

Acceleration of particles by rotating black holes: near-horizon geometry and kinematics

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Nowadays, the effect of infinite energy in the centre of mass frame due to near-horizon collisions attracts much attention. We show generality of the effect combining two seemingly completely different approaches based on properties of a particle with respect to its local light cone and calculating its velocity in the locally nonrotating frame directly. In doing so, we do not assume that particles move along geodesics. Usually, a particle reaches a horizon having the velocity equals that of light. However, there is also case of "critical" particles for which this is not so. It is just the pair of usual and critical particles that leads to the effect under discussion. The similar analysis is carried out for massless particles. Then, critical particles are distinguishable due to the finiteness of local frequency. Thus, both approach based on geometrical and kinematic properties of particles moving near the horizon, reveal the universal character of the effect.

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I. INTRODUCTION

The effect of infinite grow of the energy in the centre of mass frame due to the near-horizon collision of two particles (BSW effect) attracts now much attention [1] - [21]. At first, it was discovered for the Kerr metric [1] but later it was understood that the effect is of quite general character. It was shown in [17] that such an effect exists if orientation of a four-velocity of a massive particle with respect to its local light cone obeys some simple conditions. More

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precisely, one of coefficients in a suitable null tetrad basis (which is specified below) should vanish on the horizon ("critical" particle). From the other hand, in recent works [18], an alternative explanation was given in terms of kinematic properties of particles. It turned out that from a kinematic viewpoint, a critical particle is distinguished by the property that in the horizon limit, its velocity in the locally nonrotating frame (LNRF) [22] tends to the value which is less than that of light. Meanwhile, for typical ("usual") particles this velocity tends just to the speed of light. It is collision between a critical and usual particles that produces the effect under discussion if such a collision occurs near the horizon.

Qualitatively, it can be explained as follows. If we have two particles one of which is slow (the value of the velocity $v_1 < 1$, the speed of light $c = 1$) and the second one is fast ($v_2 \approx 1$), the relative velocity is also close to 1. Correspondingly, the Lorentz factor tends to infinity and we have the BSW effect. Actually, this is explained in terms of simple kinematics of collision in the flat space-time. Further use of the kinematic approach for investigation of rather subtle details of the BSW effect in the Kerr background can be found in Ref. [21]. If one of particles is massless, so its velocity is equal to 1 exactly, the explanation is somewhat changed. It is based on the relative role of the gravitational blueshift and the Doppler effect. In doing so, the usual and critical particles are distinguished by the property that the LNRF frequency of the massless particle is finite or infinite. It turned out that collision between massive and massless particles produce the BSW effect also for the situation when one particle is critical and the other one is usual [18].

Thus, we have two quite different explanations using different language - from the geometric point of view and on the basis of kinematics. The geometric explanation [17] was quite general whereas the kinematic one [18], was obtained for geodesic motion of particles only.

The aim of the present work is (i) to make a bridge between geometric and kinematic approaches and (ii) generalize kinematic one to an arbitrary case not requiring geodesic motion and not using equations of particles' motion at all.

II. BASIC EQUATIONS

Let us consider the space-time of a rotating black hole described by the metric

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz} dz^2. \quad (1)$$

The location of the horizon defined as a surface of infinite redshift corresponds to $N \rightarrow 0$. In what follows we will use the tetrad basis. Denoting coordinates x^μ as $x^0 = t, x^1 = l, x^2 = z, x^3 = \phi$, we choose the orthonormal tetrad vectors $h_{(a)\mu}$ in the following way:

$$h_{(0)\mu} = -N(1, 0, 0, 0), \quad (2)$$

$$h_{(1)\mu} = (0, 1, 0, 0) \quad (3)$$

$$h_{(2)\mu} = \sqrt{g_{zz}}(0, 0, 1, 0) \quad (4)$$

$$h_{(3)\mu} = \sqrt{g_{\phi\phi}}(-\omega, 0, 0, 1) \quad (5)$$

If such a tetrad is attached to an observer moving in the metric (1), it has meaning of zero angular momentum observer [22], so two abbreviations LNRF and ZAMO are used in literature. A corresponding observer "rotates with the geometry" in the sense that $\frac{d\phi}{dt} \equiv \omega$ for him. The advantage of using the tetrad components consists in that one can use the formulas of special relativity in the flat space-time tangent to any given point.

In a given context [17], a null tetrad is also convenient for the decomposition of the metric:

$$g_{\alpha\beta} = -l_\alpha N_\beta - l_\beta N_\alpha + \sigma_{\alpha\beta} \quad (6)$$

where $\sigma_{\alpha\beta} = a_\alpha a_\beta + b_\alpha b_\beta$, $l^\alpha \sigma_{\alpha\beta} = N^\alpha \sigma_{\alpha\beta} = 0$, a_μ and b_μ are spacelike vectors (see, for example, textbook [23]). For the metric (1) one can check that the null vectors can be chosen in the following way:

$$l_\mu = (-N^2, N, 0, 0), \quad (7)$$

$$N_\mu = \frac{1}{2}(-1, -\frac{1}{N}, 0, 0). \quad (8)$$

$$(Nl) = -1. \quad (9)$$

Then, it is seen that

$$h_{(0)\mu} = NN_\mu + \frac{l_\mu}{2N}, \quad (10)$$

$$h_{(1)\mu} = \frac{l_\mu}{2N} - NN_\mu. \quad (11)$$

III. CASE OF MASSIVE PARTICLES

Let us consider motion of massive particles (electrons, for brevity). Then, using our tetrad basis, we can write down the decomposition of the four-velocity,

$$u^\mu = \frac{l^\mu}{2\alpha} + \beta N^\mu + s^\mu, \quad s^\mu = Aa^\mu + Bb^\mu, \quad (12)$$

A and B are coefficients. The normalization condition $(uu) = -1$ entails

$$\alpha = \frac{\beta}{(ss) + 1}. \quad (13)$$

The vector u^μ is future-directed. We consider the vicinity of the future horizon, so vectors l^μ and N^μ are also future-directed. Therefore, in what follows the coefficients $\alpha \geq 0$, $\beta \geq 0$.

Straightforward calculations give us

$$-(uh_{(0)}) = \frac{\alpha\beta + N^2}{2N\alpha}, \quad (14)$$

$$(uh_{(1)}) = \frac{N^2 - \alpha\beta}{2\alpha N}. \quad (15)$$

$$(uh_{(2)}) = (h_{(2)}s), \quad (16)$$

$$(uh_{(3)}) = \frac{L}{\sqrt{g_{\phi\phi}}} \quad (17)$$

where $L = u_\phi$ is the angular momentum per unit mass. If the metric does not depend on ϕ , it is conserved. However, we do not exploit such a property, so our consideration is more general.

Then, we can introduce the three-velocity in this frame according to [22]:

$$v^{(i)} = v_{(i)} = \frac{u^\mu h_{\mu(i)}}{-u^\mu h_{\mu(0)}}. \quad (18)$$

The absolute value of the velocity equals

$$v^2 = [v^{(1)}]^2 + [v^{(2)}]^2 + [v^{(3)}]^2. \quad (19)$$

It is seen from (18) that $v^2 < 1$ as it should be for massive particles. Indeed, using the representation of the metric in terms of orthonormal tetrad

$$g_{\mu\nu} = -h_{(0)\mu}h_{(0)\nu} + h_{(i)\mu}h_{(i)\nu} \quad (20)$$

where summation is taken over index i and taking into account that $(uu) = -1$, one obtains that

$$v^2 = \frac{\varepsilon}{1 + \varepsilon} < 1 \quad (21)$$

where $\varepsilon = u^\mu u^\nu h_{(i)\mu} h_\nu^{(i)} > 0$. Actually, eq. (18) is nothing else than natural generalization of formulas of special relativity $v^i = \frac{u^i}{u^0}$ where $u^0 = \frac{1}{\sqrt{1-v^2}}$.

One can check that for our metric and choice of tetrads

$$v^{(1)} = \frac{N^2 - \alpha\beta}{N^2 + \alpha\beta}, \quad (22)$$

$$v^{(2)} = 2(h_{(2)s}) \frac{N\alpha}{\alpha\beta + N^2}, \quad (23)$$

$$v^{(3)} = \frac{2LN}{\sqrt{g_{\phi\phi}}} \frac{\alpha}{N^2 + \alpha\beta}. \quad (24)$$

In the particular case of the Kerr metric, the effect of infinite acceleration for geodesic particle with all nonzero components of the velocity was studied in [15].

In the horizon limit $N \rightarrow 0$ we obtain for a generic case ("usual" particles) that

$$v^{(1)} \rightarrow -1, v^{(2)} \rightarrow 0, v^{(3)} \rightarrow 0, v \rightarrow 1. \quad (25)$$

Here the sign "minus" corresponds to motion towards the horizon.

However, there is special case ("critical particles") when near the horizon the quantity $\beta \rightarrow 0$ when $N \rightarrow 0$. As for a space-like vector $(ss) > 0$, the denominator in (13) does not vanish, so α has the same order as β . We assume that the first nonvanishing term in the Taylor expansion of β has the order N . (This is confirmed by explicit calculations for the geodesic motion in the Kerr metric [17]. In general, this can be taken simply as an assumption that, by definition, distinguishes usual and critical particles.) If $\beta \approx c_1 N$, the coefficient $\alpha \approx c_2 N$ where c_1 and c_2 are some coefficients. As, as is explained above, the coefficients α and β cannot be negative and $N > 0$ by definition, the coefficients $c_{1,2} > 0$.

Then, it follows from (22), (24) that in the limit under discussion

$$|v^{(1)}| \rightarrow \frac{1 - c_1 c_2}{1 + c_1 c_2} < 1. \quad (26)$$

$$v^{(2)} \rightarrow 2(sh_{(2)}) \frac{c_2}{c_1 c_2 + 1}, \quad (27)$$

$$v^{(3)} \rightarrow \frac{2L}{\sqrt{g_{\phi\phi}}} \frac{c_2}{1 + c_1 c_2}. \quad (28)$$

Thus, both components have the same order, the particle hits the horizon nonperpendicularly (as was noticed in [1] for the Kerr metric), $v \neq 1$. Actually, this means that $v < 1$ according to the property (21).

Let us denote the absolute values of velocities as v_1 and v_2 for particles 1 and 2, respectively (not to be confused with the tetrad components). Once the properties $v_1 \rightarrow 1$, $v_2 < 1$ are established for some pairs of particles, the further analysis of their collisions which was elaborated in [18] applies to this case directly, so we reduce our problem to the known one. As a result, the relative velocity of such two particles $w \rightarrow 1$, the corresponding Lorentz factor diverges and we gain an infinite energy in the centre of mass frame (see [18] for details).

Some reservations are in order. Particles can approach the extremal horizon but in the critical case cannot reach it. Then, the proper time needed for this is infinite. If the horizon is nonextremal, the critical particle cannot penetrate the potential barrier but the near-critical one can approach the horizon as nearly as one like. Then, the energy of collision is finite but can be made as large as one wishes. These issues are already considered in [6], [9], [10], so we do not repeat details here.

IV. MASSLESS CASE

A massless particle (photon for brevity) is characterized by the wave four-vector k^μ . Then, the frequency measured by ZAMO equals

$$\omega = -k^\mu h_{\mu(0)}. \quad (29)$$

Now, the normalization condition changes to $(kk) = 0$, so instead of (13) we have

$$\alpha = \frac{\beta}{(ss)}. \quad (30)$$

Then,

$$\omega = \frac{\alpha\beta + N^2}{2N\alpha} \quad (31)$$

For usual photons, with $\alpha, \beta \neq 0$,

$$\omega \approx \frac{\beta}{2N} \rightarrow \infty. \quad (32)$$

For critical ones, again $\alpha \sim \beta$ near the horizon, $\beta \approx c_1 N$, the coefficient $\alpha \approx c_2 N$, so we obtain that

$$\omega \approx \frac{c_1 c_2 + 1}{2c_2} \quad (33)$$

remains finite. This is nothing else than, by definition, the critical photons.

Once the existence of such photons with finite ω near the horizon is established, we can use our previous results again. Namely, different combinations of collisions between an electron and a photon are considered in Sec. VI of [18] with the conclusion that pairs (critical electron, usual photon) and (critical photon, usual electron) lead to infinitely growing energies in the centre of mass frame.

V. CONCLUSION

Thus, from geometric reasonings, we deduced kinematic properties of particles moving near the horizon ($N \rightarrow 0$) both for the massive and massless cases. This revealed the role of "critical" particles as having special behavior of coefficients of expansion of the four-velocity (or the wave vector) with respect to the null version of the ZAMO basis. The effect of infinite grow if the energy in the centre of mass frame depends crucially on whether $v \rightarrow 1$ or $v \neq 1$ for massive particles near the horizon and whether $\omega \rightarrow \infty$ or ω is finite in the massless case. Once the existence of critical particle is noticed, further results follow directly from previous works [18], details of which are not repeated here.

The present approach revealed the generality of the effect under discussion. We did not use geodesic equations of motion. Moreover, we even did not use the existence of Killing vectors and did not assume that the metric coefficients (1) are independent of t and ϕ . Therefore, the results have a rather general character. They apply to any surfaces of infinite redshift (horizons) which can be characterized by the property $N \rightarrow 0$ in metric (1). Thus, pure geometric approach of [17] perfectly agrees with the kinematic ones of [18] under very general circumstances.

This generality means a challenge to attempts to restrict the effect under discussion in such collisions invoking backreaction or gravitational radiation [2], [3]. It is not clear, whether and how account for these factors can restrict the grow of the energy and in what way they can change the role of critical particles. For better understanding, dynamic analysis

should be combined with geometrical and kinematic approaches.

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