A_4 Flavor Models in Split Seesaw Mechanism

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Abstract

A seesaw mechanism in an extra-dimension, known as the split seesaw mechanism, provides a natural way to realize a splitting mass spectrum of right-handed neutrinos. It leads to one keV sterile neutrino as a dark matter candidate and two heavy right-handed neutrinos being responsible for leptogenesis to explain the observed baryon asymmetry of the Universe. We study models based on A_4 flavor symmetry in the context of the split seesaw mechanism. It is pointed out that most of known A_4 flavor models with three right-handed neutrinos being A_4 triplet suffer from a *degeneracy problem* for the bulk mass terms, which disturbs the split mechanism for right-handed neutrino mass spectrum. Then we construct a new A_4 flavor model to work in the split seesaw mechanism. In the model, the experimentally observed neutrino masses and mixing angles can be realized from both type I+II seesaw contributions. The model predicts the $\mu - \tau$ symmetry in the neutrino mass matrix at the leading order, resulting in the vanishing θ_{13} and maximal θ_{23}. The flavor symmetry A_4 is broken via the flavon vacuum alignment which can be obtained from the orbifold compactification. The model can be consistent with all data of neutrino oscillation experiments, cosmological discussions of dark matter abundance, leptogenesis, and recent astrophysical data.

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1 Introduction

Extra-dimensional theory is a fascinating approach that affords a compelling candidate solution to the hierarchy problem [1]. Indeed, the discovery of an evidence of extra-dimension as well as the Higgs particle and supersymmetry (SUSY) is one of the important missions of the CERN Large Hadron Collider (LHC) experiment. In the extra-dimensional theory, there are also some alternatives to the ordinary electroweak symmetry breaking (EWSB) mechanism in the standard model (SM), such as the gauge-Higgs unification (GHU) [2], the little Higgs [3], the Higgsless [4], and the Dirichlet Higgs [5] models, and so on. Furthermore, extra-dimensional models can give phenomenologically interesting features and predictions for physics beyond the SM, for example, the presences of Kaluza-Klein particles and a candidate for dark matter (DM) [6] from the Universal Extra-Dimensions (UED) model [7], and deviations of couplings of the Higgs [5, 8].

The elucidation of the origin of DM, which governs about 23% of the Universe [9], is one of important goals of the particle physics today. A large number of DM candidates have been discussed in literature.* One of the interesting candidates for DM is a keV sterile neutrino since some astrophysical data possibly support the existence [11, 12]. In extentions of the SM, the right-handed neutrinos can be added. The models with three right-handed neutrinos whose masses are below the EW scale have been proposed in [13, 14, 15]. In particular, the ref. [15] has pointed out that the model with one keV sterile (right-handed) neutrino and two GeV range of ones can explain simultaneously the DM and baryon asymmetry of the Universe (BAU) in addition to the results of neutrino oscillation experiments. This framework is known as νMSM [14, 15]. Furthermore, the keV sterile neutrino has been also considered in a gauge extension of the SM (left-right symmetric framework) [16]. A realization of splitting pattern of right-handed neutrino mass spectrum including keV mass state in the context of flavor symmetry has been also discussed in [17, 18, 19]. In [17], it has been shown that the degeneracy of two singlet Majorana neutrinos and the lightness (keV scale) of the third one in νMSM can be a consequence of a lepton number symmetry, which is broken in both the Yukawa and Majorana mass sectors. In [18], a keV sterile neutrino as a candidate for DM is induced from softly breaking of $L_e - L_\mu - L_\tau$ flavor symmetry. In [19], a realization of the keV sterile neutrino arises from the Froggatt-Nielsen mechanism [20]. Other interesting direction to realize the keV sterile neutrino DM and BAU has been discussed in the split seesaw mechanism [21]. This mechanism can realize a splitting mass spectrum of the right-handed neutrinos in the context of extra-dimension and account for the smallness of active neutrino masses without fine-tuning. In the mechanism, the lightest right-handed neutrino with keV mass

*See ref. [10], for a recent review, and references therein.

becomes DM candidate and the other two right-handed neutrinos make a source of BAU via leptogenesis [22].

Regarding with properties of active neutrinos, various neutrino oscillation experiments have shown that the neutrino mixing pattern is peculiar, namely, there are two large mixing angles and one small one [23]. A lot of works have been proposed to explain such a peculiar mixing pattern, e.g. $\mu-\tau$ symmetric texture [24], tri-bimaximal mixing [25], golden ratios [26], and etc.. A proposed mixing pattern can be explained by imposing a non-Abelian discrete flavor symmetry with its breaking in a specific direction. For example, $\mu-\tau$ symmetry can be explained by the group $S_3 \simeq D_3$ [27] or D_4 [28]. The tri-bimaximal mixing can be obtained from A_4 [29] or S_4 [30]. The golden ratios have been discussed with A_5 [31] and D_{10} [32].[†] Such the non-Abelian discrete symmetries might arise from the breaking of the gauge symmetries [34] or from the orbifold compactification in extra-dimensions [35]. Moreover, the vacuum alignment of the flavon field can be achieved without dealing with a relatively complicated scalar potential for A_4 [36] and S_4 [37] symmetries in the extra-dimensional theory.

In this paper, we study models based on a flavor symmetry in the context of the split seesaw mechanism which can lead to a splitting mass spectrum of the right-handed neutrinos.[‡] The paper is organized as following: In section 2, a brief review of the split seesaw mechanism is given. In section 3, we discuss the A_4 flavor models in the split seesaw mechanism. Then it will be shown how the desired flavon vacuum alignment can be obtained. We also give a comment on the leptogenesis. Section 4 is devoted to the conclusion.

2 Split Seesaw Mechanism

In this section, we give a brief review of the split seesaw mechanism [21]. The relevant terms in the Lagrangian for the canonical type I seesaw mechanism [39] reads

$$\mathcal{L} \;=\; i\bar{N}_i\gamma^\mu\partial_\mu N_i + \left(\lambda_{i\alpha}\bar{N}_i L_\alpha\,\phi - \frac{1}{2}M_{R,ij}\bar{N}_i^c N_j + \text{h.c.}\right), \tag{2.1}$$

where N_i (i=1,2,3), L_α ($\alpha = e, \mu, \tau$), and ϕ are the right-handed neutrinos, lepton doublets, and the SM Higgs doublet, respectively. The i corresponds to the generation of the right-neutrinos, and here we introduce three right-handed neutrinos to the SM. After integrating

[†] Our references for flavor models based on non-Abelian discrete symmetry are not complete. See e.g. [33] for more complete list.

[‡] The ref. [38] has studied the realization of eV scale sterile neutrinos within both seesaw mechanism and flavor symmetry. The work has shown that light sterile neutrinos can be accommodated in A_4 flavor models.

out the heavy right-handed neutrinos, the light neutrino mass matrix is given by

$$M_\nu = \lambda^T M_R^{-1} \lambda \langle \phi^0 \rangle^2, \tag{2.2}$$

where the neutrino masses depend on the Yukawa coupling λ, the right-handed neutrino mass scale, and the vacuum expectation value (VEV) of the SM Higgs field. The seesaw mechanism can lead to small active neutrino mass with the order one Yukawa coupling and heavy right-handed neutrino mass. Moreover, the decay of the right-handed neutrino can also explain the BAU, if the mass of the right-handed neutrino is at $\mathcal{O}(10^{11-12})$ GeV [40]. This process is well known as leptogenesis.

Now, let us move on to an extra-dimensional theory compactified on the orbifold $S^1/(Z_2 \times Z_2')$ with an extra-dimensional coordinate, $y \equiv x^5$. Note that the original model of split seesaw mechanism [21] has considered S^1/Z_2. An additional Z_2' in our case is imposed in order to break the flavor symmetry as we will discuss in section 3.4. However, the introduction of the additional Z_2' does not change an essential point of the splitting mechanism. The circle S^1 has a radius R. The orbifold S^1/Z_2 is obtained by modding out the Z_2 transformation, which imposes the equivalence relation, $y \sim -y$. The orbifold $S^1/(Z_2 \times Z_2')$ is obtained by modding out further the Z_2' transformation. It imposes the equivalence relation, $y' \sim -y'$, where $y' \equiv y + \pi R/2$. The fundamental region of this orbifold is given by $y \in [0, \ell \equiv \pi R/2]$. The size of the extra-dimension ℓ and the five dimensional fundamental scale M are related to the 4D reduced Planck scale as $M_{pl}^2 = M^3 \ell$. The orbifold gives two fixed points (branes). One of the branes ($y = 0$) is the SM brane where the SM particles reside, while the other ($y = \ell$) is the hidden brane.

Next, a Dirac spinor, $\Psi(y, x) = (\chi_\alpha, \bar{\psi}^{\dot{\alpha}})^T$, is introduced in the bulk with a bulk mass term m,

$$S = \int d^4x dy M (i\bar{\Psi}\Gamma^A \partial_A \Psi + m\bar{\Psi}\Psi), \tag{2.3}$$

where $A = 0, 1, 2, 3, 5$, and the five-dimensional gamma matrices Γ^A are defined as following

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \Gamma^5 = -i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.4}$$

Note that the mass scale M in (2.3) is inserted so that the mass dimension of Ψ is 3/2 as in the 4D case. The zero mode of Ψ follows the Dirac equation as

$$(i\Gamma^5 \partial_5 + m)\Psi^{(0)} = 0, \tag{2.5}$$

leading to the wavefunction profile of the zero mode in the bulk as $e^{\mp my}$ for χ and $\bar{\psi}$. In order to get the chiral fermion in 4D, we promote that the field is transformed under the orbifold parities as

$$Z_2: \ \Psi \to P\Psi = +\Psi, \quad Z_2': \ \Psi \to P'\Psi = +\Psi, \tag{2.6}$$

where $P = -i\Gamma_5$ and $P' = 1$. As the results, only $\bar{\psi}$ can have a zero mode. Note that the bulk mass term m also has to carry negative Z_2 parity, so that the action is invariant under the above parity transformations. The bulk mass term having the negative parity can be realized from the kink profile of scalar field along the extra-dimensional direction. After canonically normalizing the fermion in 4D, the zero mode of $\Psi_R = (0, \bar{\psi})^T$ can be written in term of canonically normalized field $\psi_R^{(4D)}(x)$ as

$$\Psi_R^{(0)}(y,x) = \sqrt{\frac{2m}{e^{2m\ell}-1}}\frac{1}{\sqrt{M}}e^{my}\psi_R^{(4D)}(x), \tag{2.7}$$

where $\psi_R^{(4D)}(x)$ is identified with the right-handed neutrino in 4D. The point is that the extra-dimensional wavefunction profile of $\Psi_R^{(0)}(y,x)$ with real positive m localizes on the hidden brane since the profile is expressed by the exponential function, e^{my}. Then $\psi_R^{(4D)}(x)$ couples to particles in the SM brane through exponentially suppressed couplings when $m\ell \gg 1$.

Now let us show the action for the three right-handed neutrinos,

$$\begin{aligned} S &= \int d^4x\, dy \left\{ M \left(i\bar{\Psi}_{iR}^{(0)}\Gamma^A\partial_A\Psi_{jR}^{(0)} + m_i\bar{\Psi}_{iR}^{(0)}\Psi_{iR}^{(0)}\right)\right. \\ &\qquad \left. +\delta(y)\left(\frac{\kappa_{ij}}{2}v_{\mathrm{B-L}}\bar{\Psi}_{iR}^{(0)c}\Psi_{jR}^{(0)} + \tilde{\lambda}_{i\alpha}\bar{\Psi}_{iR}^{(0)}L_\alpha\phi + \text{h.c.}\right)\right\}, \end{aligned} \tag{2.8}$$

where we work in the basis of the diagonal bulk mass term m_i. The Majorana mass of the right-handed neutrino arises from the breaking of $U(1)_{\mathrm{B-L}}$ symmetry at a high scale. Note that $v_{\mathrm{B-L}}$ determines the $B-L$ breaking scale. Inserting the result of (2.7) into (2.8), the effective 4D mass and Yukawa coupling matrices are obtained [41],

$$M_{R,ij} = \kappa_{ij}f_i f_j v_{\mathrm{B-L}}, \tag{2.9}$$

$$\lambda_{i\alpha} = \tilde{\lambda}_{i\alpha}f_i, \tag{2.10}$$

where

$$f_i \equiv \frac{1}{\sqrt{M}}\sqrt{\frac{2m_i}{e^{2m_i\ell}-1}}. \tag{2.11}$$

For the sake of the following discussion, we define a diagonal matrix F_{ij} as

$$F_{ij} \equiv \delta_{ij}f_j. \tag{2.12}$$

The effective 4D mass and Yukawa coupling matrices given in (2.9) and (2.10) can be rewritten in term of the matrix F_{ij} as

$$M_{R,ij} = F_{il}\kappa_{lm}F_{mj}v_{\mathrm{B-L}}, \tag{2.13}$$

$$\lambda_{i\alpha} = F_{in}\tilde{\lambda}_{n\alpha}. \tag{2.14}$$

After the seesaw mechanism, the light neutrino mass matrix reads

$$M_\nu = \lambda^T M_R^{-1} \lambda \langle \phi^0 \rangle^2 = \tilde{\lambda}^T \kappa^{-1} \tilde{\lambda} \frac{\langle \phi^0 \rangle^2}{v_{\rm B-L}}, \tag{2.15}$$

where the factor f is canceled out in the seesaw formula. By assigning the appropriate values for m_i, we can obtain one keV neutrino and two heavy ones naturally. For example, if we assume that the right-handed neutrino mass matrix is diagonal, we can realize a splitting mass spectrum of right-handed neutrinos, including e.g. both keV and very heavy mass scales, without fine-tuning as

$$(M_{R,1}, M_{R,2}, M_{R,3}) = (1 \text{ keV}, 10^{11} \text{ GeV}, 10^{12} \text{ GeV}), \tag{2.16}$$

for $(m_1\ell, m_2\ell, m_3\ell) \simeq (24.2, 3.64, 2.26)$ where we take $M = 5 \times 10^{17}$ GeV, $\ell^{-1} = 10^{16}$ GeV, $v_{\rm B-L} = 10^{15}$ GeV, $\kappa_{ii} = 1$ as reference values, and $M_{R,i}$ is an effective right-handed neutrino mass. Therefore, the lightest right-handed neutrino with the mass of order keV scale can be a candidate for DM and heavy right-handed neutrinos with the masses of order $\mathcal{O}(10^{11-12})$ GeV can lead to the BAU via leptogenesis if the reheating temperature is larger than $\mathcal{O}(10^{11})$ GeV or $\mathcal{O}(10^{12})$ GeV unless the heavier guys are not extremely degenerated like in (2.16). These are the essential points of the split seesaw mechanism.

3 A_4 Flavor Models in Split Seesaw Mechanism

An introduction of non-Abelian discrete flavor symmetry might be still one of fascinating approaches to explain the currently observed patterns of neutrino mixing angles rather than an approach that the current data are solely explained by determining relevant Yukawa couplings without symmetry and/or dynamics. Therefore, we attempt to embed such the symmetry into the split seesaw mechanism. The A_4 flavor symmetry is well motivated for the three generations of the SM fermions, as it is the smallest non-Abelian discrete group which has the triplet representation.

3.1 Embedding A_4 Flavor Models into Split Seesaw Mechanism

A large number of flavor models based on the A_4 symmetry are widely discussed in literatures. Then an useful classification for A_4 flavor models has been recently presented in [42]. We discuss the embedding A_4 flavor models into split seesaw mechanism with this Barry-Rodejohann (BR) classification for A_4 models, and show a simplified BR classification in Tab. 1.[§] That is classified by differences of particle assignments under A_4 symmetry for the lepton doublets L, right-handed charged leptons E, and right-handed neutrinos Ψ_R (the Higgs triplets Δ are included in the original BR classification).

[§]See [42] for the references of corresponding flavor models.

Type	L_α	$\overline{E}_\alpha$	$\overline{\Psi}_{iR}$
A	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$, $\underline{\mathbf{1}}''$	$\cdots$
B	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$, $\underline{\mathbf{1}}''$	$\underline{\mathbf{3}}$
C	$\underline{\mathbf{3}}$	$\underline{\mathbf{3}}$	$\cdots$
D	$\underline{\mathbf{3}}$	$\underline{\mathbf{3}}$	$\underline{\mathbf{3}}$
E	$\underline{\mathbf{3}}$	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$, $\underline{\mathbf{1}}''$
F	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$, $\underline{\mathbf{1}}''$	$\underline{\mathbf{3}}$	$\underline{\mathbf{3}}$
G	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$, $\underline{\mathbf{1}}''$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}'$, $\underline{\mathbf{1}}''$
H	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$	$\cdots$
I	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$
J	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$	$\underline{\mathbf{3}}$

Table 1: Simplified Barry-Rodejohann (BR) classification for A_4 flavor models

We are taking note of the right-handed neutrino mass spectrum in the split seesaw mechanism. Therefore, we discuss flavor models of Type B, D, E, F, G, I, and J because the models of Type A, C, and H do not contain the right-handed neutrinos. Moreover, it is remarked that the models of Type B, D, F, and J, which have assigned the right-handed neutrinos into the triplet $\Psi_R = (\Psi_{1R}, \Psi_{2R}, \Psi_{3R})^T$, cannot work in the context of split seesaw mechanism because the bulk mass term are degenerated,

$$m\bar{\Psi}_R\Psi_R = m(\bar{\Psi}_{1R}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{2R} + \bar{\Psi}_{3R}\Psi_{3R}). \tag{3.1}$$

Such a degenerated bulk mass spectrum cannot lead to a splitting mass spectrum of right-handed neutrinos as shown in (2.16). Therefore, we can conclude that the suitable A_4 model for the split seesaw mechanism is that the three right-handed neutrinos should not be in the same multiplet. This is one of key observations in this work.

The model of Type E has been proposed by Ma [43] and numerical analyses in the model has been presented in [44]. This model is a simple extension of the SM to a model with A_4 flavor symmetry and three right-handed neutrinos represented by singlets under the symmetry. Therefore, we will consider the model further in the next subsection. The model of Type G has been proposed in [45]. The work has pointed out that possible mass spectrum of active neutrinos under the flavor charge assignment are $m_2 \gg m_1 = m_3 = 0$ or $m_3 \gg m_1 = m_2$. It is seen from the current neutrino experiments that they are ruled out. Therefore embedding the model of Type G into the split seesaw mechanism is not attractive. Finally, the model of Type I [46] is the first complete supersymmetric model of flavor based on A_4 symmetry together with the $SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ Pati-Salam gauge symmetry. According to our simple discernment, the model can pass the above *degeneracy problem*. Since the model is supersymmetric version, which means that the model contains the lightest superparticle as a candidate for DM, embedding the mode

Field	L_α	$\overline{E}_\alpha$	$\overline{\Psi}_{1R}$	$\overline{\Psi}_{2R}$	$\overline{\Psi}_{3R}$	ϕ	$\varphi_{\nu,t}$	φ_{1l}	φ_{2l}	φ_{3l}
A_4	$\underline{\mathbf{3}}$	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}'$	$\underline{\mathbf{1}}''$	$\underline{\mathbf{1}}$	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}''$	$\underline{\mathbf{1}}'$
Z_2^{aux}	$+$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$
Z_5^{aux}	$\tilde{\omega}^3$	$\tilde{\omega}^4$	1	1	1	1	$\tilde{\omega}^2$	$\tilde{\omega}^3$	$\tilde{\omega}^3$	$\tilde{\omega}^3$

Table 2: The particle contents and charge assignment of the model: $\varphi_{\nu,t}$ and φ_{il} are flavons, which are assigned to the triplet and singlets, respectively. Note that $\tilde{\omega} \equiv e^{2\pi i/5}$.

of Type E proposed by Ma into the split seesaw mechanism is more attractive for our considerations. Therefore, we will consider a simple extension of the SM, which is similar to the model of type E, in the next subsection.

3.2 Basic Model

In this subsection, we consider an A_4 model with right-handed neutrinos assigned to singlets. The particle contents and charge assignment of the model discussed here are similar to the ones proposed by Ma in [43] except that in addition to Z_2^{aux} in Ma model, we add the auxiliary symmetry Z_5^{aux}.¶ The particle contents and charge assignment of the model are given in Tab. 2.

Now, let us first discuss the neutrino sector, the action is given in (2.8). The κ and $\hat{\lambda}$ at the brane are given by

$$\kappa = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}, \quad \tilde{\lambda} = \frac{1}{\Lambda} \begin{pmatrix} y_1^\nu & 0 & 0 \\ 0 & y_2^\nu & 0 \\ 0 & 0 & y_3^\nu \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & u_3 \end{pmatrix}, \tag{3.2}$$

and the bulk mass m becomes diagonal, $m = \text{Diag}\{m_1, m_2, m_3\}$, under the above charge assignment, where Λ is a cut-off of the theory. The coupling $\tilde{\lambda}_{i\alpha}$ arise after the flavon $\varphi_{\nu,t}$ obtaining VEV as $\langle\varphi_{\nu,t}\rangle = (u_1, u_2, u_3)^T$,

$$\left[y_1^\nu \bar{\Psi}_{1R}^{(0)} \frac{(L\varphi_\nu)}{\Lambda} + y_2^\nu \bar{\Psi}_{2R}^{(0)} \frac{(L\varphi_\nu)''}{\Lambda} + y_3^\nu \bar{\Psi}_{3R}^{(0)} \frac{(L\varphi_\nu)'}{\Lambda} \right] \phi \to \tilde{\lambda}_{i\alpha} \bar{\Psi}_{iR}^{(0)} L_\alpha \phi. \tag{3.3}$$

The neutrino Dirac mass matrix is given by

$$M_D = \lambda\langle\phi^0\rangle = F\tilde{\lambda}\langle\phi^0\rangle = \frac{1}{\Lambda} F \begin{pmatrix} y_1^\nu & 0 & 0 \\ 0 & y_2^\nu & 0 \\ 0 & 0 & y_3^\nu \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & u_3 \end{pmatrix} v_{ew}, \tag{3.4}$$

¶The auxiliary symmetry Z_5^{aux} plays an essential role in constraining the terms in the flavon scalar potential which will be shown in the following subsection.

where $\langle\phi^0\rangle = v_{ew}$. The mass matrix of right-handed neutrinos in this model reads

$$M_R = F\kappa F = \begin{pmatrix} af_1^2 & 0 & 0 \\ 0 & 0 & bf_2f_3 \\ 0 & bf_2f_3 & 0 \end{pmatrix} v_{B-L}, \tag{3.5}$$

which leads to the mass eigenvalues as

$$M_{R,1} = af_1^2 v_{B-L}, \quad M_{R,2} = M_{R,3} = bf_2f_3 v_{\mathrm{B-L}}. \tag{3.6}$$

Inserting (3.2) into (2.15), the mass matrix of light neutrinos is given by

$$M_\nu = \lambda^T\kappa^{-1}\lambda\frac{\langle\phi^0\rangle^2}{v_{\mathrm{B-L}}} = \begin{pmatrix} \frac{u_1}{\Lambda} & 0 & 0 \\ 0 & \frac{u_2}{\Lambda} & 0 \\ 0 & 0 & \frac{u_3}{\Lambda} \end{pmatrix} \begin{pmatrix} x & y & y \\ y & x & y \\ y & y & x \end{pmatrix} \begin{pmatrix} \frac{u_1}{\Lambda} & 0 & 0 \\ 0 & \frac{u_2}{\Lambda} & 0 \\ 0 & 0 & \frac{u_3}{\Lambda} \end{pmatrix} \frac{v_{ew}^2}{v_{\mathrm{B-L}}}, \tag{3.7}$$

where

$$x \equiv \frac{(y_1^\nu)^2}{a} + \frac{2y_2^\nu y_3^\nu}{b}, \quad y \equiv \frac{(y_1^\nu)^2}{a} - \frac{y_2^\nu y_3^\nu}{b}. \tag{3.8}$$

As discussed in [43], the limit where $u_1 = u_2 = u_3 = u$, the A_4 symmetry is broken to Z_3 in the neutrino sector and the eigenvalues of M_ν are

$$(x+2y)\frac{v_{ew}^2}{v_{\mathrm{B-L}}}\left(\frac{u}{\Lambda}\right)^2, \quad (x-y)\frac{v_{ew}^2}{v_{\mathrm{B-L}}}\left(\frac{u}{\Lambda}\right)^2, \quad (x-y)\frac{v_{ew}^2}{v_{\mathrm{B-L}}}\left(\frac{u}{\Lambda}\right)^2. \tag{3.9}$$

The neutrino mass matrix can be diagonalized by the tri-bimaximal mixing matrix. However, this case is not realistic since the first eigenvalue corresponds to the eigenstate $\frac{1}{\sqrt{3}}(1,1,1)$. Therefore, we have to consider the other case, that is, $u_1 \neq u_2 = u_3 = u$. In this case, the mass matrix still leads to $\theta_{13} = 0$ and $\theta_{23} = \pi/4$, while θ_{12} is determined by the parameters in the neutrino mass matrix. Using the reparametrization as $u_1 = ru$ and $u_2 = u_3 = u$, we can rewrite (3.7) as

$$M_\nu = \begin{pmatrix} r^2x & ry & ry \\ ry & x & y \\ ry & y & x \end{pmatrix} m_\nu \quad \text{with} \quad m_\nu \equiv \frac{v_{ew}^2}{v_{\mathrm{B-L}}}\left(\frac{u}{\Lambda}\right)^2. \tag{3.10}$$

This mass matrix predicts $\theta_{13} = 0$ and $\theta_{23} = \pi/4$, and the other observables are described by model parameters as follows

$$\Delta m_{21}^2 \equiv |m_2|^2 - |m_1|^2 = (x + r^2x + y)\delta_\nu m_\nu^2, \tag{3.11}$$

$$\Delta m_{31}^2 \equiv |m_3|^2 - |m_1|^2 = \left[(x-y)^2 - \frac{[(1+r^2)x + y - \delta_\nu]^2}{4}\right] m_\nu^2, \tag{3.12}$$

$$\tan^2\theta_{12} = \frac{[(1-r^2)x + y - \delta_\nu]^2}{8r^2y^2}, \tag{3.13}$$

where

$$\delta_\nu \equiv \sqrt{(1-r^2)^2x^2 + 2(1-r^2)xy + (1+8r^2)y^2}, \tag{3.14}$$

with real r, x, and y. Now, let us fit the experimental data of neutrino oscillation, Δm_{21}^2, Δm_{31}^2, and θ_{12} in this model. The best fit values [23] for the normal mass hierarchy are

$$\Delta m_{21}^2 = 7.64 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = 2.45 \times 10^{-3} \text{ eV}^2, \quad \sin^2\theta_{12} = 0.312. \tag{3.15}$$

For example, the following values can reproduce the experimental values:[‖]

$$(xm_\nu, ym_\nu, r) = (-2.23 \times 10^{-2} \text{ eV},\ 2.76 \times 10^{-2} \text{ eV},\ \pm 0.204). \tag{3.16}$$

Note that the model can also fit with the tri-bimaximal mixing, $\sin^2\theta_{12} = 1/3$. For the charged lepton sector, the action is described by

$$S_l = \int d^4x dy \delta(y) \left((\epsilon\phi^*) \left[y_1^e (\overline{E}L) \frac{\varphi_{1l}}{\Lambda} + y_2^e (\overline{E}L)' \frac{\varphi_{2l}}{\Lambda} + y_3^e (\overline{E}L)'' \frac{\varphi_{3l}}{\Lambda} \right] \right). \tag{3.17}$$

After the flavons obtain their VEVs, it leads to a diagonal charged lepton mass matrix. We can also reproduce the charged lepton masses by taking appropriate values of $y_i^e \langle \varphi_{il} \rangle / \Lambda$.

Next, let us calculate the left-right mixing in this model, which is an important quantity in the sterile neutrino DM scenario. This scenario is a decaying DM candidate. To be DM, the lifetime of the lightest right-handed neutrino should be greater than the age of the Universe. The lightest right-handed neutrino can radiatively decay into a photon and an active neutrino through the left-right mixing. This radiative decay produces a narrow line in the diffuse X-ray background, which gives a restriction on the magnitude of this mixing angle [47]. The relevant left-right mixing angle is given by

$$\theta^2 = \frac{\sum_\alpha |\lambda_{1\alpha}|^2}{M_{R,1}^2} \langle \phi_0 \rangle^2 = (2 + r^2) \frac{|y_1^\nu|^2}{a} \frac{m_\nu}{m_s} < 1.8 \times 10^{-5} \left(\frac{1 \text{ keV}}{m_s} \right)^5, \tag{3.18}$$

where $m_s = M_{R,1}$. If we use $m_s = 5$ keV, $m_\nu = 1.47 \times 10^{-3}$ eV, and $r = 0.204$, we obtain an upper bound on model parameters as $|y_1^\nu|^2/a < 9.63 \times 10^{-3}$. Note that the value of $m_\nu = 1.47 \times 10^{-3}$ eV is obtained from $v_{ew} = 174$ GeV, $v_{B-L} = 10^{15}$ GeV, and $u/\Lambda \simeq 0.22$. Using (3.8), (3.16) and the value of m_ν, we obtain $|y_1^\nu|^2/a = (x + 2y)/2 \simeq 11.3$, which is much larger than the above cosmological bound on a sterile neutrino DM scenario. The upper bound obtained here implies that the coupling to the keV sterile neutrino is too tiny and the only two heavy neutrinos can be taken into an account for the neutrino mass. Therefore, this A_4 model cannot realize a keV sterile neutrino DM scenario via the split seesaw mechanism though it can reproduce realistic mixing angles and two mass squared differences of active neutrinos. In the next subsection, we extend this A_4 flavor model to achieve the sterile neutrino DM scenario from split seesaw mechanism.

[‖]The model can also fit the inverted mass hierarchy (IH) by taking complex values of r, x, and y.

Field	Δ_L	$\varphi_{\nu,s}$	$\tilde{\varphi}_{\nu,t}$
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{3}}$
Z_2^{aux}	$+$	$+$	$+$
Z_5^{aux}	$\tilde{\omega}^2$	$\tilde{\omega}^2$	1

Table 3: Additional particle contents

3.3 Extension of the Model

In order to realize the keV sterile neutrino DM scenario in the split seesaw mechanism while conserving the realistic neutrino oscillation data, we extend the A_4 model by including the $SU(2)_L$ triplet Higgs and two gauge singlet flavons. The triplet Higgs and one of flavons are assigned to singlet under A_4 and the other flavon is a triplet. The extended particle contents is given in Tab. 3. Note that the triplet flavon $\tilde{\varphi}_{\nu,t}$ will not contribute to the neutrino mass and mixing directly, however, it is crucial to obtain a VEV alignment for the another triplet flavon $\varphi_{\nu,t}$, which will be discussed in the next subsection.

The neutrino mass in this model is obtained from the type I+II seesaw mechanism. The neutrino sector with the contribution of type II seesaw mechanism [48] is described by

$$S_\nu = S_\nu^I + S_\nu^{II}, \tag{3.19}$$

where the type I contribution, S_ν^I, is given in (2.8) and the type II contribution is given by

$$S_\nu^{II} = \int d^4x dy \delta(y) \left(\tilde{y}_1^\nu \frac{\varphi_{\nu,t}}{\Lambda} + \tilde{y}_2^\nu \frac{\varphi_{\nu,s}}{\Lambda} \right) L^T \Delta_L L. \tag{3.20}$$

As discussed previously, the left-right mixing angle puts the upper bound on the coupling of the left-handed fields to the keV sterile neutrino, $|y_1^\nu|^2/a < 9.63 \times 10^{-3}$, which is too tiny to be account for the neutrino mass scales. Keeping this constraint in mind, we obtain $x \simeq 2y_2^\nu y_3^\nu/b$ and $y \simeq -y_2^\nu y_3^\nu/b$, resulting in $x \simeq -2y$. With the constraint on the left-right mixing angle, the type I contribution alone cannot produce the realistic neutrino oscillation data in this A_4 flavor model, therefore, we introduce the type II contribution. For the sake of the discussion, we will assume that $r = 0$, that is, the VEV alignment of $\langle\varphi_{\nu,t}\rangle$ is given by $\langle\varphi_{\nu,t}\rangle = (0, u, u)^T$. We will show in the next subsection that this VEV alignment can be achieved from the orbifolding. Using $x = -2y$ and $r = 0$, the neutrino mass matrix from type I contribution (3.10) becomes

$$M_\nu^I \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2y & y \\ 0 & y & -2y \end{pmatrix} m_\nu. \tag{3.21}$$

After the triplet Higgs obtains a VEV, $v_\Delta \equiv \langle\Delta_L\rangle \simeq v_{ew}^2\mu/M_\Delta^2$, with the $B-L$ breaking scale $\mu \sim v_{\mathrm{B-L}}$ and the heavy triplet mass M_Δ, and the flavor symmetry is broken by VEV of flavons, the neutrino mass from the type II seesaw mechanism is given by

$$M_\nu^{II} = \begin{pmatrix} s & t & t \\ t & s & 0 \\ t & 0 & s \end{pmatrix} m'_\nu, \tag{3.22}$$

where

$$s \equiv \tilde{y}_2^\nu \frac{\langle \varphi_{\nu,s} \rangle / \Lambda}{u/\Lambda}, \quad t \equiv \tilde{y}_1^\nu, \quad m'_\nu \equiv v_\Delta \frac{u}{\Lambda}. \tag{3.23}$$

Assuming that $m'_\nu \simeq m_\nu$, the neutrino mass matrix reads

$$M_\nu = M_\nu^I + M_\nu^{II} \simeq \begin{pmatrix} s & t & t \\ t & s-2y & y \\ t & y & s-2y \end{pmatrix} m_\nu. \tag{3.24}$$

This mass matrix has the $\mu-\tau$ symmetry resulting in $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ at the leading order.** The other observables can be calculated at the leading order as

$$\Delta m_{21}^2 \simeq (2s - y)\sqrt{8t^2 + y^2} m_\nu^2, \tag{3.25}$$

$$\Delta m_{31}^2 \simeq \left[(s-3y)^2 - \frac{\left(2s - y - \sqrt{8t^2+y^2}\right)^2}{4} \right] m_\nu^2, \tag{3.26}$$

$$\tan^2\theta_{12} \simeq \frac{\left(y + \sqrt{8t^2+y^2}\right)^2}{8t^2}, \tag{3.27}$$

with real s, t, and y. Now, we can fit the experimental values (3.15) for the normal mass hierarchy as

$$(s, t, y) \simeq (-1.09, -2.31, \pm 2.06) \times 10^{-2}\ \mathrm{eV}/m_\nu. \tag{3.28}$$

The model can also fit the tri-bimaximal mixing angle with real s, t, and y, and the inverted mass hierarchy with complex values of the parameters. This A_4 model can reproduce realistic values of mixing angles and two mass squared differences of active neutrinos and be allowed by cosmological constraint on the left-right mixing angle in the context of keV sterile neutrino DM scenario from split seesaw mechanism.

It is well known that the keV sterile neutrino DM should not contribute to the neutrino mass via the type I seesaw mechanism to satisfy the X-ray bound. However, the A_4 flavor symmetry discussed in the previous subsection constrains on the Yukawa structure to be

**The latest T2K results have suggested $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ at 90% C.L. for NH (IH) the vanishing Dirac CP phase [49]. Then several theoretical discussions for the results have been presented [50]. Our model can give a non-vanishing contribution due to the next-leading order corrections. On the other hand, the recent MINOS results can be still consistent with the vanishing θ_{13} [51].

a specific form as given in (3.4). This means that the basic A_4 model cannot satisfy the cosmological constraint of the keV sterile neutrino DM while reproducing the observables of active neutrinos (mass squared differences and mixing angles) as we shown. However, once we introduce a contribution from the type II seesaw mechanism to the Majorana mass matrix of light neutrinos (3.22), the situation is changed, that is, the type II contribution can make the active neutrino observables realistic while realizing the keV sterile neutrino DM scenario. This is because the type II contribution to the Majorana mass matrix of light neutrinos is irrelevant to the sterile neutrino sector. This kind of scenario would be applied to any models (even with a flavor symmetry) where one of the three right-handed neutrinos is DM of keV mass (e.g. νMSM).

3.4 Flavor Symmetry Breaking from Orbifolding

In this subsection, we show how the desired flavon VEV alignment can be obtained in the extra-dimensional context. The general discussion how to obtain the vacuum alignment for the flavon by orbifolding has been discussed before in [36] for A_4 in a different A_4 basis (see [37] for S_4 case).[††]

Let us consider a flavon field in the bulk. The desired flavon VEV alignment can be obtained by properly choosing the orbifold parity P' and projecting out zero modes. Of course, there is a constraint on the choices of the orbifold parity, namely, it has to be chosen from elements of order two of A_4 group, otherwise, it does not preserve the product rule of the group. The elements of order two of A_4 group are as follows: S, T^2ST, and TST^2. To obtain the desired vacuum alignment, we choose the triplet flavon to transform under the orbifold parity Z_2' as $P' = S$.

We consider a bulk triplet flavon, $\varphi_\nu(y,x) = ((\varphi_\nu)_1, (\varphi_\nu)_2, (\varphi_\nu)_3)^T$, transformed under the orbifold parities as

$$Z_2: \ \varphi_\nu \to P\varphi_\nu = \eta\varphi_\nu, \quad Z_2': \ \varphi_\nu \to P'\varphi_\nu = \eta'\varphi_\nu, \tag{3.29}$$

where

$$P = 1, \quad P' = S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{3.30}$$

Note that the flavor symmetry with this assignment is broken at the hidden ($y = \ell$) brane, not at the SM ($y = 0$) brane. However, the flavor symmetry is also broken at the SM brane in a low energy regime since non-zero modes in the triplet whose mass is the

[††]See [52] for an alternative mechanism to flavor symmetry breaking in extra-dimensional setup. The work has proposed a general way to break flavor symmetry by imposing non-vanishing Dirichlet boundary condition on the bulk flavon(s), which can also achieve an arbitral VEV alignment.

compactification scale are absent. The VEV alignment of flavon depends on the parity η'. When the parities $\eta = +1$ and $\eta' = +1$ are given for $\tilde{\varphi}_{\nu,t}$, the zero modes become $\tilde{\varphi}_{\nu,t} = ((\tilde{\varphi}_{\nu,t})_1, 0, 0)^T$. When the parities $\eta = +1$ and $\eta' = -1$ are for $\varphi_{\nu,t}$, the zero modes read $\varphi_{\nu,t} = (0, (\varphi_{\nu,t})_2, (\varphi_{\nu,t})_3)^T$. In general, $(\varphi_{\nu,t})_2$ does not need to equal to $(\varphi_{\nu,t})_3$. But we show that $(\varphi_{\nu,t})_2 = \pm(\varphi_{\nu,t})_3$ can be obtained from the minimization of the 4D flavon scalar potential in the following discussion.

At low energy, the effective (4D) flavon scalar potential is described by the zero modes of the flavon,

$$\begin{aligned} V_f^{(4\mathrm{D})} &= \frac{m_1^2}{2}(\tilde{\varphi}_{\nu,t})_1^2 + m_2^2(|(\varphi_{\nu,t})_2|^2 + |(\varphi_{\nu,t})_3|^2) + [\mu_{123}((\tilde{\varphi}_{\nu,t})_1(\varphi_{\nu,t})_2^*(\varphi_{\nu,t})_3 + \mathrm{h.c.}] \\ &\quad + m_0^2|(\varphi_{\nu,s})|^2 + \tilde{m}_1^2|(\varphi_{1l})|^2 + \tilde{m}_2^2|(\varphi_{2l})|^2 + \tilde{m}_3^2|(\varphi_{3l})|^2 + \sum_{i,j}\lambda_{ij}X_iX_j, \end{aligned} \tag{3.31}$$

where $X = ((\tilde{\varphi}_{\nu,t})_1^2, (|(\varphi_{\nu,t})_2|^2 + |(\varphi_{\nu,t})_3|^2), |(\varphi_{\nu,s})|^2, |(\varphi_{1l})|^2, |(\varphi_{2l})|^2, |(\varphi_{3l})|^2)^T$ and λ_{ij} are dimensionless couplings. Note that the normalization constant of the wave function and the cut-off are already included into the flavon scalar potential parameters and we assume that all singlet flavons are the bulk fields.

The VEV alignment can be obtained by the minimization of the flavon scalar potential, $\partial V_f^{(4\mathrm{D})}/(\partial\varphi_i) = 0$, where φ_i are flavons in the potential. We can see that the condition $(\varphi_{\nu,t})_2 = \pm(\varphi_{\nu,t})_3$ can be achieved if μ_{123} and $(\tilde{\varphi}_{\nu,t})_1$ are non-zero. This is the reason we introduce $\tilde{\varphi}_{\nu,t}$ even it does not give the contributions to the neutrino mass and mixing at the leading order.

3.5 Leptogenesis

At the end of this paper, we comment on the realization of BAU via leptogenesis in our A_4 model.

In our model, the two heavier right-handed neutrinos are not the only source for the BAU via leptogenesis, but we also have the contribution from the triplet Higgs [53]. Leptogenesis can be realized either by the decay of the heavier right-handed neutrinos or the triplet Higgs. The CP asymmetry in the decay of those right-handed neutrinos to the leptons and SM Higgs is given by

$$\epsilon_{\Psi_{kR}} = \sum_{\alpha} \frac{\Gamma(\Psi_{kR} \to L_i + H) - \Gamma(\Psi_{kR} \to \bar{L}_i + H^\dagger)}{\Gamma(\Psi_{kR} \to L_i + H) + \Gamma(\Psi_{kR} \to \bar{L}_i + H^\dagger)}, \tag{3.32}$$

where $k = 2$ and 3 because the decay of heavier neutrinos is CP asymmetric in the split seesaw mechanism. As usual, this asymmetry is due to the interference of the tree level diagram and the three one-loop diagrams shown in Fig. 1. The left two diagrams are the

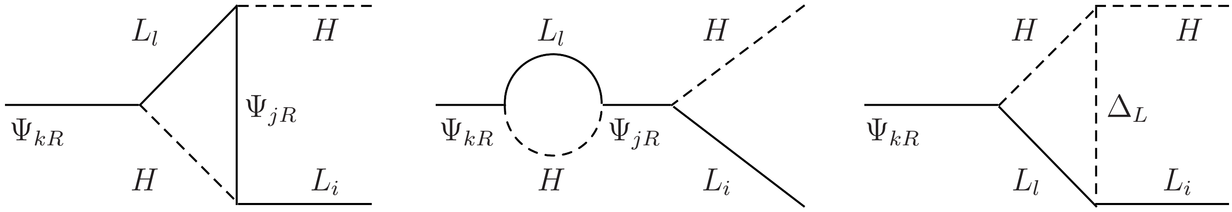

Figure 1: One-loop diagrams of the right-handed neutrino decay generating the CP asymmetry.

usual vertex and self-energy contributions, respectively, and give

$$\epsilon^I_{\Psi_{kR}} = \frac{1}{8\pi}\sum_{j\neq k}\frac{\mathrm{Im}[(\lambda\lambda^\dagger)^2_{kj}]}{\sum_i|\lambda_{ki}|^2}\sqrt{x_j}\left[1-(1+x_j)\log\left(\frac{x_j+1}{x_j}\right)+\frac{1}{1-x_j}\right], \tag{3.33}$$

where $x_j \equiv M^2_{R,j}/M^2_{R,k}$.[‡‡] The right diagram in Fig. 1 is an additional contribution to the CP asymmetry in our model, which is given by [53, 54] as

$$\epsilon^{II}_{\Psi_{kR}} = \frac{3}{8\pi}\frac{M_{R,k}v_\Delta}{v^2_{ew}}\sum_{j,l}\frac{\mathrm{Im}[\lambda^*_{kj}\lambda^*_{kl}Y^*_{\Delta jl}]}{\sum_i|\lambda_{ki}|^2}y_k\left[1-y_k\log\left(\frac{y_k+1}{y_k}\right)\right], \tag{3.34}$$

where $Y_\Delta \equiv M^{II}_\nu/v_\Delta$ and $y_k \equiv M^2_\Delta/M^2_{R,k}$. Note that there is also a decay process to generate the asymmetry, which is the decay of the triplet Higgs to two leptons. However, it is not important for our case as commented below.

There are four limiting cases for leptogenesis in the type I+II seesaw scenario [53]: [i] $M_{R,k} \ll M_\Delta$ with a dominant contribution of the right-handed neutrinos to the light neutrino masses, [ii] $M_{R,k} \ll M_\Delta$ with a dominant triplet contribution to the light neutrino masses, [iii] $M_{R,k} \gg M_\Delta$ with a dominant right-handed neutrino contribution to the neutrino masses, and [iv] $M_{R,k} \gg M_\Delta$ with dominance of the triplet contribution to the neutrino masses, where $M_{R,k}$ is the mass of the heavier right-handed neutrino. The limit [i] is almost equivalent to the ordinary leptogenesis scenario in the framework of type I seesaw mechanism, and thus this case has been extensively studied in the literatures. In the limit [ii], a constraint on the neutrino mass does not induce a violation of the out-of-equilibrium condition, $\Gamma_{M_{R,k}} < H(T)|_{T=M_{R,k}}$, where $H(T)$ is the Hubble parameter and T is the cosmic temperature, since the couplings responsible for the neutrino masses are not responsible for the tree level decay. As a result, there is no upper bound on the neutrino masses as in the usual type I leptogenesis, which is an interesting feature of the limit [ii]. The decay process of triplet Higgs to two leptons become important for the limits [iii] and [iv]. Our model naively corresponds to a mixed scenario of limits [i] and [ii], that is, $M_{R,k} \ll M_\Delta$ with

[‡‡] The mass spectrum of the right-handed neutrinos in our model is given in (3.6), which is degenerate at the leading order. However, the next leading correction, whose magnitude is typically $\mathcal{O}((u/\Lambda)^2M_{R,3})$, weaken this degeneracy to be $|M_{R,3}-M_{R,2}| \gg \Gamma_{\Psi_{3R}}$. Therefore, the hierarchical formulae for the computation of the CP asymmetry reviewed in this section are valid for our model.

comparable contributions of the right-handed neutrinos and triplet Higgs to the light neutrino mass because of (3.6), (3.10), (3.23), (3.28) and the assumption $m'_\nu \simeq m_\nu$. Because of the presence of additional contributions from the triplet Higgs decay to CP asymmetry, the bounds on the decay asymmetry and on $M_{R,2}$ become weaker than the ones obtained in [54], which discussed the limit [i]. Therefore, of course, our model with contributions from type I and II seesaw mechanisms can also realize the mixed scenario of type I+II leptogenesis. Solving the exact Boltzmann equation and searching for all allowed regions of model parameters are out of our scope in this paper.

4 Conclusion

We have studied models based on A_4 flavor symmetry in the context of the split seesaw mechanism. The split seesaw mechanism, which has been proposed an extra-dimensional theory, can provide a natural way to realize a splitting mass spectrum of right-handed neutrinos. It leads to one keV sterile neutrino as a DM candidate favored by recent cosmological observations and two heavy right-handed neutrinos being responsible for leptogenesis to explain the observed BAU.

First, we have pointed out that most of known A_4 flavor models with three right-handed neutrinos being A_4 triplet suffer from a *degeneracy problem* for the bulk mass term, which disturbs the split mechanism for right-handed neutrino mass spectrum.

Next, we have constructed a new A_4 flavor model to work in the split seesaw mechanism. The model is an extention of the model proposed by Ma. The three right-handed neutrinos are assigned into A_4 (non-trivial) singlets, and the neutrino masses and mixing angles can be realized from both type I+II seesaw contributions in the model. The model predicts the $\mu - \tau$ symmetry in the neutrino mass matrix at the leading order, resulting in the vanishing θ_{13} and maximal θ_{23}. The flavor symmetry A_4 is broken via the flavon vacuum alignment which can be obtained from the orbifold compactification. The BAU can also be realized in the mixed scenario of type I+II leptogenesis.

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Appendix

Group Theory of A_4

We give a brief knowledge of group theory of A_4 in this Appendix.

The order of A_4 group is 12. It contains 12 group elements which can be obtained from the combinations of the two group generators, S and T, which obey the generator relations,

$$S^2 = 1, \quad T^3 = 1, \quad \text{and} \quad (ST)^3 = 1.$$

The group contains three singlets and one triplet as its irreducible representations. The singlets transform under the group generators as

$$\begin{array}{lcl} \underline{\mathbf{1}} & : & S = 1, \quad T = 1, \\ \underline{\mathbf{1}}' & : & S = 1, \quad T = e^{i2\pi/3} = \omega, \\ \underline{\mathbf{1}}'' & : & S = 1, \quad T = e^{i4\pi/3} = \omega^2, \end{array}$$

and the triplet transforms as

$$\underline{\mathbf{3}} : \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

The products of singlets are given as $\underline{\mathbf{1}}' \times \underline{\mathbf{1}}' = \underline{\mathbf{1}}''$, $\underline{\mathbf{1}}' \times \underline{\mathbf{1}}'' = \underline{\mathbf{1}}$, $\underline{\mathbf{1}}'' \times \underline{\mathbf{1}}'' = \underline{\mathbf{1}}''$ etc.. When we represent the triplet as $\underline{\mathbf{3}}_a = (a_1, a_2, a_3)^T$ and $\underline{\mathbf{3}}_b = (b_1, b_2, b_3)^T$, the product of two triplets gives $\underline{\mathbf{3}}_a \times \underline{\mathbf{3}}_b = \underline{\mathbf{1}} + \underline{\mathbf{1}}' + \underline{\mathbf{1}}'' + \underline{\mathbf{3}} + \underline{\mathbf{3}}$, where

$$\begin{array}{lcl} \underline{\mathbf{1}} & = & a_1 b_1 + a_2 b_2 + a_3 b_3, \\ \underline{\mathbf{1}}' & = & a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \\ \underline{\mathbf{1}}'' & = & a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \\ \underline{\mathbf{3}} & = & (a_2 b_3, a_3 b_1, a_1 b_2)^T, \\ \underline{\mathbf{3}} & = & (a_3 b_2, a_1 b_3, a_2 b_1)^T. \end{array}$$

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