On the complementarity of pulsar timing and space laser interferometry for supermassive black hole binaries individual detection

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29 July 2011

Abstract

For the observation of an individual supermassive black hole binary (SMBHB), the complementarity between pulsar timing (PT) and space laser interferometry (SLI) has been previously suggested. Herein such complementarity is thoroughly examined. Binaries of (sub-)billion solar masses may indeed transit from PT to LSI bandwidths in a period ranging from few months to few years, depending on the chirp mass of the SMBHB and on the high frequency limit of the sensitivity band of PT systems. At the end of this transit period, SLI may detect the final moments of the same coalescence including the ringing phase, if its sensitivity at low frequency is adequate. The likelihood of sequential detection is nevertheless hindered by the current estimates on coalescence rates of (sub-)billion solar mass binaries. The bounding parameters are drawn by current PT and future SK (Square Kilometer) arrays, and by LISA (Laser Interfermeter Space Antenna) and LISA-derived configurations under consideration.

1 Introduction

1.1 Detection by pulsar timing

In the sub-micron and nano Hertz frequency band, beyond the detection of stochastic background (dominated by astrophysical rather than cosmological sources, Schutz, 2011), the individual detection of a supermassive black hole binary (SMBHB) by pulsar timing (PT) has recently received a growing attention. Simulations concord to a scenario wherein some SMBHBs stick out of the stochastic floor of unresolved SMBHBs. This perspective provides new opportunities for analysis and tests in general relativity through PT observations. The latter are otherwise limited to stochastic signal detection and to checking the different SMBHB population models proposed until now.

The top-state of the art on residual accuracy[1] from current PT arrays[2] is set in the $100 - 50$ ns domain for few pulsars; improvement by SK array[3] down to $10 - 1$ ns is expected.

Investigations on the detection of individual sources have been performed along different lines[4].

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[1] The timing residuals are computed from the phase differences between the observed time of arrivals (ToA) and the predicted ToAs based on the current model parameters.

[2] PPTA http://www.atnf.csiro.au/research/pulsar/ppta, EPTA http://www.epta.eu.org/, NANOGravhttp://nanograv.org/.

[3] SKA http://www.skatelescope.org/.

[4] In these investigations, two simplifying hypothesis have often been adopted: i) binaries are on circular orbits; ii) the mergers are gravitational waves driven.

Sesana and Vecchio (2010) evaluate the accuracy with which the parameters of a source can be measured. Corbin and Cornish (2010) show how to recover the physical parameters, like mass and distance of a SMBHB, including the distance to each pulsar as a search parameter. The latter is important for individual detection conversely to statistical background detection (Jenet et al., 2004; Finn and Lommen, 2010; Sesana et al., 2009; Lee et al., 2011). Deng and Finn (2011) use the wavefront curvature to measure the distance to, and the sky position of, the source. Corbin and Cornish (2010), Lee et al. (2011) consider that beside the so-called 'Earth term' (the gravitational wave strain at the Earth at the time when the pulse is received), it is essential to include the 'pulsar term' (the strain at the pulsar at the time when the pulse is emitted). Therefore, a SMBHB produces two quasi-monochromatic components in the PT residuals. The pulsar term is used to obtain an accurate measurement for the SMBHS location. Further, the gravitational wave amplitude is used to estimate the pulsar distance to sub-parsec accuracy.

For the maximisation of the volume of space to which pulsars are sensitive to gravitational wave sources, Burt et al. (2011) rank the efforts of finding new pulsars as slightly advantageous, and only in the presence of an existing nearby cluster of good pulsars; instead, no improvement occurs when additional new pulsars are found in otherwise void regions of the sky; further, they recommend more observing time to already low-noise pulsars to increment PT array sensitivity.

Finn and Lommen (2010) analyse burst detection coming from different sources including individual SMBHB, rather than almost mono-chromatic signals.

Turning to specific observational targets, Jenet et al. (2004) haven't found evidence of gravitational waves emitted by a proposed SMBHB in 3C 66B via the analysis of the PT residuals of PSR B1855+09. Same negative result for Lommen and Backer (2001) who were seeking evidence for a SMBHB in Sgr*A. Yardley et al. (2010) describe the observations used to produce the sensitivity curves for the Parker PT array and propose a method for detecting significant sinusoids in PT residuals.

1.2 Detection by space laser interferometry

The original LISA (Laser Interferometer Space Antenna) project[5] and LISA-derived configurations under consideration[6] refer to space mission designed to measure gravitational radiation over a broad band at low frequencies, where the Universe is richly populated by strong sources of gravitational waves, including SMBHBs. The original LISA required a measurement bandwidth from 0.1 mHz to 100 mHz and it was accompanied by the goal of extending the bandwidth down to 0.02 mHz and up to 1 Hz. Some of the studies under consolidation appear to imply a shift to higher frequencies of the sensitivity band. While such a shift would impede space laser interferometry (SLI)[7] to detect the ringing of the more massive SMBHB, the practical consequences remain marginal on the event rate of the sequential detection dealt herein.

1.3 Sequential detection

This paper wishes to explore the sequential detection of an individual SMBHB by PT and SLI. Pitkin et al. (2008) have made the first step in this direction, but we have considered necessary to improve and update upon their initial work for three basic reasons. First, we consider that the size of the SMBHB mass of interest for a sequential detection is generally larger than the fifty million solar mass SMBHB considered by Pitkin et al. (2008). Second, we build our analysis upon the findings (timing residuals, expected populations) of the recent literature not available at the times of the work of Pitkin et al. (2008). Third, we estimate the event rate of sequential detection for the first time.

The paper is structured as follows. Section 2 is devoted to the computation of transit times, coalescence and ringing frequencies for rotating SMBHBs; section 3 presents the event rates or sequential detection; section 4

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[5] LISA http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=42592.

[6] http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=48728.

[7] We adopt the acronym SLI to address laser space interferometry for gravitational wave detection in the largest sense.

sums up the conclusions. The mass M herein is normalised to dimensionless units $1+z$, where z is the redshift (Hughes, 2002).

2 Transit time and ringing

Peters and Mathews (1963), Peters (1963) (PM) have laid the foundations for the description of relativistic binaries. Their seminal work provides a basic, but for many cases sufficiently adequate, model of the relativistic two-body problem[8].

Pierro and Pinto (1996), Pierro et al. (2001) have quantified the factors underlying the PM model. The main assumptions for this model being valid are: (i) point mass, (ii) weak field, (iii) slow motion, and (iv) adiabatic evolution (negligible change of the orbital parameters over each orbit). For a presentation of LISA to particle physicists, Larson (2005) casts the PM model in a suitable way for the purpose of this paper.

The time it takes a circularised binary, of equal masses m and total mass M, to evolve between any two frequencies f_1 and f_2 is

$$\Delta t = \kappa \mathcal{M} \left(f_1^{-8/3} - f_2^{-8/3} \right), \tag{1}$$

where $\kappa = \frac{5}{256}\pi^{-8/3}\left(\frac{G}{c^3}\right)^{-5/3}$, G being the constant of gravitation, c the speed of light, and $\mathcal{M} = 2^{1/3}m^{-5/3} = 4M^{-5/3}$.

The PT high frequency limit is really just set by how often observations can be made. Daily observation would bring the high-end frequency limit to the order of 10^{-5} Hz. We have taken a semi-conservative stand by setting two values for f_2, namely $4 \cdot 10^{-7}$ Hz and $8 \cdot 10^{-7}$ Hz.

If the f_2 frequency is much larger than f_1, the latter becomes irrelevant in the computation of Δt. In this sense, a shift towards higher frequencies of the sensitivity band of SLI, does not effect the transit time. Obviously, if the space interferometer has a modest sensitivity at low frequencies, the sequential detection may be easily missed.

Figure 1 shows the transit time (in number of years) from the PT sensitivity band, supposedly limited at $4 \cdot 10^{-7}$ Hz (dashed line) or at $8 \cdot 10^{-7}$ Hz (continuous line), to the SLI sensitivity band, supposedly limited at $2 \cdot 10^{-5}$ Hz. The time of transit is displayed as function of the SMBHB total mass in billion solar mass units, normalised to a $(1+z)$ factor, z being the redshift.

The coalescence frequency (Hughes, 2002) is given by

$$f_c \simeq 2 \cdot 10^{-6} \frac{10^9\ M_\odot}{M}\ Hz. \tag{2}$$

The ring-down frequency (Echeverria, 1989; Hughes, 2002) is given by

$$f_r \simeq 3.2 \cdot 10^{-5} \frac{10^9\ M_\odot}{\eta M} [1 - 0.63(1-a)^{3/10}]\ Hz. \tag{3}$$

The coalescence frequency doesn't appear eligible for detection for the range of masses considered. The ring-down frequency, instead, falls within SLI potential configurations, especially for a high Kerr spin parameter a, Fig. 2. The parameter η takes into account the emission of gravitational radiation. For our computation $\eta = 0.94$ (Rezzolla, 2009).

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[8] For a wider and deeper introduction to the current state of the art of the two-body problem, see Blanchet et al. (2011)

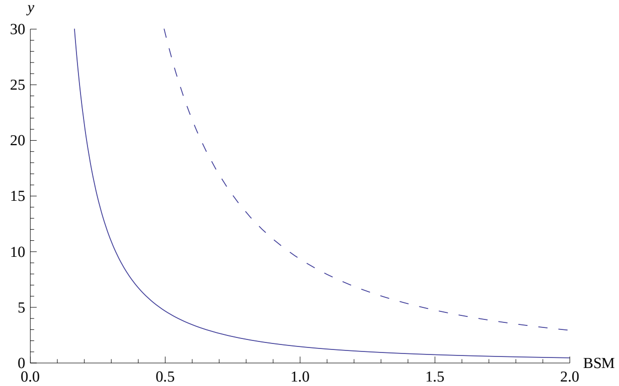

Figure 1: Time of transit (in number of years) from the PT sensitivity band, supposedly limited at $4 \cdot 10^{-7}$ Hz (dashed line) or $8 \cdot 10^{-7}$ Hz (continuous line), to SLI sensitivity band, supposedly limited at $2 \cdot 10^{-5}$ Hz (this value is scarcely influential on the time of transit). The time of transit is displayed as function of the SMBHB total mass in billion solar mass units, normalised to a $(1 + z)$ factor, z being the redshift.

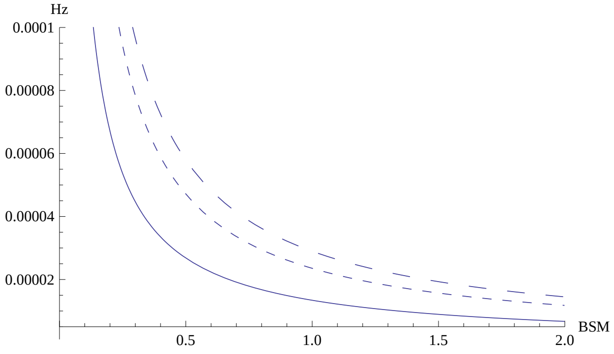

Figure 2: Frequency (Hz) of quasi-normal ringing as function of the SMBHB total mass, in billion solar mass units, normalised to a $(1+z)$ factor, z being the redshift. The dimensionless Kerr spin parameter a is set equal to 0.1(continuous line), 0.9 (medium-dashed line), or 0.99 (long-dashed line).

Transit time and ringing frequencies appear reasonable for the sequential detection to take place. Long and short transit times, from 9 months to 20 years, correspond to different astrophysical and operational scenarios determined by the cross-combinations of working PT or SK arrays, and flying SLI missions. Few (extreme) examples are given here below:

- Low SLI detection frequency for all spin values. Transit time of 18.2 years (PT $f_1 = 4 \cdot 10^{-7}\ Hz$) or 2.9 years (PT $f_1 = 8 \cdot 10^{-7}\ Hz$), corresponding to a $0.67 \cdot 10^9\ M_\odot$ SMBHB ringing at $2 \cdot 10^{-5}\ Hz$, $3.5 \cdot 10^{-5}\ Hz$, $4.3 \cdot 10^{-5}\ Hz$ for $a = 0.1$, $a = 0.9$, or $a = 0.99$, respectively.
- Rapid transit time. Transit time of 5 years (PT $f_1 = 4 \cdot 10^{-7}\ Hz$) or 9 months (PT $f_1 = 8 \cdot 10^{-7}\ Hz$), corresponding to a $1.45 \cdot 10^9\ M_\odot$ SMBHB ringing at $9.2 \cdot 10^{-6}\ Hz$, $1.6 \cdot 10^{-5}\ Hz$, $2 \cdot 10^{-5}\ Hz$ for $a = 0.1$, $a = 0.9$, or $a = 0.99$, respectively.
- Long transit time, large PT bandwidth, high ringing frequency. Transit time - beyond scenario - of 127 years (PT $f_1 = 4 \cdot 10^{-7}\ Hz$) or 20 years (PT $f_1 = 8 \cdot 10^{-7}\ Hz$), corresponding to a $0.22 \cdot 10^9\ M_\odot$ SMBHB ringing at $6.4 \cdot 10^{-5}\ Hz$, $1.1 \cdot 10^{-4}\ Hz$, $1.4 \cdot 10^{-4}\ Hz$ for $a = 0.1$, $a = 0.9$, or $a = 0.99$, respectively.

3 Event rates

Concerning the individual detection by PT, Sesana and Vecchio (2010) estimate to a handful the number of detectable binaries, including those at 5 ns effective noise level; the number of resolvable systems quickly drops if the timing precision degrades to, e.g. 50 ns. The simulations by Sesana et al. (2009), analyse a wide range of population models and indicate 5-to-15 individual sources having residuals larger than the stochastic background. The residuals are between 2 and 60 ns in the frequency range $2 \cdot 10^{-8} - 10^{-7}\ Hz$. Further, Sesana et al. (2009) identify SMBHBs of mass larger than $0.5 \cdot 10^9\ M_\odot$ and at a redshift $0.2 < z < 1.5$, most of the resolvable sources.

These findings are based on a generated statistical sample of merging massive galaxies from the online Millennium database[9] built by Springel et al. (2005). Sesana and co-workers populate the merging galaxies with central supermassive black holes according to different models. The Millennium simulation covers a comoving volume of $(500/h_{100})^3\ Mpc^3$ ($h_{100} = H_0/100\ kms^{-1}Mpc^{-1}$ is the normalised Hubble parameter), ensuring a number of massive nearby binaries adequate to construct the necessary distribution. For each model, the expected distribution of bright individual sources and the associated timing residuals are computed.

For the estimate of event rates of interest to sequential detection, the following procedure has been adopted. Numerical distributions of coalescences in the volume d^4N/dM_1dM_2dzdt, were built for this work out of the Millenium database. For every cell of such volume, it is determined the frequency of those binaries which are in an orbital phase at most 20 years before the coalescence, i.e. between 9 months and 20 years. Every cell is then integrated for 20 years and then the results of all cells are summed up, providing the average number of sources that will ring later in the SLI bandwidth. Galaxies have been populated according to the M-sigma prescription by Gultekin et al. (2009). Using different accretion models (Sesana et al., 2009), we get the following numbers for different residual levels[10]:

- For residuals of 50 ns, the event rate is estimated between $4.3 \cdot 10^{-7}$ and $8.1 \cdot 10^{-6}$.
- For residuals of 5 ns, the event rate is estimated between $2.9 \cdot 10^{-5}$ and $5.6 \cdot 10^{-3}$.
- For residuals of 1 ns, the event rate is estimated between $8.4 \cdot 10^{-4}$ and $2.5 \cdot 10^{-2}$.

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[9] Millenium http://www.g-vo.org/Millennium.

[10] From the simulations it is emerged that the value of the f_1 high frequency limit of PT is marginal. More clearly the event rate is scarcely influenced by the PT bandwidth.

The paucity of event rates for sequential detection is far from promising. Nevertheless, the true SMBHB population might not be perfectly described by current models, or come from a completely unexplored physical mechanism (Volonteri, 2011).

Obviously, the above doesn't have a bearing on event rates for separate detection of individual SMBHBs by PT, SK arrays and SLI missions.

4 Conclusions

Although the time of transit from PT to SLI and the ringing in the bandwidth of SLI are compliant with current constraints, the feebleness of event rates of sequential detection leads to refrain from any reasonable expectancy to see such events.

Still, complementarity between PT and SLI should not be dismissed at once. Approaching the two bandwidths would be helpful and efforts to increment the availability of observation time may create favourable conditions to enlarge the PT bandwidth. SLI should in parallel preserve an interest for low frequencies through the LISA-derived mission in light of the proposed higher frequency interferometers BBO and DECIGO.

It is worth mentioning the converse. Ring-down signals observed by SLI may trigger specific searches by PT (Pitkin et al., 2008) and hopefully allow to dig out the SMBHB from the background in the PT bandwidth.

Undoubtedly, a major change in astrophysical models on the population of SMBHB appears mandatory to pursue investigations any further.

Acknowledgments

A. Sesana (Golm) has contributed to the analysis of the event rates. Discussions with M. Pitkin (Glasgow) and I. Cognard (Orléans) are acknowledged.

References

- Blanchet L., Spallicci A., Whiting B., 2011. Mass and motion in general relativity, Springer, Berlin
- Burt B.J., Lommen A.N., Finn L.S., 2011, Astrophys. J., 730, 17
- Corbin V., Cornish N.J., 2010, arXiv:1008.1782v2 [astro-ph.HE]
- Deng X., Finn L.S., 2011, Mon. Not. R. Astron. Soc., 414, 50
- Echeverria F., 1989, Phys. Rev. D, 40, 3194
- ESA, 2011. Lisa assessment Study Report, ESA/SRE(2011)3
- Finn L.S., Lommen A.N., 2010, Astrophys. J., 718, 1400
- Gultekin K., Richstone D.O., Gebhardt K., Lauer T.R., Tremaine S., Aller M.C., Bender R., Dressler A., Faber S.M., Filippenko A.V., Green R., Ho L.C., Kormendy J., Magorrian J., Pinkney J., Siopis C., 2009, Astrophys. J, 698, 198
Hughes S.A., 2002, Mon. Not. R. Astron. Soc., 331, 805
- Jenet F.A., Lommen A., Larson S.L., Wen l., 2004, Astrophys. J., 606, 799
- Larson S.L., 2005, 33^{rd} SLAC Summer Institute on Particle Physics: Gravity in the quantum world and the csmos, 25 July 25 - 5 August 2005, Menlo Park (California), http://www.slac.stanford.edu/econf/C0507252/
- Lee K.J., Wex N., Kramer M., Stappers B.W., Bassa C.G., Janssen G.H., Karuppusamy R., Smits R., 2011, Mon. Not. R. Astron. Soc.
- Lommen A.N., Backer D.C., 2001, Astrophys. J., 562, 297
- Peters P.C., 1964, Phys. Rev., 136, B1224
- Peters P.C., Mathews J.,1963, Phys. Rev., 131, 435
- Pierro V., Pinto I., 1996, N.Cim. B, 111, 631

- Pierro V., Pinto I.M., Spallicci A.D., Laserra E., Recano F., 2001, Mon. Not. R. Astron. Soc., 325, 358
- Pitkin M., Clark J., Hendry M.A., Heng I.S., Messenger C., Toher J., Woan G., 2008, J. Phys. Conf. Ser., 122, 012004
- Rezzolla L., 2009, Class. Q. Grav., 26, 094023
- Sesana A., Vecchio A., 2010, Class. Q. Grav., 27, 084016
- Sesana A., Vecchio A., Volonteri M., 2009, Mon. Not. R. Astron. Soc., 394, 2255
- Schutz B.F., Cosmic Vision 2015-2025 Plan L-class missions presentation, Institut Océanographique de Paris, 3 February 2011, http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=47796
- Springel V., White S.D.M., Jenkins A., Frenk C.S., Yoshida N., Gao L., Navarro J., Thacker R., Croton D., Helly j., Peacock J.A., Cole S., Thomas P., Couchman H., Evrard A., Colberg J., Pearce F., 2005, Nat., 635, 429
- Volonteri M., Journées LISA-France, Laboratoire APC AstroParticules et Cosmologie Paris, 9-10 May 2011, http://www.apc.univ-paris7.fr/LISA-France/2011-2012_files/Volonteri.pdf
- Yardley D.R.B., Hobbs G.B., Jenet F.A., Verbiest J.P.W., Wen Z.L., Manchester R.N., Coles W.A., van Straten W., Bailes M., Bhat N.D.R., Burke-Spolaor S., Champion D.J., Hotan A.W., Sarkissian J.M., 2010, Mon. Not. R. Astron. Soc., 407, 669