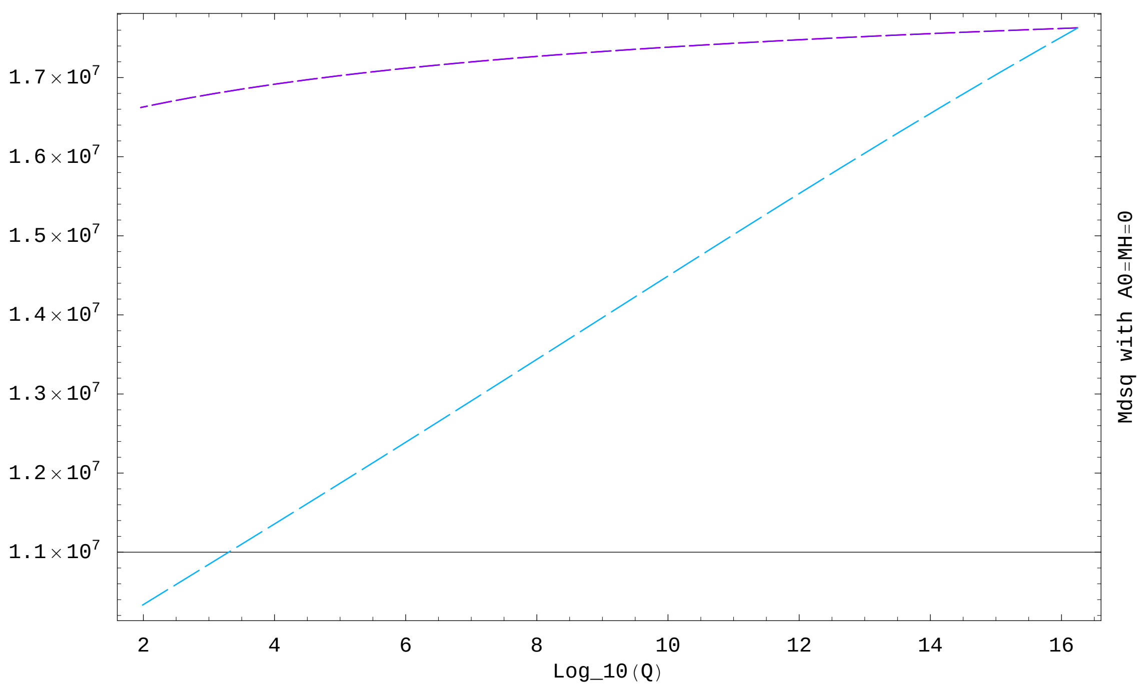


Figure 5: Little effect of $A_0$ : Hypothetical Two loop RG evolution of $M_{\tilde{d}}^2$ from $M_X^0$ to $M_Z$ with $M_H^2(M_X) = M_{\bar{H}}^2(M_X) = 0 = A_0(M_X)$ for Case I-1. Note the strong *decrease* in the the third sgeneration mass at low energies while the first two generations are unaffected. The removal of the curvilinear decrease in favour of a linear one is the only effect of putting $A_0 = 0$ in addition. The same behaviour is exhibited by all sfermions.

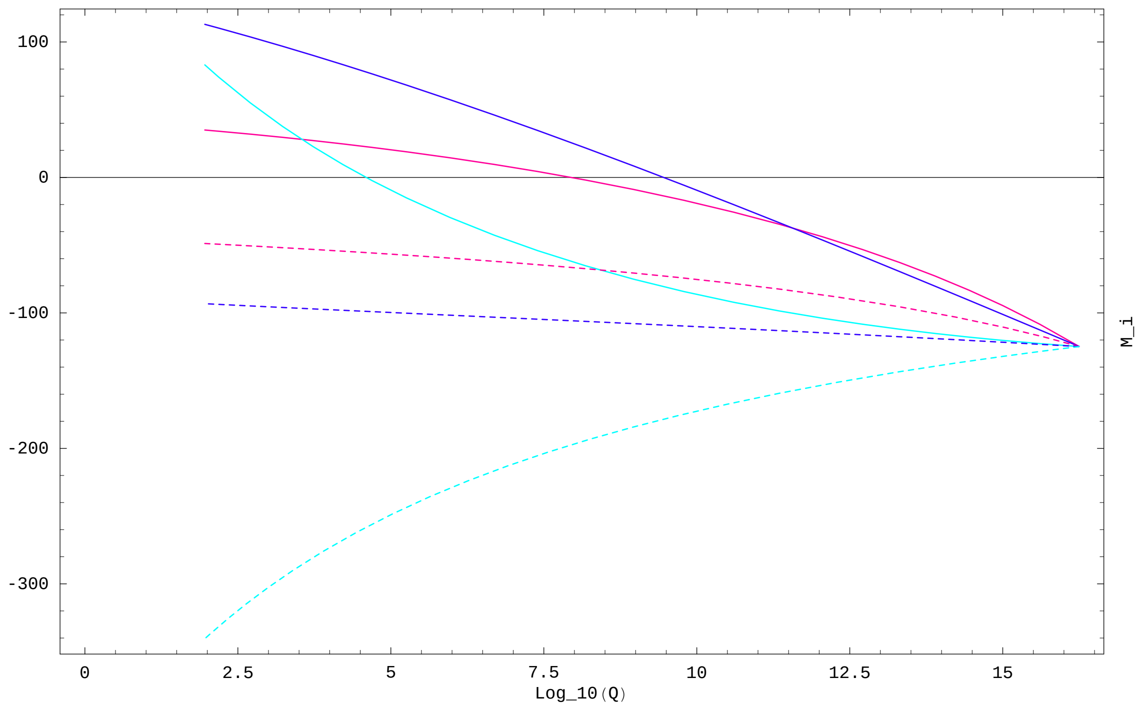


Figure 6: Hypothetical 2-loop RG evolution of gaugino masses with $A_0 \neq 0$ (full lines) and with $A_0 = 0$ (dashed lines) for Case II-1. Red:$M_1$, Blue $M_2$, Green $M_3$. Notice how the Gluino mass can even fall below the Wino mass when $A_0 \neq 0$.

The terms containing the product of the Yukawa gauge couplings and the corresponding soft trilinear couplings that are generated from $A_0$, imply that in practice the ratios can vary widely if $A_0$ is large and gluinos can even be lighter than winos : This is seen clearly from the graphs of the RG flow of the gaugino masses with and without $A_0$ (Fig. 6) and the graph of the ratios of the gaugino masses with and without $A_0$(Fig. 7).

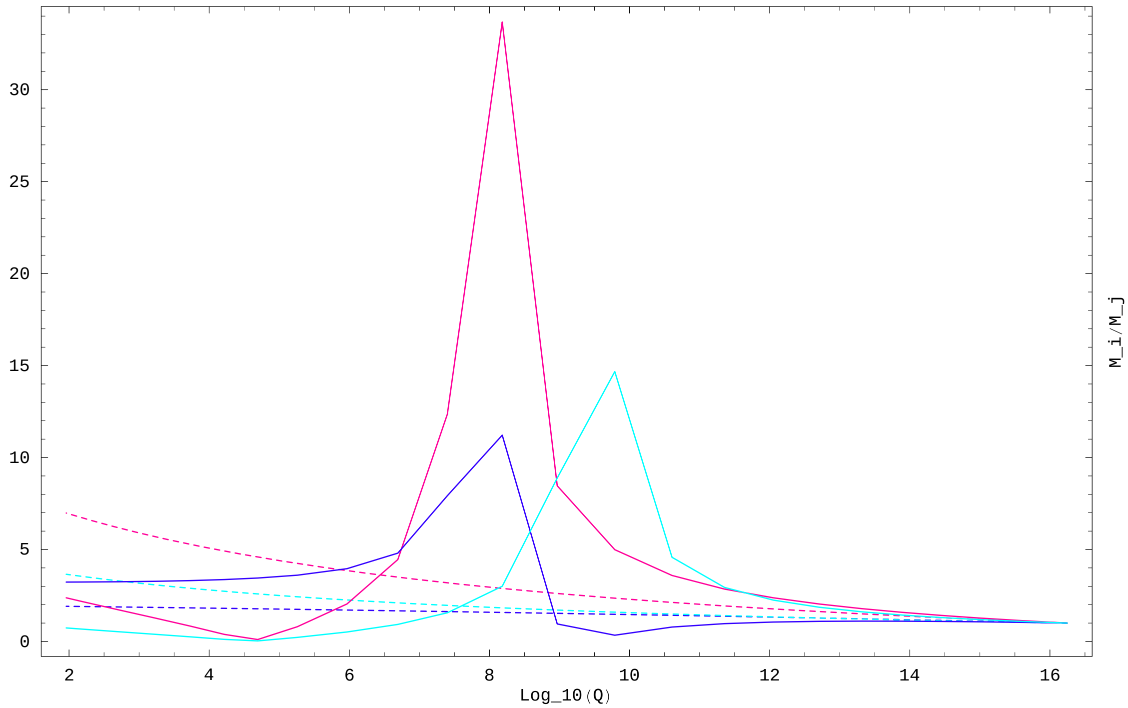


Figure 7: Hypothetical Two loop RG evolution of ratios of gaugino mass ratos with $A_0 \neq 0$ (full lines) and with $A_0 = 0$ (dashed lines) for Case II-1. Red : $M_3/M_1$, Blue: $M_2/M_1$ , Green $M_3/M_2$. In the case $A_0 = 0$, the gaugino masses follow the standard evolution to the $1:2:7$ ratio at low energies.

# Appendix C: Additional Tables of Parameter values

| Parameter | $Value$ | Field $[SU(3), SU(2), Y]$ | $Masses$ $(Units\ of 10^{16} GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0232 | $A[1,1,4]$ | 17462.07 |
| $\chi_Z$ | 0.0712 | $B[6,2,5/3]$ | 0.4028 |
| $h_{11}/10^{-6}$ | $-0.4808$ | $C[8,2,1]$ | $733.73, 2706.98, 4133.40$ |
| $h_{22}/10^{-4}$ | 0.3364 | $D[3,2,7/3]$ | $1514.69, 9791.18, 11582.78$ |
| $h_{33}$ | 0.0141 | $E[3,2,1/3]$ | $0.91, 529.34, 529.34$ |
| $f_{11}/10^{-6}$ | $-0.0989 - 0.0244i$ | | $1724.816, 5358.19, 7929.76$ |
| $f_{12}/10^{-6}$ | $1.6150 + 0.0073i$ | $F[1,1,2]$ | $202.02, 202.02$ |
| $f_{13}/10^{-5}$ | $-0.6328 - 0.0360i$ | | $1129.96, 8959.17$ |
| $f_{22}/10^{-5}$ | $-1.9062 + 0.0805i$ | $G[1,1,0]$ | $0.088, 0.73, 3.65$ |
| $f_{23}/10^{-4}$ | $1.0036 + 0.0037i$ | | $3.646, 128.52, 128.85$ |
| $f_{33}/10^{-3}$ | $-0.4254 + 0.0291i$ | $h[1,2,1]$ | $1.727, 930.24, 1430.99$ |
| $g_{12}/10^{-4}$ | $-2.8201 - 0.7277i$ | | $10152.47, 13862.15$ |
| $g_{13}/10^{-5}$ | $-57.2234 - 43.9416i$ | $I[3,1,10/3]$ | 1.02 |
| $g_{23}/10^{-4}$ | $135.6224 + 115.2576i$ | $J[3,1,4/3]$ | $1.224, 355.07, 355.07$ |
| $\lambda/10^{-2}$ | $0.0664 + 0.0432i$ | | $642.66, 7578.30$ |
| $\eta$ | $-8.7343 - 0.3535i$ | $K[3,1,8/3]$ | $1099.55, 11032.05$ |
| $\rho$ | $1.2543 - 0.6648i$ | $L[6,1,2/3]$ | $878.68, 14142.89$ |
| $k$ | $-0.0708 - 0.0060i$ | $M[6,1,8/3]$ | 14763.05 |
| $\zeta$ | $0.6792 + 1.3334i$ | $N[6,1,4/3]$ | 14078.87 |
| $\bar{\zeta}$ | $3.5786 - 0.5914i$ | $O[1,3,2]$ | 26102.41 |
| $m/10^{16}GeV$ | 0.05 | $P[3,3,2/3]$ | $1157.34, 18368.19$ |
| $m_o/10^{16}GeV$ | $-947.952e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 0.718 |
| $\gamma$ | 1.01 | $R[8,1,0]$ | $0.46, 1.53$ |
| $\bar{\gamma}$ | $-1.3309$ | $S[1,3,0]$ | 1.6833 |
| $x$ | $0.8559 + 1.0678i$ | $t[3,1,2/3]$ | $0.87, 248.68, 833.99, 1244.35$ |
| $\Delta_X$ | 2.82 | | $4579.86, 4885.32, 76873.18$ |
| $\Delta_G$ | $-19.885$ | $U[3,3,4/3]$ | 1.345 |
| $\Delta\alpha_3(M_Z)$ | $-0.011$ | $V[1,2,3]$ | 1.197 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.00, 12.97, 1184.75$ | $W[6,3,2/3]$ | 10511.05 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $0.0012, 4.56, 416.72$ | $X[3,2,5/3]$ | $0.439, 554.214, 554.214$ |
| $M_\nu(meV)$ | $2.14, 7.64, 42.49$ | $Y[6,2,1/3]$ | 0.53 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.01, 49.25, 4499.03$ | $Z[8,1,2]$ | 1.51 |
| Soft parameters | $\mathrm{m}_{\frac{1}{2}} = -387.435$ | $\mathrm{m}_0 = 2609.655$ | $\mathrm{A}_0 = -1.8398 \times 10^5$ |
| at $M_X$ | $\mu = 1.4670 \times 10^5$ | $\mathrm{B} = -1.5219 \times 10^{10}$ | $\tan\beta = 50.0000$ |
| | $\mathrm{M}^2_{\bar{\mathrm{H}}} = -1.4822 \times 10^{10}$ | $\mathrm{M}^2_{\mathrm{H}} = -1.4528 \times 10^{10}$ | $\mathrm{R}_{\frac{b\tau}{s\mu}} = 0.8725$ |
| $Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$ | $7.1909 \times 10^{-22}\ \mathrm{GeV}^{-1}$ | | |

Table 23: I-2-a: Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.106596 | 0.804720 | 2.107032 | 0.000542 |
| $y_c/10^{-3}$ | 1.026873 | 0.169434 | 1.027891 | 0.006009 |
| $y_t$ | 0.388445 | 0.015538 | 0.388409 | −0.002299 |
| $y_d/10^{-5}$ | 5.979116 | 3.485824 | 6.027227 | 0.013802 |
| $y_s/10^{-3}$ | 1.131477 | 0.534057 | 1.125268 | −0.011625 |
| $y_b$ | 0.544641 | 0.282669 | 0.545745 | 0.003905 |
| $y_e/10^{-4}$ | 1.279312 | 0.191897 | 1.279764 | 0.002359 |
| $y_\mu/10^{-2}$ | 2.593121 | 0.388968 | 2.591469 | −0.004247 |
| $y_\tau$ | 0.566253 | 0.107588 | 0.567375 | 0.010426 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | −0.0001 |
| $\sin\theta^q_{13}/10^{-4}$ | 28.5820 | 5.000000 | 28.5803 | −0.0003 |
| $\sin\theta^q_{23}/10^{-3}$ | 33.6305 | 1.300000 | 33.6299 | −0.0005 |
| $\delta^q$ | 60.0207 | 14.000000 | 59.9564 | −0.0046 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.3851 | 0.570823 | 5.3850 | −0.0002 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.7467 | 0.349345 | 1.7466 | −0.0004 |
| $\sin^2\theta^L_{12}$ | 0.2880 | 0.057607 | 0.2880 | −0.0008 |
| $\sin^2\theta^L_{23}$ | 0.4595 | 0.137850 | 0.4597 | 0.0014 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 3.44 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.119568 | 0.119717 | 0.125583 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.119570 | 0.119714 | 0.125569 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.149527 | 0.149678 | 0.155646 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.149568 | 0.149724 | 0.155689 | |
| $Eigenvalues(\Delta_Q)$ | 0.106161 | 0.106308 | 0.112283 | |
| $Eigenvalues(\Delta_L)$ | 0.136273 | 0.136419 | 0.142375 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 14.262526 | 11.287996 | | |
| $\alpha_1$ | $0.8067 + 0.0000i$ | $\bar{\alpha}_1$ | $0.9095 + 0.0000i$ | |
| $\alpha_2$ | $0.0179 - 0.0131i$ | $\bar{\alpha}_2$ | $0.0280 - 0.0024i$ | |
| $\alpha_3$ | $-0.0364 - 0.0057i$ | $\bar{\alpha}_3$ | $-0.0315 + 0.0016i$ | |
| $\alpha_4$ | $0.0024 + 0.5830i$ | $\bar{\alpha}_4$ | $0.0171 - 0.4055i$ | |
| $\alpha_5$ | $0.0486 - 0.0185i$ | $\bar{\alpha}_5$ | $0.0246 - 0.0388i$ | |
| $\alpha_6$ | $-0.0287 - 0.0629i$ | $\bar{\alpha}_6$ | $-0.0384 - 0.0525i$ | |

Table 24: I-2-bFit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0232$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.52473 | 3.07224 |
| $m_s/10^{-3}$ | 55.00000 | 9.79652 | 57.51101 |
| $m_b$ | 2.90000 | 3.24592 | 2.90989 |
| $m_e/10^{-3}$ | 0.48657 | 0.47320 | 0.48644 |
| $m_\mu$ | 0.10272 | 0.09578 | 0.10240 |
| $m_\tau$ | 1.74624 | 1.74402 | 1.74715 |
| $m_u/10^{-3}$ | 1.27000 | 1.10467 | 1.27126 |
| $m_c$ | 0.61900 | 0.53890 | 0.62018 |
| $m_t$ | 172.50000 | 148.26724 | 172.47971 |

Table 25: I-2-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} =$ 0.0712.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 104.66 | $M_{\tilde{\bar{u}}_1}$ | 3630.75 |
| $M_2$ | 353.60 | $M_{\tilde{\bar{u}}_2}$ | 3627.51 |
| $M_3$ | 268.96 | $M_{\tilde{\bar{u}}_3}$ | 20705.08 |
| $M_{\tilde{\bar{l}}_1}$ | 1887.68 | $A_{11}^{0(l)}$ | −110809.91 |
| $M_{\tilde{\bar{l}}_2}$ | 265.45 | $A_{22}^{0(l)}$ | −110677.57 |
| $M_{\tilde{\bar{l}}_3}$ | 11488.56 | $A_{33}^{0(l)}$ | −67135.87 |
| $M_{\tilde{L}_1}$ | 6440.25 | $A_{11}^{0(u)}$ | −133970.62 |
| $M_{\tilde{L}_2}$ | 6303.95 | $A_{22}^{0(u)}$ | −133969.80 |
| $M_{\tilde{L}_3}$ | 10623.40 | $A_{33}^{0(u)}$ | −66830.19 |
| $M_{\tilde{\bar{d}}_1}$ | 825.29 | $A_{11}^{0(d)}$ | −110623.57 |
| $M_{\tilde{\bar{d}}_2}$ | 819.67 | $A_{22}^{0(d)}$ | −110622.83 |
| $M_{\tilde{\bar{d}}_3}$ | 42238.45 | $A_{33}^{0(d)}$ | −37954.46 |
| $M_{\tilde{Q}_1}$ | 5443.89 | $\tan\beta$ | 50.00 |
| $M_{\tilde{Q}_2}$ | 5442.40 | $\mu(M_Z)$ | 112330.33 |
| $M_{\tilde{Q}_3}$ | 33482.05 | $B(M_Z)$ | $2.1590 \times 10^9$ |
| $M_{\bar{H}}^2$ | $-1.2007 \times 10^{10}$ | $M_H^2$ | $-1.3857 \times 10^{10}$ |

Table 26: I-2-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 268.96 |
| $M_{\chi^{\pm}}$ | 353.59, 112330.39 |
| $M_{\chi^0}$ | 104.66, 353.59, 112330.36, 112330.37 |
| $M_{\tilde{\nu}}$ | 6439.906, 6303.596, 10623.190 |
| $M_{\tilde{e}}$ | 1888.22, 6440.43, 254.80, 6304.74, 10423.50, 11670.42 |
| $M_{\tilde{u}}$ | 3630.56, 5443.61, 3627.28, 5442.15, 20701.92, 33484.59 |
| $M_{\tilde{d}}$ | 825.70, 5444.24, 820.02, 5442.76, 33474.52, 42244.47 |
| $M_A$ | 328626.38 |
| $M_{H^{\pm}}$ | 328626.39 |
| $M_{H^0}$ | 328626.36 |
| $M_{h^0}$ | 112.60 |

Table 27: I-2-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 269.14 |
| $M_{\chi^{\pm}}$ | 353.67, 112325.66 |
| $M_{\chi^0}$ | 104.69, 353.67, 112325.64, 112325.65 |
| $M_{\tilde{\nu}}$ | 6303.72, 6440.01, 10625.538 |
| $M_{\tilde{e}}$ | 296.70, 1887.75, 6294.24, 6440.47, 10487.30, 11631.85 |
| $M_{\tilde{u}}$ | 3627.98, 3631.26, 5423.60, 5444.05, 20695.60, 33484.45 |
| $M_{\tilde{d}}$ | 822.45, 828.15, 5424.23, 5444.67, 33474.38, 42241.87 |
| $M_A$ | 328751.52 |
| $M_{H^{\pm}}$ | 328751.53 |
| $M_{H^0}$ | 328751.50 |
| $M_{h^0}$ | 112.62 |

Table 28: I-2-f: Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

| Parameter | $Value$ | Field $[SU(3), SU(2), Y]$ | $Masses$ $(Units\ of 10^{16} GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0483 | $A[1,1,4]$ | 809.66 |
| $\chi_Z$ | 0.0065 | $B[6,2,5/3]$ | 0.2796 |
| $h_{11}/10^{-6}$ | 0.0210 | $C[8,2,1]$ | $43.65, 387.28, 404.49$ |
| $h_{22}/10^{-4}$ | 0.0353 | $D[3,2,7/3]$ | $42.78, 445.41, 456.44$ |
| $h_{33}$ | 0.0026 | $E[3,2,1/3]$ | $0.45, 32.26, 32.26$ |
| $f_{11}/10^{-6}$ | $0.0781 - 0.1376i$ | | $34.752, 488.08, 543.11$ |
| $f_{12}/10^{-6}$ | $-2.0155 - 0.0665i$ | $F[1,1,2]$ | $7.60, 7.60$ |
| $f_{13}/10^{-5}$ | $0.0607 + 0.0478i$ | | $31.02, 407.44$ |
| $f_{22}/10^{-5}$ | $6.5549 - 4.8873i$ | $G[1,1,0]$ | $0.070, 0.55, 0.69$ |
| $f_{23}/10^{-4}$ | $1.9973 + 2.3436i$ | | $0.692, 30.02, 30.20$ |
| $f_{33}/10^{-3}$ | $-1.0174 + 0.4408i$ | $h[1,2,1]$ | $1.176, 25.35, 41.75$ |
| $g_{12}/10^{-4}$ | $0.0608 + 0.1221i$ | | $681.54, 704.75$ |
| $g_{13}/10^{-5}$ | $-0.0465 + 1.7512i$ | $I[3,1,10/3]$ | 0.96 |
| $g_{23}/10^{-4}$ | $6.3552 + 5.8153i$ | $J[3,1,4/3]$ | $1.069, 17.56, 17.56$ |
| $\lambda/10^{-2}$ | $-0.4664 - 0.9791i$ | | $54.25, 474.36$ |
| $\eta$ | $-10.4880 + 2.4932i$ | $K[3,1,8/3]$ | $62.72, 581.65$ |
| $\rho$ | $0.6621 - 2.2887i$ | $L[6,1,2/3]$ | $29.75, 931.63$ |
| $k$ | $0.0151 - 0.0820i$ | $M[6,1,8/3]$ | 942.87 |
| $\zeta$ | $1.6335 + 0.5734i$ | $N[6,1,4/3]$ | 939.66 |
| $\bar{\zeta}$ | $0.9799 + 0.4779i$ | $O[1,3,2]$ | 1814.58 |
| $m/10^{16}GeV$ | 0.03 | $P[3,3,2/3]$ | $18.17, 1405.68$ |
| $m_o/10^{16}GeV$ | $-25.800e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 0.795 |
| $\gamma$ | 3.91 | $R[8,1,0]$ | $0.31, 1.19$ |
| $\bar{\gamma}$ | $-2.8770$ | $S[1,3,0]$ | 1.3505 |
| $x$ | $0.9444 + 0.6732i$ | $t[3,1,2/3]$ | $0.88, 24.06, 59.01, 100.44$ |
| $\Delta_X$ | 1.28 | | $307.25, 415.94, 8824.64$ |
| $\Delta_G$ | 1.218 | $U[3,3,4/3]$ | 1.139 |
| $\Delta\alpha_3(M_Z)$ | $-0.013$ | $V[1,2,3]$ | 0.808 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.01, 14.15, 596.40$ | $W[6,3,2/3]$ | 1079.74 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $0.4641, 865.17, 36452.94$ | $X[3,2,5/3]$ | $0.273, 34.505, 34.505$ |
| $M_\nu(meV)$ | $2.14, 7.63, 42.49$ | $Y[6,2,1/3]$ | 0.34 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.15, 283.31, 11937.12$ | $Z[8,1,2]$ | 1.19 |
| Soft parameters at $M_X$ | $\mathrm{m}_{\frac{1}{2}} = -193.241$ | $\mathrm{m}_0 = 4908.909$ | $\mathrm{A}_0 = -1.6043 \times 10^5$ |
| | $\mu = 1.2834 \times 10^5$ | $\mathrm{B} = -1.1313 \times 10^{10}$ | $\tan\beta = 51.5000$ |
| | $\mathrm{M}^2_{\bar{\mathrm{H}}} = -1.2330 \times 10^{10}$ | $\mathrm{M}^2_{\mathrm{H}} = -1.1748 \times 10^{10}$ | $\mathrm{R}_{\frac{b\tau}{s\mu}} = 2.6024$ |
| $Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$ | $4.9596 \times 10^{-23}\,\mathrm{GeV}^{-1}$ | | |

Table 29: I-3-a Fit : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.084307 | 0.796205 | 2.084498 | 0.000240 |
| $y_c/10^{-3}$ | 1.016016 | 0.167643 | 1.017365 | 0.008048 |
| $y_t$ | 0.383569 | 0.015343 | 0.383558 | −0.000709 |
| $y_d/10^{-5}$ | 7.238813 | 4.220228 | 7.247117 | 0.001968 |
| $y_s/10^{-3}$ | 1.372490 | 0.647815 | 1.376178 | 0.005694 |
| $y_b$ | 0.505108 | 0.262151 | 0.505457 | 0.001331 |
| $y_e/10^{-4}$ | 1.283357 | 0.192503 | 1.285487 | 0.011068 |
| $y_\mu/10^{-2}$ | 2.637591 | 0.395639 | 2.639897 | 0.005827 |
| $y_\tau$ | 0.575157 | 0.109280 | 0.570576 | −0.041920 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | 0.0002 |
| $\sin\theta^q_{13}/10^{-4}$ | 28.8487 | 5.000000 | 28.8604 | 0.0024 |
| $\sin\theta^q_{23}/10^{-3}$ | 33.9439 | 1.300000 | 33.9429 | −0.0008 |
| $\delta^q$ | 60.0208 | 14.000000 | 60.0314 | 0.0008 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.3665 | 0.568846 | 5.3642 | −0.0040 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.7445 | 0.348909 | 1.7468 | 0.0065 |
| $\sin^2\theta^L_{12}$ | 0.2875 | 0.057499 | 0.2869 | −0.0109 |
| $\sin^2\theta^L_{23}$ | 0.4576 | 0.137268 | 0.4591 | 0.0114 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 4.55 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.053857 | 0.053861 | 0.053868 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.051644 | 0.051649 | 0.051655 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.058766 | 0.058770 | 0.058776 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.065404 | 0.065409 | 0.065415 | |
| $Eigenvalues(\Delta_Q)$ | 0.050688 | 0.050692 | 0.050697 | |
| $Eigenvalues(\Delta_L)$ | 0.060022 | 0.060026 | 0.060032 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 77.639981 | 63.294285 | | |
| $\alpha_1$ | $0.7919 - 0.0000i$ | $\bar{\alpha}_1$ | $0.8754 - 0.0000i$ | |
| $\alpha_2$ | $0.0651 + 0.0139i$ | $\bar{\alpha}_2$ | $0.0558 + 0.0625i$ | |
| $\alpha_3$ | $-0.0416 - 0.0391i$ | $\bar{\alpha}_3$ | $-0.0561 - 0.0179i$ | |
| $\alpha_4$ | $-0.4597 + 0.2078i$ | $\bar{\alpha}_4$ | $0.3572 - 0.0342i$ | |
| $\alpha_5$ | $0.1484 + 0.0808i$ | $\bar{\alpha}_5$ | $0.0685 + 0.0189i$ | |
| $\alpha_6$ | $0.1097 - 0.2650i$ | $\bar{\alpha}_6$ | $0.1198 - 0.2740i$ | |

Table 30: I-3-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0483$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.62818 | 2.90275 |
| $m_s/10^{-3}$ | 55.00000 | 11.88721 | 54.95005 |
| $m_b$ | 2.90000 | 3.07412 | 2.89882 |
| $m_e/10^{-3}$ | 0.48657 | 0.47199 | 0.48604 |
| $m_\mu$ | 0.10272 | 0.09706 | 0.10272 |
| $m_\tau$ | 1.74624 | 1.73492 | 1.73563 |
| $m_u/10^{-3}$ | 1.27000 | 1.09437 | 1.26906 |
| $m_c$ | 0.61900 | 0.53416 | 0.61943 |
| $m_t$ | 172.50000 | 148.47236 | 172.66514 |

Table 31: I-3-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} = 0.0065$.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 147.27 | $M_{\tilde{\bar{u}}_1}$ | 6318.67 |
| $M_2$ | 406.23 | $M_{\tilde{\bar{u}}_2}$ | 6317.40 |
| $M_3$ | 598.97 | $M_{\tilde{\bar{u}}_3}$ | 21142.35 |
| $M_{\tilde{\bar{l}}_1}$ | 1614.43 | $A_{11}^{0(l)}$ | $-98630.83$ |
| $M_{\tilde{\bar{l}}_2}$ | 320.14 | $A_{22}^{0(l)}$ | $-98506.40$ |
| $M_{\tilde{\bar{l}}_3}$ | 16050.48 | $A_{33}^{0(l)}$ | $-59069.03$ |
| $M_{\tilde{L}_1}$ | 7827.10 | $A_{11}^{0(u)}$ | $-117145.75$ |
| $M_{\tilde{L}_2}$ | 7747.15 | $A_{22}^{0(u)}$ | $-117144.96$ |
| $M_{\tilde{L}_3}$ | 13923.16 | $A_{33}^{0(u)}$ | $-59199.13$ |
| $M_{\tilde{\bar{d}}_1}$ | 3334.46 | $A_{11}^{0(d)}$ | $-98885.51$ |
| $M_{\tilde{\bar{d}}_2}$ | 3333.11 | $A_{22}^{0(d)}$ | $-98884.62$ |
| $M_{\tilde{\bar{d}}_3}$ | 36975.96 | $A_{33}^{0(d)}$ | $-37795.22$ |
| $M_{\tilde{Q}_1}$ | 6120.46 | $\tan\beta$ | 51.50 |
| $M_{\tilde{Q}_2}$ | 6119.44 | $\mu(M_Z)$ | 101035.83 |
| $M_{\tilde{Q}_3}$ | 30209.06 | $B(M_Z)$ | $1.7444 \times 10^9$ |
| $M_{\bar{H}}^2$ | $-1.0097 \times 10^{10}$ | $M_H^2$ | $-1.1107 \times 10^{10}$ |

Table 32: I-3-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 598.97 |
| $M_{\chi^{\pm}}$ | 406.23, 101035.90 |
| $M_{\chi^0}$ | 147.27, 406.23, 101035.87, 101035.87 |
| $M_{\tilde{\nu}}$ | 7826.822, 7746.869, 13922.999 |
| $M_{\tilde{e}}$ | 1615.07, 7827.26, 316.41, 7747.59, 13877.17, 16090.39 |
| $M_{\tilde{u}}$ | 6120.21, 6318.57, 6119.06, 6317.42, 21138.66, 30212.30 |
| $M_{\tilde{d}}$ | 3334.56, 6120.76, 3333.19, 6119.76, 30199.68, 36983.67 |
| $M_A$ | 299785.40 |
| $M_{H^{\pm}}$ | 299785.42 |
| $M_{H^0}$ | 299785.39 |
| $M_{h^0}$ | 114.58 |

Table 33: I-3-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 599.14 |
| $M_{\chi^{\pm}}$ | 406.47, 100968.77 |
| $M_{\chi^0}$ | 147.34, 406.47, 100968.75, 100968.75 |
| $M_{\tilde{\nu}}$ | 7744.59, 7824.46, 14034.822 |
| $M_{\tilde{e}}$ | 393.14, 1632.84, 7745.08, 7824.89, 13992.95, 16284.21 |
| $M_{\tilde{u}}$ | 6108.76, 6117.73, 6311.63, 6312.80, 21005.60, 30236.11 |
| $M_{\tilde{d}}$ | 3331.17, 3332.57, 6109.44, 6118.29, 30223.69, 37095.48 |
| $M_A$ | 300060.73 |
| $M_{H^{\pm}}$ | 300060.74 |
| $M_{H^0}$ | 300060.72 |
| $M_{h^0}$ | 114.85 |

Table 34: I-3-f: Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

| Parameter | $Value$ | Field $[SU(3),SU(2),Y]$ | $Masses$ $(Units\ of 10^{16} GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0771 | $A[1,1,4]$ | 168.86 |
| $\chi_Z$ | 0.0225 | $B[6,2,5/3]$ | 0.3197 |
| $h_{11}/10^{-6}$ | 0.0156 | $C[8,2,1]$ | $5.54, 205.24, 206.53$ |
| $h_{22}/10^{-4}$ | $-0.0131$ | $D[3,2,7/3]$ | $4.90, 110.54, 138.09$ |
| $h_{33}$ | 0.0033 | $E[3,2,1/3]$ | $0.38, 3.89, 6.18$ |
| $f_{11}/10^{-6}$ | $0.0198 + 0.1542i$ | | $6.184, 165.12, 192.29$ |
| $f_{12}/10^{-6}$ | $-1.2260 + 0.3835i$ | $F[1,1,2]$ | $1.15, 1.15$ |
| $f_{13}/10^{-5}$ | $0.3221 + 0.1153i$ | | $3.20, 85.63$ |
| $f_{22}/10^{-5}$ | $5.6438 - 6.8723i$ | $G[1,1,0]$ | $0.089, 0.28, 0.28$ |
| $f_{23}/10^{-4}$ | $2.0515 + 3.1055i$ | | $0.722, 16.74, 17.12$ |
| $f_{33}/10^{-3}$ | $-1.0661 + 0.7067i$ | $h[1,2,1]$ | $1.325, 3.74, 5.08$ |
| $g_{12}/10^{-4}$ | $0.0506 - 0.0960i$ | | $145.15, 190.66$ |
| $g_{13}/10^{-5}$ | $-1.5789 - 2.0450i$ | $I[3,1,10/3]$ | 1.29 |
| $g_{23}/10^{-4}$ | $2.9105 + 6.4235i$ | $J[3,1,4/3]$ | $1.261, 3.10, 3.10$ |
| $\lambda/10^{-2}$ | $-6.6788 - 1.6361i$ | | $8.01, 130.00$ |
| $\eta$ | $-14.9636 + 0.9886i$ | $K[3,1,8/3]$ | $8.83, 154.03$ |
| $\rho$ | $0.4151 - 1.6189i$ | $L[6,1,2/3]$ | $4.56, 252.21$ |
| $k$ | $0.0254 - 0.0733i$ | $M[6,1,8/3]$ | 258.11 |
| $\zeta$ | $0.9369 + 0.5584i$ | $N[6,1,4/3]$ | 249.35 |
| $\bar{\zeta}$ | $1.5736 - 0.4378i$ | $O[1,3,2]$ | 456.61 |
| $m/10^{16}GeV$ | 0.03 | $P[3,3,2/3]$ | $1.32, 401.98$ |
| $m_o/10^{16}GeV$ | $-3.749e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 1.138 |
| $\gamma$ | 4.49 | $R[8,1,0]$ | $0.34, 1.34$ |
| $\bar{\gamma}$ | $-2.4523$ | $S[1,3,0]$ | 1.5523 |
| $x$ | $0.8697 + 0.4711i$ | $t[3,1,2/3]$ | $0.94, 3.03, 8.58, 10.22$ |
| $\Delta_X$ | 0.45 | | $162.05, 171.60, 3725.03$ |
| $\Delta_G$ | 19.546 | $U[3,3,4/3]$ | 1.367 |
| $\Delta\alpha_3(M_Z)$ | $-0.008$ | $V[1,2,3]$ | 0.875 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.01, 6.59, 279.67$ | $W[6,3,2/3]$ | 382.85 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $5.5373, 3939.17, 167187.45$ | $X[3,2,5/3]$ | $0.293, 6.732, 6.732$ |
| $M_\nu(meV)$ | $2.30, 7.53, 41.51$ | $Y[6,2,1/3]$ | 0.36 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.46, 326.47, 13856.04$ | $Z[8,1,2]$ | 1.34 |
| Soft parameters | $\mathrm{m}_{\frac{1}{2}} = -221.928$ | $\mathrm{m}_0 = 5063.109$ | $\mathrm{A}_0 = -1.4400 \times 10^5$ |
| at $M_X$ | $\mu = 1.2715 \times 10^5$ | $\mathrm{B} = -9.2002 \times 10^9$ | $\tan\beta = 51.0000$ |
| | $\mathrm{M}^2_{\bar{\mathrm{H}}} = -1.4452 \times 10^{10}$ | $\mathrm{M}^2_{\mathrm{H}} = -1.3796 \times 10^{10}$ | $\mathrm{R}_{\frac{b\tau}{s\mu}} = 3.6620$ |
| $Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$ | $4.5150 \times 10^{-22}\ \mathrm{GeV}^{-1}$ | | |

Table 35: I-4-a Fit : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.099660 | 0.802070 | 2.099948 | 0.000359 |
| $y_c/10^{-3}$ | 1.023488 | 0.168875 | 1.022975 | −0.003035 |
| $y_t$ | 0.368075 | 0.014723 | 0.368096 | 0.001417 |
| $y_d/10^{-5}$ | 6.248810 | 3.643056 | 6.265778 | 0.004658 |
| $y_s/10^{-3}$ | 1.185571 | 0.559590 | 1.201681 | 0.028788 |
| $y_b$ | 0.424469 | 0.220299 | 0.436945 | 0.056636 |
| $y_e/10^{-4}$ | 1.173101 | 0.175965 | 1.171800 | −0.007392 |
| $y_\mu/10^{-2}$ | 2.478866 | 0.371830 | 2.481095 | 0.005994 |
| $y_\tau$ | 0.527577 | 0.100240 | 0.523402 | −0.041647 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | −0.0001 |
| $\sin\theta^q_{13}/10^{-4}$ | 29.4359 | 5.000000 | 29.4173 | −0.0037 |
| $\sin\theta^q_{23}/10^{-3}$ | 34.6341 | 1.300000 | 34.6323 | −0.0013 |
| $\delta^q$ | 60.0209 | 14.000000 | 60.0406 | 0.0014 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.1465 | 0.545532 | 5.1456 | −0.0016 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.6659 | 0.333190 | 1.6666 | 0.0019 |
| $\sin^2\theta^L_{12}$ | 0.2885 | 0.057707 | 0.2883 | −0.0043 |
| $\sin^2\theta^L_{23}$ | 0.4613 | 0.138393 | 0.4616 | 0.0023 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 4.29 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.011860 | 0.011862 | 0.011863 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.008409 | 0.008412 | 0.008413 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.002387 | 0.002389 | 0.002390 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.012738 | 0.012740 | 0.012741 | |
| $Eigenvalues(\Delta_Q)$ | 0.014518 | 0.014520 | 0.014520 | |
| $Eigenvalues(\Delta_L)$ | 0.011946 | 0.011948 | 0.011948 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 59.178650 | 51.533539 | | |
| $\alpha_1$ | $0.7545 - 0.0000i$ | $\bar{\alpha}_1$ | $0.8064 - 0.0000i$ | |
| $\alpha_2$ | $0.0491 + 0.0409i$ | $\bar{\alpha}_2$ | $-0.0007 + 0.0768i$ | |
| $\alpha_3$ | $0.0199 - 0.0535i$ | $\bar{\alpha}_3$ | $-0.0051 - 0.0324i$ | |
| $\alpha_4$ | $-0.5232 - 0.0719i$ | $\bar{\alpha}_4$ | $0.3297 + 0.1532i$ | |
| $\alpha_5$ | $0.2001 + 0.0065i$ | $\bar{\alpha}_5$ | $0.1175 - 0.0307i$ | |
| $\alpha_6$ | $0.0512 - 0.3191i$ | $\bar{\alpha}_6$ | $0.0653 - 0.4377i$ | |

Table 36: I-4-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0771$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.57869 | 2.88556 |
| $m_s/10^{-3}$ | 55.00000 | 11.09336 | 55.29916 |
| $m_b$ | 2.90000 | 2.96813 | 2.93512 |
| $m_e/10^{-3}$ | 0.48657 | 0.45919 | 0.48201 |
| $m_\mu$ | 0.10272 | 0.09719 | 0.10198 |
| $m_\tau$ | 1.74624 | 1.72539 | 1.72437 |
| $m_u/10^{-3}$ | 1.27000 | 1.11423 | 1.27070 |
| $m_c$ | 0.61900 | 0.54254 | 0.61873 |
| $m_t$ | 172.50000 | 147.09707 | 172.15913 |

Table 37: I-4-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} =$ 0.0225.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 104.16 | $M_{\tilde{\bar{u}}_1}$ | 6018.96 |
| $M_2$ | 302.05 | $M_{\tilde{\bar{u}}_2}$ | 6018.69 |
| $M_3$ | 350.55 | $M_{\tilde{\bar{u}}_3}$ | 32268.70 |
| $M_{\tilde{\bar{l}}_1}$ | 115.35 | $A_{11}^{0(l)}$ | −92737.91 |
| $M_{\tilde{\bar{l}}_2}$ | 123.64 | $A_{22}^{0(l)}$ | −92630.01 |
| $M_{\tilde{\bar{l}}_3}$ | 27768.92 | $A_{33}^{0(l)}$ | −58880.24 |
| $M_{\tilde{L}_1}$ | 8173.98 | $A_{11}^{0(u)}$ | −105676.88 |
| $M_{\tilde{L}_2}$ | 8174.04 | $A_{22}^{0(u)}$ | −105676.19 |
| $M_{\tilde{L}_3}$ | 21289.08 | $A_{33}^{0(u)}$ | −55613.06 |
| $M_{\tilde{\bar{d}}_1}$ | 1701.52 | $A_{11}^{0(d)}$ | −92749.44 |
| $M_{\tilde{\bar{d}}_2}$ | 1702.61 | $A_{22}^{0(d)}$ | −92748.73 |
| $M_{\tilde{\bar{d}}_3}$ | 40148.17 | $A_{33}^{0(d)}$ | −41722.79 |
| $M_{\tilde{Q}_1}$ | 5628.19 | $\tan\beta$ | 51.00 |
| $M_{\tilde{Q}_2}$ | 5628.21 | $\mu(M_Z)$ | 106300.71 |
| $M_{\tilde{Q}_3}$ | 36452.35 | $B(M_Z)$ | $1.8340 \times 10^9$ |
| $M_{\bar{H}}^2$ | $-1.1556 \times 10^{10}$ | $M_H^2$ | $-1.2234 \times 10^{10}$ |

Table 38: I-4-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | $350.55$ |
| $M_{\chi^\pm}$ | $302.05, 106300.78$ |
| $M_{\chi^0}$ | $104.16, 302.05, 106300.75, 106300.76$ |
| $M_{\tilde{\nu}}$ | $8173.708, 8173.768, 21288.979$ |
| $M_{\tilde{e}}$ | $123.89, 8174.12, 114.16, 8174.45, 21282.54, 27774.02$ |
| $M_{\tilde{u}}$ | $5627.92, 6018.85, 5627.87, 6018.64, 32265.13, 36456.04$ |
| $M_{\tilde{d}}$ | $1701.72, 5628.53, 1702.77, 5628.56, 36439.70, 40159.71$ |
| $M_A$ | $305894.52$ |
| $M_{H^\pm}$ | $305894.54$ |
| $M_{H^0}$ | $305894.52$ |
| $M_{h^0}$ | $128.29$ |

Table 39: I-4-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | $350.89$ |
| $M_{\chi^\pm}$ | $302.21, 106276.46$ |
| $M_{\chi^0}$ | $104.22, 302.21, 106276.43, 106276.44$ |
| $M_{\tilde{\nu}}$ | $8173.67, 8173.76, 21295.664$ |
| $M_{\tilde{e}}$ | $89.25, 117.38, 8174.09, 8174.44, 21289.49, 27784.15$ |
| $M_{\tilde{u}}$ | $5627.05, 5648.66, 6018.00, 6018.22, 32278.73, 36462.50$ |
| $M_{\tilde{d}}$ | $1699.10, 1700.14, 5627.66, 5649.32, 36446.20, 40166.59$ |
| $M_A$ | $306124.85$ |
| $M_{H^\pm}$ | $306124.86$ |
| $M_{H^0}$ | $306124.84$ |
| $M_{h^0}$ | $128.38$ |

Table 40: I-4-f: Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

| Parameter | $Value$ | Field $[SU(3), SU(2), Y]$ | $Masses$ $(Units\ of 10^{16} GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0050 | $A[1,1,4]$ | 948.38 |
| $\chi_Z$ | 0.0038 | $B[6,2,5/3]$ | 0.2867 |
| $h_{11}/10^{-6}$ | 0.0200 | $C[8,2,1]$ | 54.37, 436.41, 458.35 |
| $h_{22}/10^{-4}$ | 0.0348 | $D[3,2,7/3]$ | 52.04, 521.13, 532.76 |
| $h_{33}$ | 0.0027 | $E[3,2,1/3]$ | 0.47, 38.02, 38.02 |
| $f_{11}/10^{-6}$ | $0.0796 - 0.1364i$ | | 43.611, 563.34, 624.40 |
| $f_{12}/10^{-6}$ | $-2.0359 - 0.0639i$ | $F[1,1,2]$ | 9.05, 9.05 |
| $f_{13}/10^{-5}$ | $0.0591 + 0.0506i$ | | 36.80, 477.20 |
| $f_{22}/10^{-5}$ | $6.5418 - 4.7649i$ | $G[1,1,0]$ | 0.072, 0.56, 0.77 |
| $f_{23}/10^{-4}$ | $1.9886 + 2.3416i$ | | 0.767, 32.75, 32.92 |
| $f_{33}/10^{-3}$ | $-1.0141 + 0.4463i$ | $h[1,2,1]$ | 1.220, 30.91, 50.43 |
| $g_{12}/10^{-4}$ | $0.0574 + 0.1266i$ | | 793.49, 821.05 |
| $g_{13}/10^{-5}$ | $-0.0387 + 1.6989i$ | $I[3,1,10/3]$ | 0.98 |
| $g_{23}/10^{-4}$ | $6.4300 + 5.9592i$ | $J[3,1,4/3]$ | 1.093, 20.78, 20.78 |
| $\lambda/10^{-2}$ | $-0.3569 - 0.8704i$ | | 64.58, 550.84 |
| $\eta$ | $-10.3588 + 2.2825i$ | $K[3,1,8/3]$ | 74.44, 677.26 |
| $\rho$ | $0.6169 - 2.2892i$ | $L[6,1,2/3]$ | 36.04, 1081.79 |
| $k$ | $0.0102 - 0.0863i$ | $M[6,1,8/3]$ | 1094.90 |
| $\zeta$ | $1.5786 + 0.4937i$ | $N[6,1,4/3]$ | 1091.44 |
| $\bar{\zeta}$ | $1.0902 + 0.4822i$ | $O[1,3,2]$ | 2107.18 |
| $m/10^{16}GeV$ | 0.03 | $P[3,3,2/3]$ | 21.00, 1627.16 |
| $m_o/10^{16}GeV$ | $-31.348e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 0.808 |
| $\gamma$ | 3.89 | $R[8,1,0]$ | 0.32, 1.22 |
| $\bar{\gamma}$ | $-2.8834$ | $S[1,3,0]$ | 1.3822 |
| $x$ | $0.9453 + 0.6806i$ | $t[3,1,2/3]$ | 0.90, 29.04, 70.15, 120.87 |
| $\Delta_X$ | 1.36 | | 348.42, 478.70, 10025.59 |
| $\Delta_G$ | $-0.114$ | $U[3,3,4/3]$ | 1.164 |
| $\Delta\alpha_3(M_Z)$ | $-0.010$ | $V[1,2,3]$ | 0.830 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | 0.00, 15.70, 660.27 | $W[6,3,2/3]$ | 1241.12 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | 0.2212, 725.65, 30521.19 | $X[3,2,5/3]$ | 0.280, 40.637, 40.637 |
| $M_\nu(meV)$ | 2.90, 7.98, 43.17 | $Y[6,2,1/3]$ | 0.35 |
| $\{\text{Evals[f]}\}/10^{-7}$ | 0.09, 283.52, 11924.94 | $Z[8,1,2]$ | 1.21 |
| Soft parameters | $\mathrm{m}_{\frac{1}{2}} = -170.786$ | $\mathrm{m}_0 = 1582.439$ | $\mathrm{A}_0 = -7.8096 \times 10^4$ |
| at $M_X$ | $\mu = 6.8314 \times 10^4$ | $\mathrm{B} = -3.0341 \times 10^9$ | $\tan\beta = 51.5000$ |
| | $\mathrm{M}^2_{\bar{\mathrm{H}}} = -3.4294 \times 10^9$ | $\mathrm{M}^2_{\mathrm{H}} = -3.4202 \times 10^9$ | $\mathrm{R}_{\frac{b\tau}{s\mu}} = 2.4959$ |
| $Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$ | $4.4479 \times 10^{-23}\,\mathrm{GeV}^{-1}$ | | |

Table 41: II-3-a : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.100774 | 0.802496 | 2.100767 | −0.000009 |
| $y_c/10^{-3}$ | 1.023520 | 0.168881 | 1.023538 | 0.000104 |
| $y_t$ | 0.393877 | 0.015755 | 0.393880 | 0.000182 |
| $y_d/10^{-5}$ | 7.305382 | 4.259037 | 7.317435 | 0.002830 |
| $y_s/10^{-3}$ | 1.384369 | 0.653422 | 1.382982 | −0.002123 |
| $y_b$ | 0.516686 | 0.268160 | 0.516774 | 0.000329 |
| $y_e/10^{-4}$ | 1.317579 | 0.197637 | 1.317708 | 0.000656 |
| $y_\mu/10^{-2}$ | 2.780859 | 0.417129 | 2.781086 | 0.000546 |
| $y_\tau$ | 0.583110 | 0.110791 | 0.582736 | −0.003378 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | −0.0001 |
| $\sin\theta^q_{13}/10^{-4}$ | 28.7161 | 5.000000 | 28.7167 | 0.0001 |
| $\sin\theta^q_{23}/10^{-3}$ | 33.7880 | 1.300000 | 33.7881 | 0.0001 |
| $\delta^q$ | 60.0207 | 14.000000 | 60.0113 | −0.0007 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.5328 | 0.586474 | 5.5328 | 0.0000 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.8000 | 0.359999 | 1.8000 | 0.0000 |
| $\sin^2\theta^L_{12}$ | 0.2873 | 0.057461 | 0.2873 | 0.0000 |
| $\sin^2\theta^L_{23}$ | 0.4569 | 0.137073 | 0.4569 | 0.0000 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 6.88 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.057990 | 0.057995 | 0.058002 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.055908 | 0.055913 | 0.055920 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.064334 | 0.064339 | 0.064345 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.070580 | 0.070585 | 0.070592 | |
| $Eigenvalues(\Delta_Q)$ | 0.054281 | 0.054286 | 0.054292 | |
| $Eigenvalues(\Delta_L)$ | 0.064789 | 0.064794 | 0.064799 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 78.095163 | 63.896767 | | |
| $\alpha_1$ | $0.7879 - 0.0000i$ | $\bar{\alpha}_1$ | $0.8695 - 0.0000i$ | |
| $\alpha_2$ | $0.0649 + 0.0142i$ | $\bar{\alpha}_2$ | $0.0583 + 0.0634i$ | |
| $\alpha_3$ | $-0.0413 - 0.0387i$ | $\bar{\alpha}_3$ | $-0.0548 - 0.0158i$ | |
| $\alpha_4$ | $-0.4579 + 0.2315i$ | $\bar{\alpha}_4$ | $0.3691 - 0.0278i$ | |
| $\alpha_5$ | $0.1505 + 0.0742i$ | $\bar{\alpha}_5$ | $0.0719 + 0.0175i$ | |
| $\alpha_6$ | $0.0984 - 0.2655i$ | $\bar{\alpha}_6$ | $0.1081 - 0.2814i$ | |

Table 42: II-3-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0050$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m+\Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.62793 | 2.90404 |
| $m_s/10^{-3}$ | 55.00000 | 11.86773 | 54.92429 |
| $m_b$ | 2.90000 | 3.08544 | 2.90008 |
| $m_e/10^{-3}$ | 0.48657 | 0.48023 | 0.48631 |
| $m_\mu$ | 0.10272 | 0.10130 | 0.10265 |
| $m_\tau$ | 1.74624 | 1.74579 | 1.74474 |
| $m_u/10^{-3}$ | 1.27000 | 1.09336 | 1.26989 |
| $m_c$ | 0.61900 | 0.53271 | 0.61905 |
| $m_t$ | 172.50000 | 149.44924 | 171.97519 |

Table 43: II-3-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} =$ 0.0038.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 43.51 | $M_{\tilde{\bar{u}}_1}$ | 205.86 |
| $M_2$ | 144.40 | $M_{\tilde{\bar{u}}_2}$ | 200.37 |
| $M_3$ | 89.92 | $M_{\tilde{\bar{u}}_3}$ | 15100.44 |
| $M_{\tilde{\bar{l}}_1}$ | 2153.08 | $A_{11}^{0(l)}$ | $-47638.60$ |
| $M_{\tilde{\bar{l}}_2}$ | 2072.60 | $A_{22}^{0(l)}$ | $-47572.19$ |
| $M_{\tilde{\bar{l}}_3}$ | 12763.49 | $A_{33}^{0(l)}$ | $-28011.67$ |
| $M_{\tilde{L}_1}$ | 2923.85 | $A_{11}^{0(u)}$ | $-56421.60$ |
| $M_{\tilde{L}_2}$ | 2894.78 | $A_{22}^{0(u)}$ | $-56421.22$ |
| $M_{\tilde{L}_3}$ | 9407.34 | $A_{33}^{0(u)}$ | $-27834.22$ |
| $M_{\tilde{\bar{d}}_1}$ | 455.68 | $A_{11}^{0(d)}$ | $-47530.07$ |
| $M_{\tilde{\bar{d}}_2}$ | 455.15 | $A_{22}^{0(d)}$ | $-47529.64$ |
| $M_{\tilde{\bar{d}}_3}$ | 20922.14 | $A_{33}^{0(d)}$ | $-17493.87$ |
| $M_{\tilde{Q}_1}$ | 2445.28 | $\tan\beta$ | 51.50 |
| $M_{\tilde{Q}_2}$ | 2445.00 | $\mu(M_Z)$ | 52905.86 |
| $M_{\tilde{Q}_3}$ | 18336.34 | $B(M_Z)$ | $4.0145 \times 10^8$ |
| $M_{\bar{H}}^2$ | $-2.6746 \times 10^9$ | $M_H^2$ | $-3.0610 \times 10^9$ |

Table 44: II-3-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 89.92 |
| $M_{\chi^{\pm}}$ | $144.39, 52905.99$ |
| $M_{\chi^0}$ | $43.51, 144.39, 52905.94, 52905.95$ |
| $M_{\tilde{\nu}}$ | $2923.095, 2894.022, 9407.108$ |
| $M_{\tilde{e}}$ | $2153.55, 2924.25, 2068.45, 2898.51, 9391.01, 12775.68$ |
| $M_{\tilde{u}}$ | $202.54, 2444.65, 196.56, 2444.41, 15095.44, 18341.55$ |
| $M_{\tilde{d}}$ | $456.42, 2446.04, 455.69, 2445.80, 18317.31, 20938.90$ |
| $M_A$ | 143814.30 |
| $M_{H^{\pm}}$ | 143814.33 |
| $M_{H^0}$ | 143814.29 |
| $M_{h^0}$ | 123.80 |

Table 45: II-3-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 90.00 |
| $M_{\chi^{\pm}}$ | $144.43, 52903.84$ |
| $M_{\chi^0}$ | $43.52, 144.43, 52903.79, 52903.80$ |
| $M_{\tilde{\nu}}$ | $2894.06, 2923.13, 9407.538$ |
| $M_{\tilde{e}}$ | $2065.45, 2153.46, 2899.51, 2924.33, 9392.27, 12775.95$ |
| $M_{\tilde{u}}$ | $198.31, 204.25, 2444.78, 2451.67, 15094.69, 18340.05$ |
| $M_{\tilde{d}}$ | $456.03, 456.94, 2446.17, 2453.03, 18315.87, 20938.32$ |
| $M_A$ | 143872.09 |
| $M_{H^{\pm}}$ | 143872.11 |
| $M_{H^0}$ | 143872.08 |
| $M_{h^0}$ | 123.91 |

Table 46: II-3-f: Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 6 for explanation.

| Parameter | $Value$ | Field $[SU(3), SU(2), Y]$ | $Masses$ $(Units\ of 10^{16} GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0061 | $A[1,1,4]$ | 137.29 |
| $\chi_Z$ | 0.0179 | $B[6,2,5/3]$ | 0.3082 |
| $h_{11}/10^{-6}$ | 0.0277 | $C[8,2,1]$ | $4.55, 166.93, 168.04$ |
| $h_{22}/10^{-4}$ | $-0.0081$ | $D[3,2,7/3]$ | $3.93, 95.60, 117.42$ |
| $h_{33}$ | 0.0034 | $E[3,2,1/3]$ | $0.37, 3.21, 5.35$ |
| $f_{11}/10^{-6}$ | $0.0242 + 0.1635i$ | | $5.346, 138.97, 160.78$ |
| $f_{12}/10^{-6}$ | $-1.6733 + 0.4769i$ | $F[1,1,2]$ | $0.96, 0.96$ |
| $f_{13}/10^{-5}$ | $0.3397 + 0.1181i$ | | $2.66, 69.79$ |
| $f_{22}/10^{-5}$ | $5.3022 - 7.2813i$ | $G[1,1,0]$ | $0.087, 0.26, 0.26$ |
| $f_{23}/10^{-4}$ | $2.0313 + 3.1221i$ | | $0.698, 14.94, 15.31$ |
| $f_{33}/10^{-3}$ | $-1.0321 + 0.6903i$ | $h[1,2,1]$ | $1.233, 3.11, 4.16$ |
| $g_{12}/10^{-4}$ | $0.0375 - 0.1054i$ | | $126.31, 161.82$ |
| $g_{13}/10^{-5}$ | $-1.5603 - 1.8606i$ | $I[3,1,10/3]$ | 1.27 |
| $g_{23}/10^{-4}$ | $3.3223 + 6.3071i$ | $J[3,1,4/3]$ | $1.249, 2.62, 2.62$ |
| $\lambda/10^{-2}$ | $-7.6198 - 1.9488i$ | | $6.73, 105.91$ |
| $\eta$ | $-14.4375 + 1.9014i$ | $K[3,1,8/3]$ | $7.33, 124.51$ |
| $\rho$ | $0.3809 - 1.5817i$ | $L[6,1,2/3]$ | $3.76, 205.78$ |
| $k$ | $0.0229 - 0.0773i$ | $M[6,1,8/3]$ | 210.07 |
| $\zeta$ | $0.9769 + 0.4323i$ | $N[6,1,4/3]$ | 203.97 |
| $\bar{\zeta}$ | $1.4935 - 0.4959i$ | $O[1,3,2]$ | 386.98 |
| $m/10^{16}GeV$ | 0.03 | $P[3,3,2/3]$ | $1.11, 339.57$ |
| $m_o/10^{16}GeV$ | $-3.093e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 1.117 |
| $\gamma$ | 4.63 | $R[8,1,0]$ | $0.33, 1.33$ |
| $\bar{\gamma}$ | $-2.3784$ | $S[1,3,0]$ | 1.5325 |
| $x$ | $0.8833 + 0.4731i$ | $t[3,1,2/3]$ | $0.94, 2.51, 7.17, 9.19$ |
| $\Delta_X$ | 0.39 | | $134.01, 142.61, 3033.21$ |
| $\Delta_G$ | 20.063 | $U[3,3,4/3]$ | 1.351 |
| $\Delta\alpha_3(M_Z)$ | $-0.012$ | $V[1,2,3]$ | 0.851 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.01, 7.43, 250.87$ | $W[6,3,2/3]$ | 320.01 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $5.8487, 6146.10, 207404.09$ | $X[3,2,5/3]$ | $0.284, 5.796, 5.796$ |
| $M_\nu(meV)$ | $4.27, 8.56, 43.09$ | $Y[6,2,1/3]$ | 0.35 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.38, 400.40, 13511.64$ | $Z[8,1,2]$ | 1.33 |
| Soft parameters at $M_X$ | $m_{\frac{1}{2}} = -118.386$ | $m_0 = 1773.201$ | $A_0 = -5.9163 \times 10^4$ |
| | $\mu = 5.4671 \times 10^4$ | $B = -1.7502 \times 10^9$ | $\tan\beta = 51.0000$ |
| | $M^2_{\bar{H}} = -2.5185 \times 10^9$ | $M^2_H = -2.4818 \times 10^9$ | $R_{\frac{b\tau}{s\mu}} = 3.5550$ |
| $Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$ | $5.4994 \times 10^{-22}\ \text{GeV}^{-1}$ | | |

Table 47: II-4-a: Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.169056 | 0.828579 | 2.169068 | 0.000015 |
| $y_c/10^{-3}$ | 1.057321 | 0.174458 | 1.057338 | 0.000102 |
| $y_t$ | 0.390644 | 0.015626 | 0.390643 | −0.000072 |
| $y_d/10^{-5}$ | 6.611935 | 3.854758 | 6.614046 | 0.000548 |
| $y_s/10^{-3}$ | 1.255361 | 0.592530 | 1.256890 | 0.002581 |
| $y_b$ | 0.452379 | 0.234785 | 0.453431 | 0.004481 |
| $y_e/10^{-4}$ | 1.247646 | 0.187147 | 1.247511 | −0.000719 |
| $y_\mu/10^{-2}$ | 2.635432 | 0.395315 | 2.635821 | 0.000983 |
| $y_\tau$ | 0.542972 | 0.103165 | 0.542667 | −0.002953 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | 0.0001 |
| $\sin\theta^q_{13}/10^{-4}$ | 29.1115 | 5.000000 | 29.1102 | −0.0003 |
| $\sin\theta^q_{23}/10^{-3}$ | 34.2526 | 1.300000 | 34.2523 | −0.0002 |
| $\delta^q$ | 60.0206 | 14.000000 | 60.0202 | 0.0000 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.5012 | 0.583126 | 5.5012 | 0.0000 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.7832 | 0.356636 | 1.7832 | 0.0000 |
| $\sin^2\theta^L_{12}$ | 0.2882 | 0.057639 | 0.2882 | 0.0000 |
| $\sin^2\theta^L_{23}$ | 0.4601 | 0.138033 | 0.4601 | −0.0001 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 6.89 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.008011 | 0.008014 | 0.008014 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.004482 | 0.004484 | 0.004485 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.002764 | 0.002765 | 0.002767 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.007822 | 0.007824 | 0.007825 | |
| $Eigenvalues(\Delta_Q)$ | 0.011230 | 0.011232 | 0.011232 | |
| $Eigenvalues(\Delta_L)$ | 0.007511 | 0.007513 | 0.007513 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 58.805443 | 52.078952 | | |
| $\alpha_1$ | $0.7626 + 0.0000i$ | $\bar{\alpha}_1$ | $0.8079 + 0.0000i$ | |
| $\alpha_2$ | $0.0468 + 0.0455i$ | $\bar{\alpha}_2$ | $-0.0039 + 0.0795i$ | |
| $\alpha_3$ | $0.0221 - 0.0516i$ | $\bar{\alpha}_3$ | $-0.0018 - 0.0322i$ | |
| $\alpha_4$ | $-0.4860 - 0.1021i$ | $\bar{\alpha}_4$ | $0.2917 + 0.1584i$ | |
| $\alpha_5$ | $0.2131 + 0.0060i$ | $\bar{\alpha}_5$ | $0.1288 - 0.0330i$ | |
| $\alpha_6$ | $0.0562 - 0.3402i$ | $\bar{\alpha}_6$ | $0.0759 - 0.4541i$ | |

Table 48: II-4-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0061$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.60092 | 2.89886 |
| $m_s/10^{-3}$ | 55.00000 | 11.41936 | 55.02309 |
| $m_b$ | 2.90000 | 2.98616 | 2.90724 |
| $m_e/10^{-3}$ | 0.48657 | 0.48126 | 0.48642 |
| $m_\mu$ | 0.10272 | 0.10163 | 0.10271 |
| $m_\tau$ | 1.74624 | 1.74575 | 1.74469 |
| $m_u/10^{-3}$ | 1.27000 | 1.12845 | 1.25425 |
| $m_c$ | 0.61900 | 0.55007 | 0.61140 |
| $m_t$ | 172.50000 | 150.12439 | 172.45759 |

Table 49: II-4-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} =$ 0.0179.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 35.00 | $M_{\tilde{\bar{u}}_1}$ | 1355.00 |
| $M_2$ | 110.00 | $M_{\tilde{\bar{u}}_2}$ | 1354.97 |
| $M_3$ | 82.43 | $M_{\tilde{\bar{u}}_3}$ | 15017.39 |
| $M_{\tilde{\bar{l}}_1}$ | 1668.31 | $A_{11}^{0(l)}$ | $-37671.35$ |
| $M_{\tilde{\bar{l}}_2}$ | 1686.32 | $A_{22}^{0(l)}$ | $-37622.42$ |
| $M_{\tilde{\bar{l}}_3}$ | 12622.76 | $A_{33}^{0(l)}$ | $-23300.14$ |
| $M_{\tilde{L}_1}$ | 2873.66 | $A_{11}^{0(u)}$ | $-42600.14$ |
| $M_{\tilde{L}_2}$ | 2878.88 | $A_{22}^{0(u)}$ | $-42599.84$ |
| $M_{\tilde{L}_3}$ | 9306.38 | $A_{33}^{0(u)}$ | $-21238.50$ |
| $M_{\tilde{\bar{d}}_1}$ | 362.45 | $A_{11}^{0(d)}$ | $-37604.10$ |
| $M_{\tilde{\bar{d}}_2}$ | 363.78 | $A_{22}^{0(d)}$ | $-37603.79$ |
| $M_{\tilde{\bar{d}}_3}$ | 17343.17 | $A_{33}^{0(d)}$ | $-16155.25$ |
| $M_{\tilde{Q}_1}$ | 2110.14 | $\tan\beta$ | 51.00 |
| $M_{\tilde{Q}_2}$ | 2110.24 | $\mu(M_Z)$ | 44372.34 |
| $M_{\tilde{Q}_3}$ | 16267.76 | $B(M_Z)$ | $2.6446 \times 10^8$ |
| $M_{\bar{H}}^2$ | $-1.9734 \times 10^9$ | $M_H^2$ | $-2.1327 \times 10^9$ |

Table 50: II-4-d : Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | $82.43$ |
| $M_{\chi^{\pm}}$ | $109.99, 44372.49$ |
| $M_{\chi^0}$ | $35.00, 109.99, 44372.43, 44372.44$ |
| $M_{\tilde{\nu}}$ | $2872.898, 2878.121, 9306.146$ |
| $M_{\tilde{e}}$ | $1668.92, 2874.07, 1683.95, 2881.03, 9294.77, 12631.49$ |
| $M_{\tilde{u}}$ | $1354.50, 2109.42, 1354.39, 2109.57, 15008.78, 16276.95$ |
| $M_{\tilde{d}}$ | $363.38, 2111.02, 364.49, 2111.16, 16229.75, 17378.88$ |
| $M_A$ | $116157.09$ |
| $M_{H^{\pm}}$ | $116157.12$ |
| $M_{H^0}$ | $116157.08$ |
| $M_{h^0}$ | $127.41$ |

Table 51: II-4-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | $82.49$ |
| $M_{\chi^{\pm}}$ | $110.02, 44370.87$ |
| $M_{\chi^0}$ | $35.01, 110.02, 44370.81, 44370.82$ |
| $M_{\tilde{\nu}}$ | $2872.91, 2878.14, 9306.355$ |
| $M_{\tilde{e}}$ | $1668.80, 1681.53, 2874.11, 2881.63, 9295.61, 12631.53$ |
| $M_{\tilde{u}}$ | $1354.49, 1354.60, 2109.47, 2123.10, 15008.73, 16275.15$ |
| $M_{\tilde{d}}$ | $363.58, 364.64, 2111.08, 2124.64, 16228.06, 17378.66$ |
| $M_A$ | $116199.01$ |
| $M_{H^{\pm}}$ | $116199.03$ |
| $M_{H^0}$ | $116199.00$ |
| $M_{h^0}$ | $127.43$ |

Table 52: II-4-f: Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

| Parameter | $Value$ | Field $[SU(3),SU(2),Y]$ | $Masses$ $(Units\ of 10^{16}GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0143 | $A[1,1,4]$ | 16007.39 |
| $\chi_Z$ | 0.0066 | $B[6,2,5/3]$ | 0.3294 |
| $h_{11}/10^{-6}$ | $-0.4946$ | $C[8,2,1]$ | 671.99, 2566.01, 3875.58 |
| $h_{22}/10^{-4}$ | 0.3413 | $D[3,2,7/3]$ | 1412.23, 9043.81, 10626.05 |
| $h_{33}$ | 0.0143 | $E[3,2,1/3]$ | 0.74, 488.82, 488.82 |
| $f_{11}/10^{-6}$ | $-0.1019-0.0216i$ | | 1600.508, 4925.64, 7365.53 |
| $f_{12}/10^{-6}$ | $1.6275-0.0739i$ | $F[1,1,2]$ | 186.39, 186.39 |
| $f_{13}/10^{-5}$ | $-0.6414-0.0365i$ | | 1045.25, 8230.98 |
| $f_{22}/10^{-5}$ | $-1.8755+0.1518i$ | $G[1,1,0]$ | 0.072, 0.60, 3.18 |
| $f_{23}/10^{-4}$ | $1.0186-0.0082i$ | | 3.179, 111.33, 111.60 |
| $f_{33}/10^{-3}$ | $-0.4301+0.0171i$ | $h[1,2,1]$ | 1.448, 865.06, 1334.27 |
| $g_{12}/10^{-4}$ | $-2.8487-0.6757i$ | | 9342.01, 12728.76 |
| $g_{13}/10^{-5}$ | $-57.2472-43.8940i$ | $I[3,1,10/3]$ | 0.84 |
| $g_{23}/10^{-4}$ | $139.1403+111.6681i$ | $J[3,1,4/3]$ | 1.001, 327.80, 327.80 |
| $\lambda/10^{-2}$ | $0.0590+0.0380i$ | | 585.72, 6966.80 |
| $\eta$ | $-8.6784-0.3384i$ | $K[3,1,8/3]$ | 1019.21, 10139.63 |
| $\rho$ | $1.2622-0.6215i$ | $L[6,1,2/3]$ | 816.96, 12975.96 |
| $k$ | $-0.0720-0.0067i$ | $M[6,1,8/3]$ | 13542.30 |
| $\zeta$ | $0.6375+1.3223i$ | $N[6,1,4/3]$ | 12916.72 |
| $\bar{\zeta}$ | $3.7447-0.5884i$ | $O[1,3,2]$ | 23941.32 |
| $m/10^{16}GeV$ | 0.04 | $P[3,3,2/3]$ | 1079.14, 16853.68 |
| $m_o/10^{16}GeV$ | $-882.138e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 0.588 |
| $\gamma$ | 0.99 | $R[8,1,0]$ | 0.38, 1.25 |
| $\bar{\gamma}$ | $-1.3849$ | $S[1,3,0]$ | 1.3766 |
| $x$ | $0.8562+1.0667i$ | $t[3,1,2/3]$ | 0.71, 234.75, 767.39, 1155.96 |
| $\Delta_X$ | 2.82 | | 4209.99, 4493.21, 70106.74 |
| $\Delta_G$ | $-20.827$ | $U[3,3,4/3]$ | 1.100 |
| $\Delta\alpha_3(M_Z)$ | $-0.011$ | $V[1,2,3]$ | 0.979 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | 0.00, 12.52, 1043.05 | $W[6,3,2/3]$ | 9640.83 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | 0.0015, 5.50, 458.44 | $X[3,2,5/3]$ | 0.359, 511.842, 511.842 |
| $M_\nu(meV)$ | 2.40, 7.72, 42.52 | $Y[6,2,1/3]$ | 0.43 |
| $\{\text{Evals[f]}\}/10^{-7}$ | 0.01, 54.51, 4543.19 | $Z[8,1,2]$ | 1.24 |
| Soft parameters at $M_X$ | $m_{\frac{1}{2}}=-397.337$<br>$\mu=1.4564\times10^{5}$<br>$M^2_{\bar{H}}=-1.4614\times10^{10}$ | $m_0=2547.245$<br>$B=-1.4999\times10^{10}$<br>$M^2_H=-1.4329\times10^{10}$ | $A_0=-1.8262\times10^{5}$<br>$\tan\beta=50.0000$<br>$R_{\frac{b\tau}{s\mu}}=0.8775$ |
| $Max(\|L_{ABCD}\|,\|R_{ABCD}\|)$ | $7.8076\times10^{-22}\ GeV^{-1}$ | | |

Table 53: III-2-a : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.114293 | 0.807660 | 2.114551 | 0.000320 |
| $y_c/10^{-3}$ | 1.030627 | 0.170053 | 1.030427 | −0.001178 |
| $y_t$ | 0.388741 | 0.015550 | 0.388744 | 0.000205 |
| $y_d/10^{-5}$ | 5.990999 | 3.492753 | 6.024454 | 0.009578 |
| $y_s/10^{-3}$ | 1.132757 | 0.534661 | 1.128624 | −0.007730 |
| $y_b$ | 0.546568 | 0.283669 | 0.545182 | −0.004887 |
| $y_e/10^{-4}$ | 1.282003 | 0.192301 | 1.282314 | 0.001614 |
| $y_\mu/10^{-2}$ | 2.593678 | 0.389052 | 2.594359 | 0.001750 |
| $y_\tau$ | 0.567317 | 0.107790 | 0.566957 | −0.003338 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | −0.0004 |
| $\sin\theta^q_{13}/10^{-4}$ | 28.5695 | 5.000000 | 28.5621 | −0.0015 |
| $\sin\theta^q_{23}/10^{-3}$ | 33.6158 | 1.300000 | 33.6166 | 0.0006 |
| $\delta^q$ | 60.0207 | 14.000000 | 59.9838 | −0.0026 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.3891 | 0.571243 | 5.3891 | −0.0001 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.7482 | 0.349632 | 1.7482 | 0.0001 |
| $\sin^2\theta^L_{12}$ | 0.2880 | 0.057602 | 0.2880 | −0.0003 |
| $\sin^2\theta^L_{23}$ | 0.4594 | 0.137826 | 0.4595 | 0.0009 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 4.86 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.117591 | 0.117741 | 0.123579 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.117423 | 0.117569 | 0.123393 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.146546 | 0.146698 | 0.152632 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.147096 | 0.147254 | 0.153189 | |
| $Eigenvalues(\Delta_Q)$ | 0.104428 | 0.104576 | 0.110528 | |
| $Eigenvalues(\Delta_L)$ | 0.133889 | 0.134036 | 0.139953 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 14.257424 | 11.302210 | | |
| $\alpha_1$ | $0.7967 + 0.0000i$ | $\bar{\alpha}_1$ | $0.8979 + 0.0000i$ | |
| $\alpha_2$ | $0.0175 - 0.0129i$ | $\bar{\alpha}_2$ | $0.0271 - 0.0023i$ | |
| $\alpha_3$ | $-0.0376 - 0.0062i$ | $\bar{\alpha}_3$ | $-0.0325 + 0.0015i$ | |
| $\alpha_4$ | $0.0013 + 0.5966i$ | $\bar{\alpha}_4$ | $0.0147 - 0.4306i$ | |
| $\alpha_5$ | $0.0485 - 0.0181i$ | $\bar{\alpha}_5$ | $0.0250 - 0.0387i$ | |
| $\alpha_6$ | $-0.0286 - 0.0628i$ | $\bar{\alpha}_6$ | $-0.0382 - 0.0523i$ | |

Table 54: III-2-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0143$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m + \Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.52468 | 2.91331 |
| $m_s/10^{-3}$ | 55.00000 | 9.82943 | 54.75545 |
| $m_b$ | 2.90000 | 3.24471 | 2.89957 |
| $m_e/10^{-3}$ | 0.48657 | 0.47433 | 0.48705 |
| $m_\mu$ | 0.10272 | 0.09592 | 0.10272 |
| $m_\tau$ | 1.74624 | 1.74370 | 1.74662 |
| $m_u/10^{-3}$ | 1.27000 | 1.10824 | 1.26917 |
| $m_c$ | 0.61900 | 0.54005 | 0.61848 |
| $m_t$ | 172.50000 | 148.32422 | 172.51404 |

Table 55: III-2-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} =$ 0.0066.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 98.92 | $M_{\tilde{\bar{u}}_1}$ | 3553.79 |
| $M_2$ | 341.43 | $M_{\tilde{\bar{u}}_2}$ | 3550.53 |
| $M_3$ | 232.18 | $M_{\tilde{\bar{u}}_3}$ | 20687.29 |
| $M_{\tilde{\bar{l}}_1}$ | 1868.65 | $A_{11}^{0(l)}$ | $-110036.65$ |
| $M_{\tilde{\bar{l}}_2}$ | 204.32 | $A_{22}^{0(l)}$ | $-109904.91$ |
| $M_{\tilde{\bar{l}}_3}$ | 11397.10 | $A_{33}^{0(l)}$ | $-66704.24$ |
| $M_{\tilde{L}_1}$ | 6369.22 | $A_{11}^{0(u)}$ | $-132899.38$ |
| $M_{\tilde{L}_2}$ | 6233.07 | $A_{22}^{0(u)}$ | $-132898.56$ |
| $M_{\tilde{L}_3}$ | 10527.00 | $A_{33}^{0(u)}$ | $-66235.28$ |
| $M_{\tilde{\bar{d}}_1}$ | 712.29 | $A_{11}^{0(d)}$ | $-109811.44$ |
| $M_{\tilde{\bar{d}}_2}$ | 705.83 | $A_{22}^{0(d)}$ | $-109810.71$ |
| $M_{\tilde{\bar{d}}_3}$ | 41913.64 | $A_{33}^{0(d)}$ | $-37711.03$ |
| $M_{\tilde{Q}_1}$ | 5387.22 | $\tan\beta$ | 50.00 |
| $M_{\tilde{Q}_2}$ | 5385.73 | $\mu(M_Z)$ | 111532.64 |
| $M_{\tilde{Q}_3}$ | 33269.50 | $B(M_Z)$ | $2.1254 \times 10^9$ |
| $M_{\bar{H}}^2$ | $-1.1842 \times 10^{10}$ | $M_H^2$ | $-1.3659 \times 10^{10}$ |

Table 56: III-2-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | $232.18$ |
| $M_{\chi^\pm}$ | $341.42, 111532.70$ |
| $M_{\chi^0}$ | $98.92, 341.42, 111532.68, 111532.68$ |
| $M_{\tilde{\nu}}$ | $6368.872, 6232.714, 10526.788$ |
| $M_{\tilde{e}}$ | $1869.20, 6369.40, 190.06, 6233.87, 10327.33, 11578.52$ |
| $M_{\tilde{u}}$ | $3553.60, 5386.93, 3550.29, 5385.48, 20684.11, 33272.06$ |
| $M_{\tilde{d}}$ | $712.77, 5387.56, 706.24, 5386.09, 33261.88, 41919.73$ |
| $M_A$ | $326055.13$ |
| $M_{H^\pm}$ | $326055.14$ |
| $M_{H^0}$ | $326055.11$ |
| $M_{h^0}$ | $112.76$ |

Table 57: III-2-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | $232.36$ |
| $M_{\chi^\pm}$ | $341.50, 111528.02$ |
| $M_{\chi^0}$ | $98.94, 341.50, 111528.00, 111528.00$ |
| $M_{\tilde{\nu}}$ | $6232.83, 6368.97, 10529.123$ |
| $M_{\tilde{e}}$ | $342.67, 1868.73, 6223.30, 6369.45, 10391.14, 11540.07$ |
| $M_{\tilde{u}}$ | $3550.99, 3554.30, 5367.30, 5387.36, 20677.92, 33271.88$ |
| $M_{\tilde{d}}$ | $709.00, 715.54, 5367.93, 5387.99, 33261.71, 41917.16$ |
| $M_A$ | $326179.32$ |
| $M_{H^\pm}$ | $326179.33$ |
| $M_{H^0}$ | $326179.31$ |
| $M_{h^0}$ | $112.78$ |

Table 58: III-2-f: Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

| Parameter | $Value$ | Field $[SU(3),SU(2),Y]$ | $Masses$ $(Units\ of 10^{16} GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0031 | $A[1,1,4]$ | 689.26 |
| $\chi_Z$ | 0.0012 | $B[6,2,5/3]$ | 0.2457 |
| $h_{11}/10^{-6}$ | 0.0205 | $C[8,2,1]$ | $38.27, 330.90, 346.79$ |
| $h_{22}/10^{-4}$ | 0.0364 | $D[3,2,7/3]$ | $36.96, 399.26, 408.77$ |
| $h_{33}$ | 0.0027 | $E[3,2,1/3]$ | $0.40, 27.84, 27.84$ |
| $f_{11}/10^{-6}$ | $0.0774 - 0.1295i$ | | $30.102, 433.12, 477.24$ |
| $f_{12}/10^{-6}$ | $-2.0249 - 0.0687i$ | $F[1,1,2]$ | $6.43, 6.43$ |
| $f_{13}/10^{-5}$ | $0.0636 + 0.0485i$ | | $26.27, 346.91$ |
| $f_{22}/10^{-5}$ | $6.5620 - 4.8133i$ | $G[1,1,0]$ | $0.063, 0.48, 0.60$ |
| $f_{23}/10^{-4}$ | $2.0322 + 2.3187i$ | | $0.600, 26.21, 26.37$ |
| $f_{33}/10^{-3}$ | $-1.0357 + 0.4496i$ | $h[1,2,1]$ | $1.019, 21.54, 35.95$ |
| $g_{12}/10^{-4}$ | $0.0555 + 0.1219i$ | | $600.38, 620.06$ |
| $g_{13}/10^{-5}$ | $-0.0665 + 1.8279i$ | $I[3,1,10/3]$ | 0.85 |
| $g_{23}/10^{-4}$ | $6.3795 + 5.6374i$ | $J[3,1,4/3]$ | $0.959, 14.94, 14.94$ |
| $\lambda/10^{-2}$ | $-0.4433 - 1.0332i$ | | $46.51, 403.62$ |
| $\eta$ | $-10.5295 + 2.6182i$ | $K[3,1,8/3]$ | $53.75, 492.39$ |
| $\rho$ | $0.6481 - 2.2964i$ | $L[6,1,2/3]$ | $25.26, 794.20$ |
| $k$ | $0.0149 - 0.0810i$ | $M[6,1,8/3]$ | 802.14 |
| $\zeta$ | $1.6389 + 0.5058i$ | $N[6,1,4/3]$ | 802.65 |
| $\bar{\zeta}$ | $1.0205 + 0.5627i$ | $O[1,3,2]$ | 1583.99 |
| $m/10^{16}GeV$ | 0.03 | $P[3,3,2/3]$ | $15.91, 1232.28$ |
| $m_o/10^{16}GeV$ | $-21.872e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 0.711 |
| $\gamma$ | 3.97 | $R[8,1,0]$ | $0.27, 1.07$ |
| $\bar{\gamma}$ | $-2.9091$ | $S[1,3,0]$ | 1.2049 |
| $x$ | $0.9564 + 0.6718i$ | $t[3,1,2/3]$ | $0.78, 20.40, 50.58, 87.07$ |
| $\Delta_X$ | 1.21 | | $275.21, 368.92, 7729.25$ |
| $\Delta_G$ | 1.734 | $U[3,3,4/3]$ | 1.018 |
| $\Delta\alpha_3(M_Z)$ | $-0.016$ | $V[1,2,3]$ | 0.714 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.01, 13.27, 525.55$ | $W[6,3,2/3]$ | 948.60 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $0.6555, 1070.44, 42380.30$ | $X[3,2,5/3]$ | $0.241, 29.690, 29.690$ |
| $M_\nu(meV)$ | $2.42, 7.81, 43.04$ | $Y[6,2,1/3]$ | 0.30 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.19, 306.30, 12126.94$ | $Z[8,1,2]$ | 1.06 |
| Soft parameters at $M_X$ | $\mathrm{m}_{\frac{1}{2}} = -143.002$ <br> $\mu = 8.8928 \times 10^4$ <br> $\mathrm{M}^2_{\bar{\mathrm{H}}} = -5.8300 \times 10^9$ | $\mathrm{m}_0 = 3347.253$ <br> $\mathrm{B} = -5.4965 \times 10^9$ <br> $\mathrm{M}^2_{\mathrm{H}} = -5.5566 \times 10^9$ | $\mathrm{A}_0 = -1.0946 \times 10^5$ <br> $\tan\beta = 51.5000$ <br> $\mathrm{R}_{\frac{b\tau}{s\mu}} = 2.6676$ |
| $Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$ | $5.5297 \times 10^{-23}\ \mathrm{GeV}^{-1}$ | | |

Table 59: III-3-a : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.121322 | 0.810345 | 2.121299 | −0.000028 |
| $y_c/10^{-3}$ | 1.034062 | 0.170620 | 1.033951 | −0.000648 |
| $y_t$ | 0.392382 | 0.015695 | 0.392383 | 0.000076 |
| $y_d/10^{-5}$ | 7.238271 | 4.219912 | 7.243490 | 0.001237 |
| $y_s/10^{-3}$ | 1.372396 | 0.647771 | 1.371099 | −0.002002 |
| $y_b$ | 0.512784 | 0.266135 | 0.513218 | 0.001628 |
| $y_e/10^{-4}$ | 1.291572 | 0.193736 | 1.291652 | 0.000409 |
| $y_\mu/10^{-2}$ | 2.645649 | 0.396847 | 2.645665 | 0.000038 |
| $y_\tau$ | 0.580184 | 0.110235 | 0.580135 | −0.000444 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | −0.0001 |
| $\sin\theta^q_{13}/10^{-4}$ | 28.7475 | 5.000000 | 28.7473 | 0.0000 |
| $\sin\theta^q_{23}/10^{-3}$ | 33.8250 | 1.300000 | 33.8252 | 0.0002 |
| $\delta^q$ | 60.0207 | 14.000000 | 60.0163 | −0.0003 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.5085 | 0.583902 | 5.5085 | 0.0000 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.7916 | 0.358315 | 1.7916 | 0.0000 |
| $\sin^2\theta^L_{12}$ | 0.2874 | 0.057476 | 0.2874 | 0.0002 |
| $\sin^2\theta^L_{23}$ | 0.4571 | 0.137144 | 0.4571 | −0.0004 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 5.41 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.050027 | 0.050031 | 0.050037 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.047655 | 0.047660 | 0.047665 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.053435 | 0.053439 | 0.053445 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.060550 | 0.060554 | 0.060560 | |
| $Eigenvalues(\Delta_Q)$ | 0.047387 | 0.047391 | 0.047396 | |
| $Eigenvalues(\Delta_L)$ | 0.055538 | 0.055543 | 0.055547 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 78.220842 | 64.106181 | | |
| $\alpha_1$ | $0.7970 + 0.0000i$ | $\bar{\alpha}_1$ | $0.8786 + 0.0000i$ | |
| $\alpha_2$ | $0.0641 + 0.0154i$ | $\bar{\alpha}_2$ | $0.0540 + 0.0624i$ | |
| $\alpha_3$ | $-0.0400 - 0.0369i$ | $\bar{\alpha}_3$ | $-0.0557 - 0.0165i$ | |
| $\alpha_4$ | $-0.4642 + 0.1746i$ | $\bar{\alpha}_4$ | $0.3473 - 0.0021i$ | |
| $\alpha_5$ | $0.1479 + 0.0832i$ | $\bar{\alpha}_5$ | $0.0692 + 0.0216i$ | |
| $\alpha_6$ | $0.1139 - 0.2641i$ | $\bar{\alpha}_6$ | $0.1278 - 0.2750i$ | |

Table 60: III-3-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0031$. Target values, at $M_X$ of the fermion yukawa couplings and mixing parameters, together with the estimated uncertainties, achieved values and pulls. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m+\Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.62377 | 2.90229 |
| $m_s/10^{-3}$ | 55.00000 | 11.80718 | 54.95706 |
| $m_b$ | 2.90000 | 3.08261 | 2.90002 |
| $m_e/10^{-3}$ | 0.48657 | 0.47238 | 0.48647 |
| $m_\mu$ | 0.10272 | 0.09671 | 0.10272 |
| $m_\tau$ | 1.74624 | 1.74579 | 1.74575 |
| $m_u/10^{-3}$ | 1.27000 | 1.10616 | 1.26997 |
| $m_c$ | 0.61900 | 0.53916 | 0.61901 |
| $m_t$ | 172.50000 | 149.41400 | 172.44284 |

Table 61: III-3-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} = 0.0012$.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 98.22 | $M_{\tilde{\bar{u}}_1}$ | 4257.81 |
| $M_2$ | 273.31 | $M_{\tilde{\bar{u}}_2}$ | 4256.95 |
| $M_3$ | 386.79 | $M_{\tilde{\bar{u}}_3}$ | 15648.62 |
| $M_{\tilde{\bar{l}}_1}$ | 1055.61 | $A_{11}^{0(l)}$ | $-66907.20$ |
| $M_{\tilde{\bar{l}}_2}$ | 149.79 | $A_{22}^{0(l)}$ | $-66822.57$ |
| $M_{\tilde{\bar{l}}_3}$ | 12367.95 | $A_{33}^{0(l)}$ | $-39462.25$ |
| $M_{\tilde{L}_1}$ | 5350.98 | $A_{11}^{0(u)}$ | $-79457.41$ |
| $M_{\tilde{L}_2}$ | 5300.00 | $A_{22}^{0(u)}$ | $-79456.87$ |
| $M_{\tilde{L}_3}$ | 10339.21 | $A_{33}^{0(u)}$ | $-39415.43$ |
| $M_{\tilde{\bar{d}}_1}$ | 2146.46 | $A_{11}^{0(d)}$ | $-67055.77$ |
| $M_{\tilde{\bar{d}}_2}$ | 2145.58 | $A_{22}^{0(d)}$ | $-67055.16$ |
| $M_{\tilde{\bar{d}}_3}$ | 25986.72 | $A_{33}^{0(d)}$ | $-25007.87$ |
| $M_{\tilde{Q}_1}$ | 4117.71 | $\tan\beta$ | 51.50 |
| $M_{\tilde{Q}_2}$ | 4117.04 | $\mu(M_Z)$ | 69115.80 |
| $M_{\tilde{Q}_3}$ | 21502.97 | $B(M_Z)$ | $7.5656 \times 10^8$ |
| $M_{\bar{H}}^2$ | $-4.7132 \times 10^9$ | $M_H^2$ | $-5.2004 \times 10^9$ |

Table 62: III-3-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 386.79 |
| $M_{\chi^{\pm}}$ | $273.31, 69115.90$ |
| $M_{\chi^0}$ | $98.22, 273.31, 69115.86, 69115.87$ |
| $M_{\tilde{\nu}}$ | $5350.564, 5299.581, 10338.994$ |
| $M_{\tilde{e}}$ | $1056.57, 5351.20, 141.75, 5300.63, 10298.54, 12402.01$ |
| $M_{\tilde{u}}$ | $4117.34, 4257.65, 4116.48, 4256.98, 15643.84, 21507.38$ |
| $M_{\tilde{d}}$ | $2146.61, 4118.16, 2145.70, 4117.51, 21489.76, 25997.73$ |
| $M_A$ | 197426.75 |
| $M_{H^{\pm}}$ | 197426.76 |
| $M_{H^0}$ | 197426.73 |
| $M_{h^0}$ | 117.60 |

Table 63: III-3-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | 386.90 |
| $M_{\chi^{\pm}}$ | $273.35, 69113.11$ |
| $M_{\chi^0}$ | $98.23, 273.35, 69113.07, 69113.08$ |
| $M_{\tilde{\nu}}$ | $5299.63, 5350.60, 10339.851$ |
| $M_{\tilde{e}}$ | $118.46, 1056.41, 5300.46, 5351.24, 10300.80, 12402.58$ |
| $M_{\tilde{u}}$ | $4113.23, 4117.53, 4257.15, 4257.85, 15641.39, 21506.49$ |
| $M_{\tilde{d}}$ | $2145.97, 2146.92, 4114.23, 4118.35, 21488.89, 25996.33$ |
| $M_A$ | 197503.48 |
| $M_{H^{\pm}}$ | 197503.50 |
| $M_{H^0}$ | 197503.46 |
| $M_{h^0}$ | 117.80 |

Table 64: III-3-f : Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

| Parameter | $Value$ | Field $[SU(3),SU(2),Y]$ | $Masses$ $(Units\ of 10^{16}GeV)$ |
|---|---|---|---|
| $\chi_X$ | 0.0469 | $A[1,1,4]$ | 153.03 |
| $\chi_Z$ | 0.0139 | $B[6,2,5/3]$ | 0.3445 |
| $h_{11}/10^{-6}$ | $-0.0162$ | $C[8,2,1]$ | $5.06, 191.08, 192.38$ |
| $h_{22}/10^{-4}$ | $-0.0156$ | $D[3,2,7/3]$ | $4.50, 104.56, 129.69$ |
| $h_{33}$ | 0.0033 | $E[3,2,1/3]$ | $0.41, 3.63, 5.83$ |
| $f_{11}/10^{-6}$ | $0.0145 + 0.1575i$ | | $5.828, 154.26, 178.97$ |
| $f_{12}/10^{-6}$ | $-1.2719 + 0.4383i$ | $F[1,1,2]$ | $1.06, 1.06$ |
| $f_{13}/10^{-5}$ | $0.3213 + 0.1092i$ | | $2.94, 77.81$ |
| $f_{22}/10^{-5}$ | $5.9014 - 6.6842i$ | $G[1,1,0]$ | $0.097, 0.28, 0.28$ |
| $f_{23}/10^{-4}$ | $2.0445 + 3.1164i$ | | $0.783, 16.65, 17.07$ |
| $f_{33}/10^{-3}$ | $-1.0245 + 0.7289i$ | $h[1,2,1]$ | $1.382, 3.46, 4.74$ |
| $g_{12}/10^{-4}$ | $0.0428 - 0.0951i$ | | $134.98, 176.94$ |
| $g_{13}/10^{-5}$ | $-1.5145 - 2.2227i$ | $I[3,1,10/3]$ | 1.41 |
| $g_{23}/10^{-4}$ | $2.9856 + 6.1462i$ | $J[3,1,4/3]$ | $1.370, 2.90, 2.90$ |
| $\lambda/10^{-2}$ | $-7.7575 - 1.6636i$ | | $7.29, 119.06$ |
| $\eta$ | $-14.6064 + 1.2176i$ | $K[3,1,8/3]$ | $7.99, 140.68$ |
| $\rho$ | $0.4221 - 1.5473i$ | $L[6,1,2/3]$ | $4.16, 230.75$ |
| $k$ | $0.0267 - 0.0731i$ | $M[6,1,8/3]$ | 236.13 |
| $\zeta$ | $1.0476 + 0.4885i$ | $N[6,1,4/3]$ | 228.10 |
| $\bar{\zeta}$ | $1.5631 - 0.4286i$ | $O[1,3,2]$ | 421.09 |
| $m/10^{16}GeV$ | 0.03 | $P[3,3,2/3]$ | $1.17, 372.51$ |
| $m_o/10^{16}GeV$ | $-3.437e^{-iArg(\lambda)}$ | $Q[8,3,0]$ | 1.242 |
| $\gamma$ | 4.58 | $R[8,1,0]$ | $0.37, 1.45$ |
| $\bar{\gamma}$ | $-2.2542$ | $S[1,3,0]$ | 1.6823 |
| $x$ | $0.8706 + 0.4654i$ | $t[3,1,2/3]$ | $1.02, 2.78, 7.81, 9.33$ |
| $\Delta_X$ | 0.43 | | $152.15, 160.68, 3371.92$ |
| $\Delta_G$ | 20.355 | $U[3,3,4/3]$ | 1.485 |
| $\Delta\alpha_3(M_Z)$ | $-0.009$ | $V[1,2,3]$ | 0.944 |
| $\{M^{\nu^c}/10^{11}GeV\}$ | $0.01, 7.01, 280.82$ | $W[6,3,2/3]$ | 356.14 |
| $\{M^{\nu}_{II}/10^{-12}eV\}$ | $6.3962, 5020.83, 201135.16$ | $X[3,2,5/3]$ | $0.316, 6.340, 6.340$ |
| $M_\nu(meV)$ | $2.32, 7.54, 41.49$ | $Y[6,2,1/3]$ | 0.39 |
| $\{\text{Evals[f]}\}/10^{-7}$ | $0.43, 340.95, 13658.39$ | $Z[8,1,2]$ | 1.45 |
| Soft parameters at $M_X$ | $\mathrm{m}_{\frac{1}{2}} = -258.670$ <br> $\mu = 1.2975 \times 10^5$ <br> $\mathrm{M}^2_{\bar{\mathrm{H}}} = -1.4950 \times 10^{10}$ | $\mathrm{m}_0 = 4903.144$ <br> $\mathrm{B} = -9.5887 \times 10^9$ <br> $\mathrm{M}^2_{\mathrm{H}} = -1.4315 \times 10^{10}$ | $\mathrm{A}_0 = -1.4665 \times 10^5$ <br> $\tan\beta = 51.0000$ <br> $\mathrm{R}_{\frac{b\tau}{s\mu}} = 3.7388$ |
| $Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$ | $4.7911 \times 10^{-22}\ \mathrm{GeV}^{-1}$ | | |

Table 65: III-4-a : Column 1 contains values of the NMSGUT-SUGRY-NUHM parameters at $M_X$ derived from an accurate fit to all 18 fermion data and compatible with RG constraints. See caption to Table 2 for explanation.

| Parameter | $Target = \bar{O}_i$ | $Uncert. = \delta_i$ | $Achieved = O_i$ | $Pull = (O_i - \bar{O}_i)/\delta_i$ |
|---|---|---|---|---|
| $y_u/10^{-6}$ | 2.106535 | 0.804696 | 2.106685 | 0.000186 |
| $y_c/10^{-3}$ | 1.026841 | 0.169429 | 1.027073 | 0.001369 |
| $y_t$ | 0.368035 | 0.014721 | 0.368036 | 0.000048 |
| $y_d/10^{-5}$ | 6.294474 | 3.669678 | 6.323679 | 0.007958 |
| $y_s/10^{-3}$ | 1.194492 | 0.563800 | 1.201918 | 0.013170 |
| $y_b$ | 0.431150 | 0.223767 | 0.439556 | 0.037566 |
| $y_e/10^{-4}$ | 1.180231 | 0.177035 | 1.179590 | −0.003620 |
| $y_\mu/10^{-2}$ | 2.495809 | 0.374371 | 2.496108 | 0.000799 |
| $y_\tau$ | 0.530719 | 0.100837 | 0.528387 | −0.023127 |
| $\sin\theta^q_{12}$ | 0.2210 | 0.001600 | 0.2210 | 0.0001 |
| $\sin\theta^q_{13}/10^{-4}$ | 29.3942 | 5.000000 | 29.3933 | −0.0002 |
| $\sin\theta^q_{23}/10^{-3}$ | 34.5852 | 1.300000 | 34.5835 | −0.0013 |
| $\delta^q$ | 60.0209 | 14.000000 | 60.0050 | −0.0011 |
| $(m^2_{12})/10^{-5}(eV)^2$ | 5.1427 | 0.545127 | 5.1428 | 0.0002 |
| $(m^2_{23})/10^{-3}(eV)^2$ | 1.6651 | 0.333025 | 1.6650 | −0.0003 |
| $\sin^2\theta^L_{12}$ | 0.2885 | 0.057694 | 0.2885 | −0.0003 |
| $\sin^2\theta^L_{23}$ | 0.4611 | 0.138326 | 0.4610 | −0.0005 |
| $\theta^L_{13}(degrees)$ | 3.7 | 3.7 | 4.54 | |
| $Eigenvalues(\Delta_{\bar{u}})$ | 0.010307 | 0.010309 | 0.010310 | |
| $Eigenvalues(\Delta_{\bar{d}})$ | 0.006885 | 0.006887 | 0.006888 | |
| $Eigenvalues(\Delta_{\bar{\nu}})$ | 0.000492 | 0.000494 | 0.000494 | |
| $Eigenvalues(\Delta_{\bar{e}})$ | 0.010759 | 0.010761 | 0.010762 | |
| $Eigenvalues(\Delta_Q)$ | 0.013199 | 0.013201 | 0.013201 | |
| $Eigenvalues(\Delta_L)$ | 0.010228 | 0.010230 | 0.010230 | |
| $\Delta_{\bar{H}}, \Delta_H$ | 58.562855 | 50.761700 | | |
| $\alpha_1$ | $0.7564 + 0.0000i$ | $\bar{\alpha}_1$ | $0.8100 + 0.0000i$ | |
| $\alpha_2$ | $0.0475 + 0.0440i$ | $\bar{\alpha}_2$ | $-0.0078 + 0.0789i$ | |
| $\alpha_3$ | $0.0250 - 0.0486i$ | $\bar{\alpha}_3$ | $-0.0002 - 0.0292i$ | |
| $\alpha_4$ | $-0.5034 - 0.0920i$ | $\bar{\alpha}_4$ | $0.2831 + 0.1536i$ | |
| $\alpha_5$ | $0.2065 + 0.0095i$ | $\bar{\alpha}_5$ | $0.1215 - 0.0343i$ | |
| $\alpha_6$ | $0.0602 - 0.3353i$ | $\bar{\alpha}_6$ | $0.0906 - 0.4570i$ | |

Table 66: III-4-b: Fit with $\chi_X = \sqrt{\sum_{i=1}^{17}(O_i - \bar{O}_i)^2/\delta_i^2} = 0.0469$. See caption to Table 3 for explanation.

| Parameter | $SM(M_Z)$ | $m^{GUT}(M_Z)$ | $m^{MSSM} = (m+\Delta m)^{GUT}(M_Z)$ |
|---|---|---|---|
| $m_d/10^{-3}$ | 2.90000 | 0.58241 | 2.89785 |
| $m_s/10^{-3}$ | 55.00000 | 11.06967 | 55.04827 |
| $m_b$ | 2.90000 | 2.97388 | 2.91949 |
| $m_e/10^{-3}$ | 0.48657 | 0.46121 | 0.48338 |
| $m_\mu$ | 0.10272 | 0.09755 | 0.10210 |
| $m_\tau$ | 1.74624 | 1.73358 | 1.73257 |
| $m_u/10^{-3}$ | 1.27000 | 1.11823 | 1.26917 |
| $m_c$ | 0.61900 | 0.54517 | 0.61876 |
| $m_t$ | 172.50000 | 147.05975 | 172.17090 |

Table 67: III-4-c: Values of standard model fermion masses in GeV at $M_Z$ compared with the masses obtained from values of GUT derived yukawa couplings run down from $M_X^0$ to $M_Z$ both before and after threshold corrections. Fit with $\chi_Z = \sqrt{\sum_{i=1}^{9}(m_i^{MSSM} - m_i^{SM})^2/(m_i^{MSSM})^2} =$ 0.0139.

| Parameter | $Value$ | Parameter | $Value$ |
|---|---|---|---|
| $M_1$ | 94.02 | $M_{\tilde{\bar{u}}_1}$ | 5778.49 |
| $M_2$ | 284.49 | $M_{\tilde{\bar{u}}_2}$ | 5778.19 |
| $M_3$ | 269.37 | $M_{\tilde{\bar{u}}_3}$ | 32851.51 |
| $M_{\tilde{\bar{l}}_1}$ | 100.27 | $A_{11}^{0(l)}$ | $-94185.07$ |
| $M_{\tilde{\bar{l}}_2}$ | 122.47 | $A_{22}^{0(l)}$ | $-94074.32$ |
| $M_{\tilde{\bar{l}}_3}$ | 28577.12 | $A_{33}^{0(l)}$ | $-59361.23$ |
| $M_{\tilde{L}_1}$ | 8090.35 | $A_{11}^{0(u)}$ | $-107534.77$ |
| $M_{\tilde{L}_2}$ | 8090.50 | $A_{22}^{0(u)}$ | $-107534.06$ |
| $M_{\tilde{L}_3}$ | 21787.08 | $A_{33}^{0(u)}$ | $-56550.41$ |
| $M_{\tilde{\bar{d}}_1}$ | 1167.18 | $A_{11}^{0(d)}$ | $-94095.80$ |
| $M_{\tilde{\bar{d}}_2}$ | 1168.84 | $A_{22}^{0(d)}$ | $-94095.08$ |
| $M_{\tilde{\bar{d}}_3}$ | 40997.04 | $A_{33}^{0(d)}$ | $-42029.00$ |
| $M_{\tilde{Q}_1}$ | 5546.60 | $\tan\beta$ | 51.00 |
| $M_{\tilde{Q}_2}$ | 5546.62 | $\mu(M_Z)$ | 108204.37 |
| $M_{\tilde{Q}_3}$ | 37188.99 | $B(M_Z)$ | $1.9013 \times 10^9$ |
| $M_{\bar{H}}^2$ | $-1.1922 \times 10^{10}$ | $M_H^2$ | $-1.2689 \times 10^{10}$ |

Table 68: III-4-d: Values (GeV) in of the soft Susy parameters at $M_Z$ (evolved from the soft SUGRY-NUHM parameters at $M_X$). See caption to Table 5 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | $269.37$ |
| $M_{\chi^{\pm}}$ | $284.48, 108204.44$ |
| $M_{\chi^0}$ | $94.02, 284.48, 108204.41, 108204.42$ |
| $M_{\tilde{\nu}}$ | $8090.077, 8090.228, 21786.977$ |
| $M_{\tilde{e}}$ | $110.00, 8090.50, 111.64, 8090.93, 21780.86, 28581.94$ |
| $M_{\tilde{u}}$ | $5546.33, 5778.37, 5546.23, 5778.19, 32848.10, 37192.53$ |
| $M_{\tilde{d}}$ | $1167.48, 5546.94, 1169.07, 5546.97, 37176.73, 41008.21$ |
| $M_A$ | $311453.73$ |
| $M_{H^{\pm}}$ | $311453.74$ |
| $M_{H^0}$ | $311453.72$ |
| $M_{h^0}$ | $128.46$ |

Table 69: III-4-e: Spectra of supersymmetric partners calculated ignoring generation mixing effects. See caption to Table 6 for explanation.

| Field | $Mass(GeV)$ |
|---|---|
| $M_{\tilde{G}}$ | $269.50$ |
| $M_{\chi^{\pm}}$ | $284.54, 108200.79$ |
| $M_{\chi^0}$ | $94.04, 284.54, 108200.77, 108200.77$ |
| $M_{\tilde{\nu}}$ | $8090.11, 8090.27, 21787.520$ |
| $M_{\tilde{e}}$ | $88.50, 108.38, 8090.53, 8090.99, 21781.65, 28582.63$ |
| $M_{\tilde{u}}$ | $5546.48, 5569.26, 5778.35, 5778.53, 32847.38, 37188.53$ |
| $M_{\tilde{d}}$ | $1168.04, 1169.61, 5547.10, 5569.98, 37172.75, 41007.52$ |
| $M_A$ | $311542.91$ |
| $M_{H^{\pm}}$ | $311542.92$ |
| $M_{H^0}$ | $311542.91$ |
| $M_{h^0}$ | $128.51$ |

Table 70: Spectra of supersymmetric partners calculated including generation mixing effects. See caption to Table 7 for explanation.

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