arXiv:1107.4063 KA–TP–19–2011 (v4)

Inflation and the cosmological constant

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Abstract

A particular compensation-type solution of the main cosmological constant problem has been proposed recently, with two massless vector fields dynamically canceling an arbitrary cosmological constant Λ. The naive expectation is that such a compensation mechanism does not allow for the existence of an inflationary phase in the very early Universe. However, it is shown that certain boundary conditions on the vector fields can in fact give rise to an inflationary epoch.

PACS numbers: 04.20.Cv, 98.80.Cq, 98.80.Es

Keywords: general relativity, inflationary universe, cosmological constant

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I. INTRODUCTION

Inflation [1, 2], an epoch of exponential expansion, may have played an important role in the evolution of the very early universe (see Ref. [3] for an incomplete list of precursor papers and Ref. [4] for further references and discussion). The mechanism relies, however, on one crucial assumption (stated, for example, a few lines below Eq. (3.7) in Ref. [1]): the minimum of the total scalar potential is set to zero, i.e., the corresponding vacuum energy density (effective cosmological constant Λ) is assumed to vanish. In other words, it is taken for granted that a solution has been found to the main cosmological constant problem [5] (CCP1): why is the present value of $|\Lambda|^{1/4}$ negligible when compared with the known energy scales of elementary particle physics? [The next cosmological constant problem (CCP2) is, of course, to explain the measured value $\Lambda^{\text{exp}} \sim (2\,\text{meV})^4$, but this question lies outside the scope of the present article.]

Following an earlier suggestion to consider vector fields [6] and using the insights from the q–theory approach [7] to CCP1, a special model of two massless vector fields has been presented in Ref. [8]. The massless vector fields of the model cancel dynamically an arbitrary initial (bare) cosmological constant Λ without upsetting the local Newtonian dynamics (a potential problem discussed in Ref. [9]).

But, if any cosmological constant can be canceled dynamically, what happens to inflation in the very early universe?

In order to address this issue, we investigate the simplest possible extension of the two-vector-field model by adding a fundamental scalar field with a quadratic potential, while keeping an initial cosmological constant Λ. The question is, then, whether or not it is possible to have an inflationary phase of finite duration.

II. MODEL

The model of Ref. [8] has two massless vector fields $A_\alpha(x)$ and $B_\alpha(x)$. Now, a fundamental complex scalar field $\Sigma(x)$ is added. Equivalently, it is possible to work with two real scalars by defining $\Sigma(x) \equiv [\phi_1(x) + i\,\phi_2(x)]/\sqrt{2}$. The relevant effective action is ($\hbar = c = 1$):

$$S_{\text{eff}}[A,\,B,\,\Sigma,\,g] \;=\; -\int d^4x\,\sqrt{-g}\,\Big(\tfrac{1}{2}\,(E_{\text{Planck}})^2\,R + \epsilon(Q_A,\,Q_B,\,\Sigma) + \Lambda$$
$$-\partial_\alpha\Sigma\;\partial^\alpha\Sigma^\star + U\big(|\Sigma|\big)\Big), \tag{1a}$$

$$Q_A \;\equiv\; \sqrt{A_{\alpha;\beta}\;A^{\alpha;\beta}}\,, \quad Q_B \equiv \sqrt{B_{\alpha;\beta}\;B^{\alpha;\beta}}\,, \tag{1b}$$

$$E_{\text{Planck}} \;\equiv\; (8\pi\,G_N)^{-1/2} \approx 2.44\times 10^{18}\ \text{GeV}\,, \tag{1c}$$

for a scalar potential $U\big(|\Sigma|\big) \geq 0$ with $U(|\Sigma_{\text{min}}|) = 0$. Specifically, the following two functions U and ϵ are used:

$$U\big(|\Sigma|\big) \;=\; M^2\,|\Sigma|^2\,, \tag{1d}$$

$$\epsilon(Q_A,\,Q_B,\,\Sigma) \;=\; (E_{\text{Planck}})^4\;\frac{Q_A^4 - Q_B^4}{(E_{\text{Planck}})^8\,\delta_{\text{eff}}(\Sigma) + Q_A^2\,Q_B^2}\;, \tag{1e}$$

$$\delta_{\text{eff}}(\Sigma) \;\equiv\; \delta\;\frac{|\Sigma|^2}{|\Sigma|^2 + (E_{\text{Planck}})^2\,\eta}\;, \tag{1f}$$

for $0 < M \ll E_{\text{Planck}}$ and (small) positive constants δ and η. The motivation of using the particular function (1f) is that, even for a fixed positive value of δ, the inverse vacuum compressibility χ^{-1} vanishes if $\Sigma \to 0$ and the standard local Newtonian dynamics may be recovered (see Ref. [8] for further discussion).

The constant Λ in the effective action (1a) includes the vacuum-energy-density contributions from the zero-point energies of the standard-model quantum fields (not shown explicitly). In principle, this effective cosmological constant Λ can be of arbitrary sign and have a magnitude of order $(E_{\text{Planck}})^4$. For further discussion and references on the effective-action method, see Ref. [7, (c)].

It is also possible to write (1a) in terms of the total potential,

$$U_{\text{tot}}\big(|\Sigma|,\,\Lambda\big) \;=\; U\big(|\Sigma|\big) + \Lambda = M^2\,|\Sigma|^2 + \Lambda\,. \tag{1g}$$

As mentioned in the first paragraph of Sec. I, this quantity U_{tot} has been used in the previous discussions of inflation, with Λ set to zero by hand. Here, we keep Λ arbitrary but introduce vector fields which have the potentiality to cancel it. In order to provide this cancellation of the effective cosmological constant Λ, the vector fields must appear in (1a) via the derivative terms contained in ϵ, at least, within the q–theory framework [7, (a,c)].

The isotropic *Ansatz* [6] for the vector fields $A_\alpha(x)$ and $B_\beta(x)$, the scalar $\Sigma(x)$, and the metric $g_{\alpha\beta}(x)$ is:

$$A_0 \;=\; A_0(t) \equiv V(t)\,, \qquad A_1 = A_2 = A_3 = 0\,, \tag{2a}$$

$$B_0 \;=\; B_0(t) \equiv W(t)\,, \qquad B_1 = B_2 = B_3 = 0\,, \tag{2b}$$

$$\Sigma \;=\; \Sigma(t)\,, \tag{2c}$$

$$(g_{\alpha\beta}) \;=\; \text{diag}\big(1,\,-a(t),\,-a(t),\,-a(t)\big)\,, \tag{2d}$$

where t is the cosmic time of a spatially flat Friedmann–Robertson–Walker (FRW) universe, with cosmic scale factor $a(t)$ and Hubble parameter $H(t) \equiv [da(t)/dt]/a(t)$.

Using appropriate powers of the reduced Planck energy (1c) without additional numerical factors, the above dimensionful variables can be replaced by the following dimensionless variables:

$$\big\{\Lambda,\,M,\,U,\,\epsilon\,,\,t,\,H\big\} \;\to\; \big\{\lambda,\,m,\,u,\,e,\,\tau,\,h\big\}\,, \tag{3a}$$

$$\big\{Q_A,\,Q_B,\,V,\,W,\,\Sigma\big\} \;\to\; \big\{q_A,\,q_B,\,v,\,w,\,\sigma\big\}\,. \tag{3b}$$

From now on, an overdot stands for differentiation with respect to τ, for example, $h(\tau) \equiv \dot{a}(\tau)/a(\tau)$.

In terms of these dimensionless variables, the *Ansatz* (2) reduces the field equations from (1a) to a set of coupled ordinary differential equations (ODEs) for $v(\tau)$, $w(\tau)$, $h(\tau)$, and $\sigma(\tau)$. Three of these ODEs have already been given in (3.11) of Ref. [8], except that (3.11c) now contains a dimensionless pressure term from the scalar field, specifically, $p_\sigma = |\dot{\sigma}|^2 - u(|\sigma|)$. The fourth ODE is simply the standard FRW Klein–Gordon equation for $\sigma(\tau)$.

The corresponding Friedmann equation is given by

$$3\,h^2 \;=\; \lambda + r_\sigma + \Big[\, \widetilde{e}(q_A,\, q_B,\, \sigma)\,\Big]_{q_A=\sqrt{\dot{v}^2+3\,h^2\,v^2},\;\, q_B=\sqrt{\dot{w}^2+3\,h^2\,w^2}}\,, \tag{4a}$$

$$\widetilde{e} \;\equiv\; e - q_A\,\frac{de}{dq_A} - q_B\,\frac{de}{dq_B} = \frac{\left(q_A^2\,q_B^2 - 3\,\delta_\text{eff}\right)\left(q_A^4 - q_B^4\right)}{\left(\delta_\text{eff} + q_A^2\,q_B^2\right)^2}\,, \tag{4b}$$

$$r_\sigma \;=\; |\dot{\sigma}|^2 + u(|\sigma|)\,, \tag{4c}$$

with $\delta_\text{eff} \equiv \delta\,|\sigma|^2/(|\sigma|^2+\eta)$ from (1f) and r_σ corresponding to the dimensionless energy density from the scalar field. In conjunction with the four ODEs mentioned in the previous paragraph, (4a) acts as a constraint equation [8]: if (4a) is satisfied by the boundary conditions, it is satisfied always.

Observe that, in terms of dimensionful variables, the vacuum energy density $\widetilde{\epsilon}$ on the right-hand side of (4a) differs from the vacuum energy density ϵ entering the action (1a). The possible difference of $\widetilde{\epsilon}$ and ϵ is one of the main results of q–theory (see the original article Ref. [7, (a)] or the one-page summary of App. A in Ref. [10]).

III. RESULTS

Numerical solutions of the reduced field equations are presented in four figures. All of these results are obtained from a single model, specified by the action (1a) and *Ansatz* (2), and are differentiated only by their model parameters (e.g., Λ zero or not) and initial boundary conditions (e.g., initial vector-field values zero or not). In principle, the numerical calculation must be performed for a small but nonzero value of η (for example, $\eta = 10^{-4}$), but the numerical calculation at large values of τ is speeded up by taking the value $\eta = 0$.

Figure 1 shows an inflationary epoch, followed by a standard FRW-like expansion phase with $h \sim (2/3)\,\tau^{-1}$ due to the fact that $\sigma(\tau)$ rapidly spirals inward towards the minimum $\sigma_\text{min} = 0$.[1] (The main characteristics of this particular type of slow-roll inflation, for the case of a single real scalar field, are discussed in Sec. 5.4.1 of Ref. [4].) Here, the total scalar potential U_tot in (1g) has its minimum energy fine-tuned to zero, that is $\Lambda = 0$.

[1] Generically, $\sigma(\tau)$ does not hit 0 at a finite value of τ and the same holds for δ_eff from (1f). This is the reason for using a complex scalar rather than a single real scalar which passes through 0 many times.

Figure 2 shows that removing the fine-tuning by changing Λ to a positive value leads to eternal inflation without a subsequent FRW-like phase, due to the presence of a nonzero value of the vacuum energy density (effective cosmological constant) even if $\sigma(t) \to 0$.

Figure 3 shows that having small but nonzero initial values of the vector fields leads to the termination of the inflationary phase by the eventual vector-field cancellation of the initial cosmological constant $\Lambda > 0$. Changing the value of $\lambda \equiv \Lambda/(E_{\text{Planck}})^4$ from 0.01 to 0.03 or to 0.0075 gives similar results. Returning to $\lambda = 0.01$, it has also been verified that setting $\eta = 10^{-4}$ gives essentially the same results as for $\eta = 0$ up to $\tau = 450$.

Figure 4 shows the absence of a significant inflationary phase for large enough initial values of the vector fields, due to the immediate and complete vector-field cancellation of $\lambda + u(|\sigma|)$. Again, it has been verified that setting $\eta = 10^{-4}$ gives essentially the same results as for $\eta = 0$ up to $\tau = 10^4$.

Two final comments are as follows. First, it is seen that $\tau^{-1}\, v(\tau)$ in Figs. 3 or 4 peaks when $\tau\, h(\tau)$ first drops to 1, the position and height of the $\tau^{-1}\, v(\tau)$ peak depending on the initial conditions. Strictly speaking, this observation also holds for Fig. 2, with the position of the peak moved off towards infinity.

Second, extending the numerical solutions of Figs. 3 and 4 to $\tau = 10^6$, the asymptotic behavior appears to be $v \sim (q_{A0}/2)\,\tau$, $w \sim (q_{B0}/2)\,\tau$, and $h \sim 1/\tau$. If confirmed, this asymptotic behavior would correspond to a different branch than the one found numerically in Ref. [8] (for the theory without scalars) and would in fact correspond to the standard q–theory branch [7, (c)] with constant $q_A = q_{A0}$ and $q_B = q_{B0}$.

IV. DISCUSSION

The results of Figs. 2 and 4 were to be expected. The surprising (and encouraging?) results are those of Fig. 3, with an inflationary phase of some ten e-foldings of $a(\tau)$ for the model and parameters chosen. (Different initial conditions have been seen to give some thirty e-foldings and Fig. 2 can be interpreted as having infinitely many e-foldings.) Qualitatively, the Hubble parameter $h(\tau)$ of the top right panel in Fig. 3 ($\lambda \neq 0$) resembles that of Fig. 1 ($\lambda = 0$), even though the detailed behavior differs as shown by the respective bottom right panels.

The results of Fig. 3 are, of course, only exploratory. It remains, for example, to analyze the nonlinear dynamics displayed in Fig. 3 and to rigorously establish the $\tau \to \infty$ limit for the $\eta = 10^{-4}$ case, corresponding respectively to an early phase with inflation[2] and a late (FRW-like) phase with standard local Newtonian dynamics. The physical origin of the massless vector fields themselves also needs to be clarified.

[2] Possible observable effects may come from the dynamics of density perturbations with near-horizon wavelengths (cf. Ref. [4]), as the dynamics can be expected to be modified by the interaction between the scalar (inflaton) field and the vector fields needed for the cancelation of the bare cosmological constant Λ.

ACKNOWLEDGMENTS

It is a pleasure to thank G.E. Volovik for numerous discussions on the cosmological constant problem, S. Thambyahpillai for helpful comments on the manuscript, and V. Emelyanov for a valuable suggestion to use the scalar field in the ϵ function.

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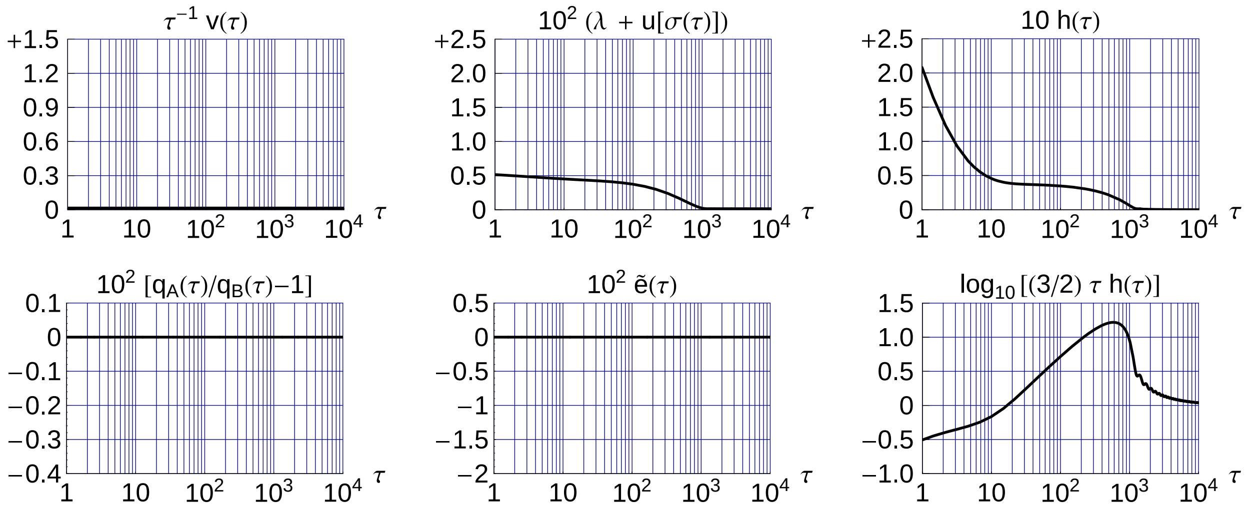

FIG. 1: Numerical solution of the reduced field equations for model (1) and *Ansatz* (2). The dimensionless model parameters are $m = 0.01$, $\delta = 10^{-6}$, $\eta = 0$, and $\lambda = 0$. The boundary conditions are $a(1) = 1$, $\{\varphi_1(1), \dot{\varphi}_1(1), \varphi_2(1), \dot{\varphi}_2(1)\} = \{10, -0.25, 0, -0.433013\}$, and $v(1) = \dot{v}(1) = w(1) = \dot{w}(1) = 0$. [The real scalars φ_n are defined by $\sigma \equiv (\varphi_1 + i\,\varphi_2)/\sqrt{2}$.] The value of $h(1)$ follows from the Friedmann equation (4a). With these boundary conditions, the reduced field equations have the exact solution $v(\tau) = w(\tau) = 0$ for $\tau \geq 1$. The top right panel shows an $h(\tau)$ plateau corresponding to an inflationary phase. The bottom right panel shows that, long after inflation, $h(\tau)$ asymptotically goes as $(2/3)\,\tau^{-1}$.

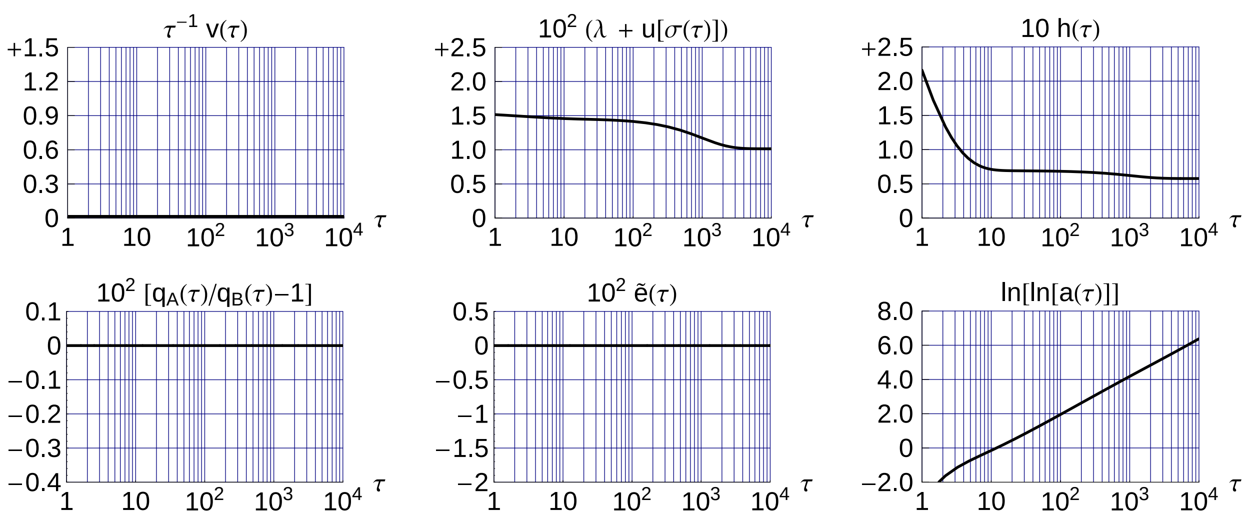

FIG. 2: Same as Fig. 1, but now with nonzero cosmological constant, $\lambda = 0.01$. Same boundary conditions as Fig. 1, so that the exact solution $v(\tau) = w(\tau) = 0$ persists. The Friedmann equation (4a) gives the asymptotic value $h(\infty) = \sqrt{\lambda/3} \approx 0.05774$.

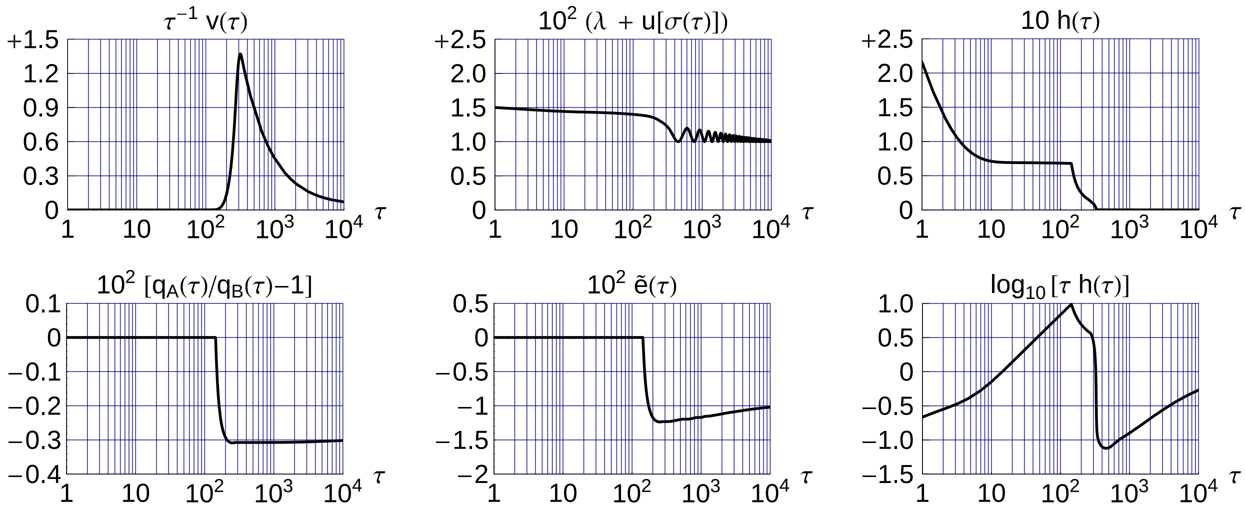

FIG. 3: Same parameters and boundary conditions as Fig. 2 (e.g., $\lambda = 0.01$), except for small but nonzero starting values of v and w. Specifically, the boundary conditions at $\tau = 1$ are: $\{a,\, v,\, \dot{v},\, w,\, \dot{w},\, h,\, \varphi_1,\, \dot{\varphi}_1,\, \varphi_2,\, \dot{\varphi}_2\} = \{1,\, 2\times10^{-4},\, 2\times10^{-4},\, 2\times10^{-4},\, 2\times10^{-4},\, 0.216025,\, 10,\, -0.25,\, 0,\, -0.433013\}$. The total vacuum energy density entering the right-hand side of the Friedmann equation (4a) is given by the sum of the two panels in the middle column and vanishes asymptotically.

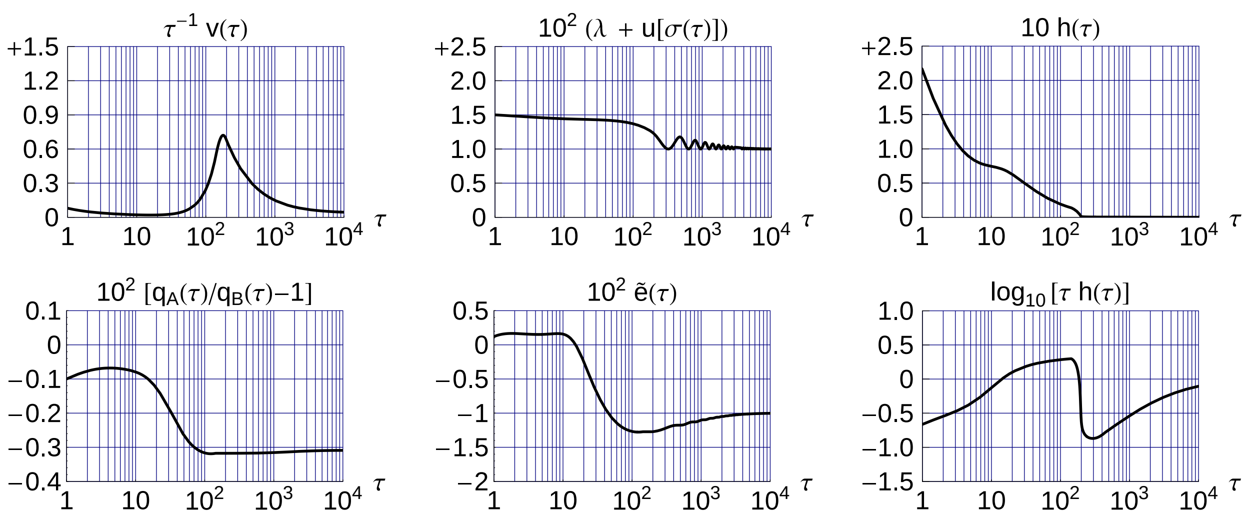

FIG. 4: Same parameters and boundary conditions as Fig. 3 (e.g., $\lambda = 0.01$), except for relatively large starting values of v and w. Specifically, the boundary conditions at $\tau = 1$ are: $\{a,\, v,\, \dot{v},\, w,\, \dot{w},\, h,\, \varphi_1,\, \dot{\varphi}_1,\, \varphi_2,\, \dot{\varphi}_2\} = \{1,\, 0.0799201,\, 0.01998,\, 0.08,\, 0.02,\, 0.216961,\, 10,\, -0.25,\, 0,\, -0.433013\}$.