

Life might be rare despite its early emergence on Earth: a Bayesian analysis of the probability of abiogenesis

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Life arose on Earth sometime in the first few hundred million years after the young planet had cooled to the point that it could support water-based organisms on its surface. The early emergence of life on Earth has been taken as evidence that the probability of abiogenesis is high, if starting from young-Earth-like conditions. We revisit this argument quantitatively in a Bayesian statistical framework. By constructing a simple model of the probability of abiogenesis, we calculate a Bayesian estimate of its posterior probability, given the data that life emerged fairly early in Earth's history and that, billions of years later, sentient creatures noted this fact and considered its implications. We find that, given only this very limited empirical information, the choice of Bayesian prior for the abiogenesis probability parameter has a dominant influence on the computed posterior probability. Thus, although life began on this planet fairly soon after the Earth became habitable, this fact is consistent with an arbitrarily low intrinsic probability of abiogenesis for plausible uninformative priors, and therefore with life being arbitrarily rare in the Universe.

Astrobiology

Abbreviations: Gyr, gigayear (10^9 years); PDF, probability density function; CDF, cumulative distribution function

Introduction

The current fundamental questions in astrobiology are the issues of whether or not extraterrestrial life exists and, if so, of how abundant it is in the Universe. The most direct and promising approach to answering these questions is surely empirical, the search for life on other bodies in the Solar System [1, 2] and beyond in other planetary systems [3, 4]. Nevertheless, a theoretical approach is possible in principle and could provide a useful complement to the more direct lines of investigation.

In particular, if we knew the probability per unit time and per unit volume of abiogenesis in a pre-biotic environment as a function of its physical and chemical conditions and if we could determine or estimate the prevalence of such environments in the Universe, we could make a statistical estimate of the abundance of extraterrestrial life. This relatively straightforward approach is, of course, thwarted by our great ignorance regarding both inputs to the argument at present.

There does, however, appear to be one possible way of finessing our lack of detailed knowledge concerning both the process of abiogenesis and the occurrence of suitable pre-biotic environments (whatever they might be) in the Universe. Namely, we can try to use our knowledge that life arose at least once in an environment (whatever it was) on the early Earth to try to infer something about the probability per unit time of abiogenesis on an Earth-like planet without the need (or ability) to say how Earth-like it need be or in what ways. We will hereinafter refer to this probability density, which can also be considered a rate, as λ or simply the “probability of abiogenesis.”

Any inferences about the probability of life arising (given the conditions present on the early Earth) must be informed by how long it took for the first living creatures to evolve. By definition, improbable events generally happen infrequently. It follows that the duration between events provides a metric (however imperfect) of the probability or rate of the events. The time-span between when Earth achieved pre-biotic conditions suitable for abiogenesis plus generally habitable climatic conditions [5, 6, 7] and when life first arose, therefore, seems to serve as a basis for estimating λ . Revisiting and quantifying this analysis is the subject of this paper.

We note one previous attempt to address this issue quantitatively by [8]; we compare our treatment and significantly different results to theirs below. Perhaps more importantly however, an argument of this general sort has been widely used in a qualitative and even intuitive way to conclude that λ is unlikely to be extremely small because it would then be surprising for abiogenesis to have occurred as quickly as it did on Earth [9, 10, 11, 12, 13, 14, 15]. Indeed, the early emergence of life on Earth is often taken as significant supporting evidence for “optimism” about the existence of extra-terrestrial life (*i.e.*, for the view that it is fairly common) [16, 17, 8]. The major motivation of this paper is to determine the quantitative validity of this inference.

A Bayesian Formulation of the Calculation

Bayes's theorem [18] can be written as

$$P[\mathcal{M}|\mathcal{D}] = \frac{P[\mathcal{D}|\mathcal{M}] \times P_{\text{prior}}[\mathcal{M}]}{P[\mathcal{D}]} . \quad [1]$$

Here, we take \mathcal{M} to be a model and \mathcal{D} to be data. In order to use eq. (1) to evaluate the posterior probability of abiogenesis, we must specify appropriate \mathcal{M} and \mathcal{D} .

A Poisson or Uniform Rate Model. In considering the development of life on a planet, we suggest that a reasonable, if simplistic, model is that it is a Poisson process during a period of time from t_{min} until t_{max} . In this model, the conditions on a young planet preclude the development of life for a time

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period of t_{\min} after its formation. Furthermore, if the planet remains lifeless until t_{\max} has elapsed, it will remain lifeless thereafter as well because conditions no longer permit life to arise. For a planet around a solar-type star, t_{\max} is almost certainly $\lesssim 10$ Gyr (10 billion years, the main sequence lifetime of the Sun) and could easily be a substantially shorter period of time if there is something about the conditions on a young planet that are necessary for abiogenesis. Between these limiting times, we posit that there is a certain probability per unit time (λ) of life developing. For $t_{\min} < t < t_{\max}$, then, the probability of life arising n times in time t is

$$P[\lambda, n, t] = P_{\text{Poisson}}[\lambda, n, t] = e^{-\lambda(t-t_{\min})} \frac{\{\lambda(t-t_{\min})\}^n}{n!}, \quad [2]$$

where t is the time since the formation of the planet.

This formulation could well be questioned on a number of grounds. Perhaps most fundamentally, it treats abiogenesis as though it were a single instantaneous event and implicitly assumes that it can occur in only a single way (*i.e.*, by only a single process or mechanism) and only in one type of physical environment. It is, of course, far more plausible that abiogenesis is actually the result of a complex chain of events that take place over some substantial period of time and perhaps via different pathways and in different environments. However, knowledge of the actual origin of life on Earth, to say nothing of other possible ways in which it might originate, is so limited that a more complex model is not yet justified. In essence, the simple Poisson event model used in this paper attempts to “integrate out” all such details and treat abiogenesis as a “black box” process: certain chemical and physical conditions as input produce a certain probability of life emerging as an output.

Another issue is that λ , the probability per unit time, could itself be a function of time. In fact, the claim that life could not have arisen outside the window (t_{\min}, t_{\max}) is tantamount to saying that $\lambda = 0$ for $t \leq t_{\min}$ and for $t \geq t_{\max}$. Instead of switching from 0 to a fixed value instantaneously, λ could exhibit a complicated variation with time. If so, however, $P[\lambda, n, t]$ is not represented by the Poisson distribution and eq. (2) is not valid. Unless a particular (non top-hat-function) time-variation of λ is suggested on theoretical grounds, it seems unwise to add such unconstrained complexity.

A further criticism is that λ could be a function of n : it could be that life arising once (or more) changes the probability per unit time of life arising again. Since we are primarily interested in the probability of life arising *at all* – *i.e.*, the probability of $n \neq 0$ – we can define λ simply to be the value appropriate for a prebiotic planet (whatever that value may be) and remain agnostic as to whether it differs for $n \geq 1$.

Thus, within the adopted model, the probability of life arising is one minus the probability of it not arising:

$$P_{\text{life}} = 1 - P_{\text{Poisson}}[\lambda, 0, t] = 1 - e^{-\lambda(t-t_{\min})}. \quad [3]$$

A Minimum Evolutionary Time Constraint. Naively, the single datum informing our calculation of the posterior of λ appears to be simply that life arose on Earth at least once, approximately 3.8 billion years ago (give or take a few hundred million years). There is additional significant context for this datum, however. Recall that the standard claim is that, since life arose early on the only habitable planet that we have examined for inhabitants, the probability of abiogenesis is probably high (in our language, λ is probably large). This standard argument neglects a potentially important selection effect, namely: On Earth, it took nearly 4 Gyr for evolution

Models of $t_0 = 4.5$ Gyr-Old Planets

Model	Hypothetical	Conserv.1	Conserv.2	Optimistic
t_{\min}	0.5	0.5	0.5	0.5
t_{emerge}	0.51	1.3	1.3	0.7
t_{\max}	10	1.4	10	10
δt_{evolve}	1	2	3.1	1
t_{required}	3.5	1.4	1.4	3.5
Δt_1	0.01	0.80	0.80	0.20
Δt_2	3.00	0.90	0.90	3.00
\mathcal{R}	300	1.1	1.1	15

All times are in Gyr. Two “Conservative” (Conserv.) models are shown, to indicate that t_{required} may be limited either by a small value of t_{\max} (“Conserv.1”), or by a large value of δt_{evolve} (“Conserv.2”).

to lead to organisms capable of pondering the probability of life elsewhere in the Universe. If this is a necessary duration, then it would be impossible for us to find ourselves on, for example, a (~ 4.5 -Gyr old) planet on which life first arose only after the passage of 3.5 billion years. On such planets there would not yet have been enough time for creatures capable of such contemplations to evolve. In other words, if evolution *requires* 3.5 Gyr for life to evolve from the simplest forms to sentient, questioning beings, then we *had* to find ourselves on a planet on which life arose relatively early, regardless of the value of λ .

In order to introduce this constraint into the calculation we define δt_{evolve} as the minimum amount of time required after the emergence of life for cosmologically curious creatures to evolve, t_{emerge} as the age of the Earth when life first emerged (~ 0.7 Gyr or so), and t_0 as the current age of the Earth. The data, then, are that life arose on Earth at least once, approximately 3.8 billion years ago, *and* that this emergence was early enough that human beings had the opportunity subsequently to evolve and to wonder about their origins and the possibility of life elsewhere in the Universe. In equation form:

$$t_{\text{emerge}} < t_0 - \delta t_{\text{evolve}}. \quad [4]$$

The Likelihood Term. We now may substitute in for the $P[\mathcal{D}|\mathcal{M}]$ term in eq. (1). Let $t_{\text{required}} \equiv \min[t_0 - \delta t_{\text{evolve}}, t_{\max}]$. Our existence on Earth requires that life appeared within t_{required} . In other words, t_{required} is the maximum age that the Earth could have had at the origin of life in order for humanity to have a chance of showing up by the present. We define $\mathcal{S}_{\mathcal{E}}$ to be the set of all Earth-like worlds of age approximately t_0 in a large, unbiased volume and $L[t]$ to be the subset of $\mathcal{S}_{\mathcal{E}}$ on which life has emerged within a time t . $L[t_{\text{required}}]$ is the set of planets on which life emerged early enough that sentient beings could have evolved before the present (t_0), and, presuming $t_{\text{emerge}} < t_{\text{required}}$ (which we know was the case for Earth), $L[t_{\text{emerge}}]$ is the subset of $L[t_{\text{required}}]$ on which life emerged as quickly as it did on Earth. Correspondingly, $N_{\mathcal{S}_{\mathcal{E}}}$, N_{t_r} , and N_{t_e} are the respective numbers of planets in sets $\mathcal{S}_{\mathcal{E}}$, $L[t_{\text{required}}]$, and $L[t_{\text{emerge}}]$. The fractions $\varphi_{t_r} \equiv N_{t_r}/N_{\mathcal{S}_{\mathcal{E}}}$ and $\varphi_{t_e} \equiv N_{t_e}/N_{\mathcal{S}_{\mathcal{E}}}$ are, respectively, the fraction of Earth-like planets on which life arose

¹An alternative way to derive equation (5) is to let $E =$ “abiogenesis occurred between t_{\min} and t_{emerge} ” and $R =$ “abiogenesis occurred between t_{\min} and t_{required} .” We then have, from the rules of conditional probability, $P[E|R, \mathcal{M}] = P[E, R|\mathcal{M}]/P[R|\mathcal{M}]$. Since E entails R , the numerator on the right-hand side is simply equal to $P[E|\mathcal{M}]$, which means that the previous equation reduces to equation (5).

within t_{required} and the fraction on which life emerged within t_{emerge} . The ratio $r \equiv \varphi_{t_e}/\varphi_{t_r} = N_{t_e}/N_{t_r}$ is the fraction of L_{t_r} on which life arose as soon as it did on Earth. Given that we *had to* find ourselves on such a planet in the set L_{t_r} in order to write and read about this topic, the ratio r characterizes the probability of the data given the model if the probability of intelligent observers arising is independent of the time of abiogenesis (so long as abiogenesis occurs before t_{required}). (This last assumption might seem strange or unwarranted, but the effect of relaxing this assumption is to make it more likely that we would find ourselves on a planet with early abiogenesis and therefore to reduce our limited ability to infer anything about λ from our observations.) Since $\varphi_{t_e} = 1 - \text{Poisson}[\lambda, 0, t_{\text{emerge}}]$ and $\varphi_{t_r} = 1 - \text{Poisson}[\lambda, 0, t_{\text{required}}]$, we may write that

$$P[\mathcal{D}|\mathcal{M}] = \frac{1 - \exp[-\lambda(t_{\text{emerge}} - t_{\text{min}})]}{1 - \exp[-\lambda(t_{\text{required}} - t_{\text{min}})]} \quad [5]$$

if $t_{\text{min}} < t_{\text{emerge}} < t_{\text{required}}$ (and $P[\mathcal{D}|\mathcal{M}] = 0$ otherwise). This is called the “likelihood function,” and represents the probability of the observation(s), given a particular model.¹ It is via this function that the data “condition” our prior beliefs about λ in standard Bayesian terminology.

Limiting Behavior of the Likelihood. It is instructive to consider the behavior of equation (5) in some interesting limits. For $\lambda(t_{\text{required}} - t_{\text{min}}) \ll 1$, the numerator and denominator of equation (5) each go approximately as the argument of the exponential function; therefore, in this limit, the likelihood function is approximately constant:

$$P[\mathcal{D}|\mathcal{M}] \approx \frac{t_{\text{emerge}} - t_{\text{min}}}{t_{\text{required}} - t_{\text{min}}}. \quad [6]$$

This result is intuitively easy to understand as follows: If λ is sufficiently small, it is overwhelmingly likely that abiogenesis occurred only once in the history of the Earth, and by the assumptions of our model, the one event is equally likely to occur at any time during the interval between t_{min} and t_{required} . The chance that this will occur by t_{emerge} is then just the fraction of that total interval that has passed by t_{emerge} , just the result given in equation (6).

In the other limit, when $\lambda(t_{\text{emerge}} - t_{\text{min}}) \gg 1$, the numerator and denominator of equation (5) are both approximately 1. In this case, the likelihood function is also approximately constant (and equal to unity). This result is even more intuitively obvious since a very large value of λ implies that abiogenesis events occur at a high rate (given suitable conditions) and are thus likely to have occurred very early in the interval between t_{min} and t_{required} .

These two limiting cases then already reveal a key conclusion of the analysis, namely that the resulting posterior distribution of λ for both very large and very small values will have the shape of the prior, just scaled by different constants. Only when λ is neither very large nor very small – or, more precisely, when $\lambda(t_{\text{emerge}} - t_{\text{min}}) \approx 1$ – do the data and the prior both inform the posterior probability at a roughly equal level.

The Bayes Factor. In this context, note that the probability in equation (5) depends crucially on two time differences, $\Delta t_1 \equiv t_{\text{emerge}} - t_{\text{min}}$ and $\Delta t_2 \equiv t_{\text{required}} - t_{\text{min}}$, and that the ratio of the likelihood function at large λ to its value at small λ goes roughly as

$$\mathcal{R} \equiv \frac{P[\text{data}|\text{large}\lambda]}{P[\text{data}|\text{small}\lambda]} \approx \frac{\Delta t_2}{\Delta t_1}. \quad [7]$$

\mathcal{R} is called the Bayes factor or Bayes ratio and is sometimes employed for model selection purposes. In one conventional interpretation [19], $\mathcal{R} \leq 10$ implies no strong reason *in the*

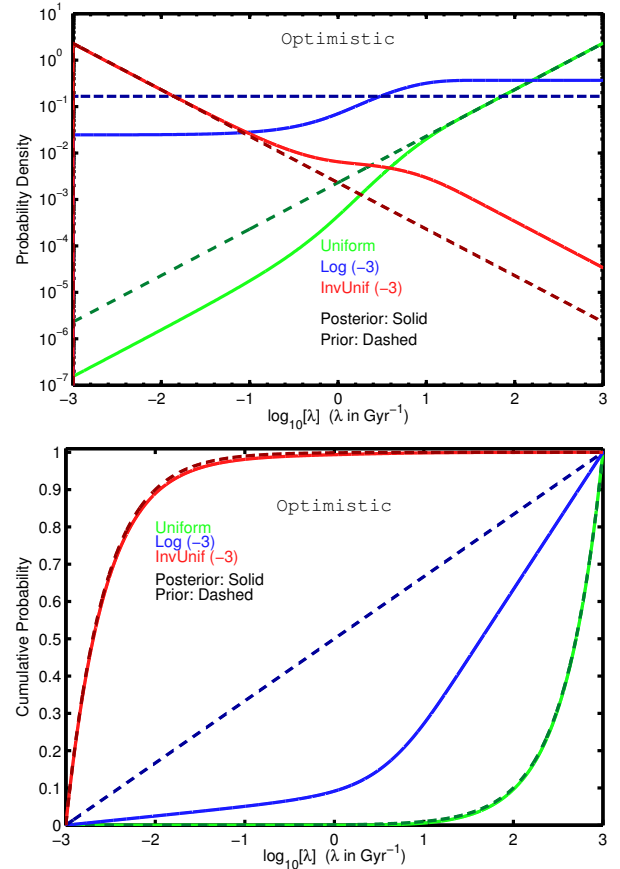


Fig. 1. PDF and CDF of λ for uniform, logarithmic, and inverse-uniform priors, for model **Optimistic**, with $\lambda_{\text{min}} = 10^{-3}\text{Gyr}^{-1}$ and $\lambda_{\text{max}} = 10^3\text{Gyr}^{-1}$. *Top:* The dashed and solid curves represent, respectively, the prior and posterior probability distribution functions (PDFs) of λ under three different assumptions about the nature of the prior. The green curves are for a prior that is uniform on the range $0\text{Gyr}^{-1} \leq \lambda \leq \lambda_{\text{max}}$ (“Uniform”); the blue are for a prior that is uniform in the log of λ on the range $-3 \leq \log \lambda \leq 3$ (“Log (-3)”); and the red are for a prior that is uniform in λ^{-1} on the interval $10^{-3}\text{Gyr} \leq \lambda^{-1} \leq 10^3\text{Gyr}$ (“InvUnif (-3)”). *Bottom:* The curves represent the cumulative distribution functions (CDFs) of λ . The ordinate on each curve represents the integrated probability from 0 to the abscissa (color and line-style schemes are the same as in the top panel). For a uniform prior, the posterior CDF traces the prior almost exactly. In this case, the posterior judgment that λ is probably large simply reflects the prior judgment of the distribution of λ . For the prior that is uniform in λ^{-1} (InvUnif), the posterior judgment is quite opposite – namely, that λ is probably quite small – but this judgment is also foretold by the prior, which is traced nearly exactly by the posterior. For the logarithmic prior, the datum (that life on Earth arose within a certain time window) does influence the posterior assessment of λ , shifting it in the direction of making greater values of λ more probable. Nevertheless, the posterior probability is $\sim 12\%$ that $\lambda < 1\text{Gyr}^{-1}$. Lower λ_{min} and/or lower λ_{max} would further increase the posterior probability of very low λ , for any of the priors.

data alone to prefer the model in the numerator over the one in the denominator. For the problem at hand, this means that the datum of the early appearance of life on Earth does not justify preference for a large value of λ over an arbitrarily small one unless evaluation of equation (7) gives a result larger than roughly ten (or unless equation 6 gives a result smaller than roughly a tenth).

The Prior Term. To compute the desired posterior probability, what remains to be specified is $P_{\text{prior}}[\mathcal{M}]$, the prior joint probability density function (PDF) of λ , t_{min} , t_{max} , and δt_{evolve} . Taking these parameters to be independent, we have

$$P_{\text{prior}}[\mathcal{M}] = f_{\lambda}[\lambda]f_{t_s}[t_{\text{min}}]f_{t_m}[t_{\text{max}}]f_{\delta t_e}[\delta t_{\text{evolve}}], \quad [8]$$

where f_{λ} , f_{t_s} , f_{t_m} , and $f_{\delta t_e}$ are the probability density functions of λ , t_{min} , t_{max} , and δt_{evolve} , respectively.

Since we lack any theory for the probability density functions of t_{min} , t_{max} , and δt_{evolve} , we will simply take these to be delta functions at several theoretically interesting values: a purely hypothetical value in which life arose extremely quickly, a most conservative value, and a value in between that is also optimistic but for which there does exist some evidence (see Table 1). For the values in Table 1, the likelihood ratio \mathcal{R} varies from ~ 1.1 to 300, with the parameters of the “optimistic” model giving a borderline significance value of $\mathcal{R} = 15$. Thus, only the hypothetical case gives a decisive preference for large λ by the Bayes factor metric, and we emphasize that there is no evidence or claim in the literature that abiogenesis on Earth occurred that early, only 10 million years after conditions first permitted it!

We also lack a first-principles theory or other solid prior information for λ . We therefore take three different functional forms for the prior – uniform in λ , uniform in λ^{-1} (equivalent to saying that the mean time until life appears is uniformly distributed), and uniform in $\log_{10} \lambda$. The uniform prior may be written

$$f_{\lambda,u}[\lambda] = \frac{1}{\lambda_{\text{max}}} \quad [9]$$

for $0 \leq \lambda \leq \lambda_{\text{max}}$ (and =0 otherwise), where λ_{max} is the maximum possible rate of abiogenesis. We take $\lambda_{\text{max}} = 100 \text{Gyr}^{-1}$. The inverse-uniform prior may be written

$$f_{\lambda,iu}[\lambda] = \frac{1}{\lambda^2} \times \frac{1}{\lambda_{\text{min}}^{-1} - \lambda_{\text{max}}^{-1}} \quad [10]$$

for $\lambda_{\text{min}} \leq \lambda \leq \lambda_{\text{max}}$ (and =0 otherwise). The logarithmic prior may be written

$$f_{\lambda,l}[\lambda] = \frac{1}{\lambda} \times \frac{\log_{10} e}{\log_{10} \lambda_{\text{max}} - \log_{10} \lambda_{\text{min}}} \quad [11]$$

for $\lambda_{\text{min}} \leq \lambda \leq \lambda_{\text{max}}$ (and =0 otherwise), where λ_{min} is the lower bound on λ (in the logarithmic prior case). For $f_{\lambda,iu}$ and $f_{\lambda,l}$, we take three values of λ_{min} : 10^{-22}Gyr^{-1} , 10^{-11}Gyr^{-1} , and 10^{-3}Gyr^{-1} , corresponding roughly to life occurring once in the observable Universe, once in our galaxy, and once per 200 stars (assuming one Earth-like planet per star).

In standard Bayesian terminology, both the uniform in λ and the uniform in λ^{-1} priors are said to be highly “informative.” This means that they strongly favor large and small, respectively, values of λ in advance, *i.e.*, on some basis other than the empirical evidence represented by the likelihood term. For example, the uniform in λ prior is equivalent to asserting that we know on some other basis (other than the early emergence of life on Earth) that it is a hundred times less likely that λ is less than 10^{-3}Gyr^{-1} than that it is less than 0.1Gyr^{-1} . The uniform in λ^{-1} prior has the equivalent sort of preference for small λ values. By contrast, the logarithmic prior is a relatively “uninformative” one in standard Bayesian terminology and is equivalent to asserting that we have no prior information that informs us of even the order-of-magnitude of λ .

In our opinion, the logarithmic prior is the most appropriate one given our current lack of knowledge of the process(es) of abiogenesis, but it is nevertheless instructive to carry all three priors through the calculation of the posterior distribution of λ because they vividly illuminate the extent to which the result depends on the data vs the assumed prior.

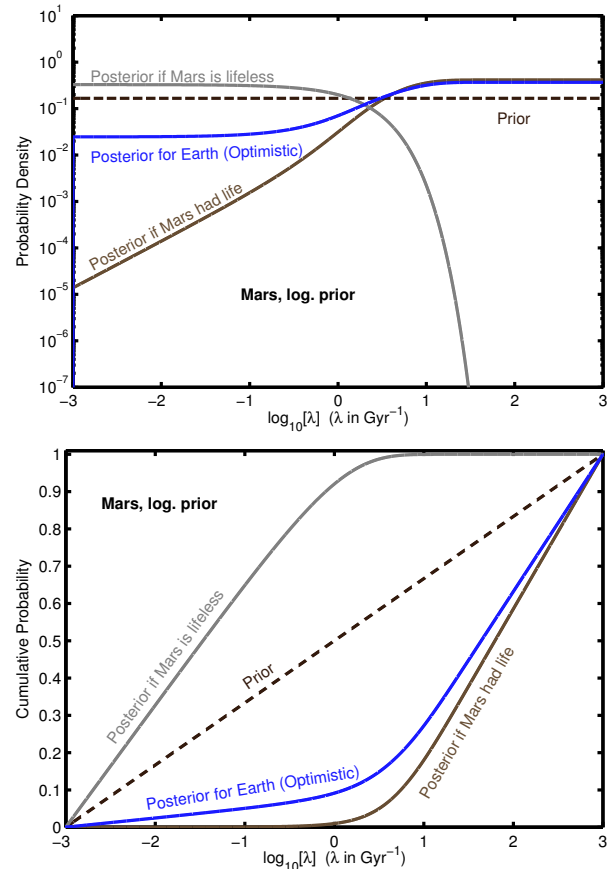


Fig. 2. PDF (left) and CDF (right) of λ for Mars, for logarithmic prior. $\lambda_{\text{min}} = 10^{-3} \text{Gyr}^{-1}$, $\lambda_{\text{max}} = 10^3 \text{Gyr}^{-1}$. In contrast to the case for Earth, where the datum that life has arisen here shifts the posterior away from the prior in the direction of making higher values of λ more likely, a lifeless planet would shift the posterior in the direction of making lower values of λ more likely. Finding out that life arose independently on Mars and Earth would significantly reduce the posterior probability of low λ .

Comparison with Previous Analysis. Using a binomial probability analysis, Lineweaver & Davis [8] attempted to quantify q , the probability that life would arise within the first billion years on an Earth-like planet. Although the binomial distribution typically applies to discrete situations (in contrast to the continuous passage of time, during which life might arise), there is a simple correspondence between their analysis and the Poisson model described above. The probability that life would arise at least once within a billion years (what [8] call q) is a simple transformation of λ , obtained from equation (3), with $\Delta t_1 = 1 \text{Gyr}$:

$$q = 1 - e^{-(\lambda)(1\text{Gyr})} \quad \text{or} \quad \lambda = \ln[1 - q]/(1\text{Gyr}). \quad [12]$$

In the limit of $\lambda(1\text{Gyr}) \ll 1$, equation (12) implies that q is equal to $\lambda(1\text{Gyr})$. Though not cast in Bayesian terms, the Lineweaver & Davis [8] calculation is based on an implicit prior that is uniform in q . As a result, it is equivalent to our uniform- λ prior for small values of λ (or q).

The Posterior Probability of Abiogenesis

We compute the normalized product of the probability of the data given λ (equation 5) with each of the three priors (uniform, logarithmic, and inverse uniform; equations 9-11). This

gives us the Bayesian posterior PDF of λ , which we also derive for each model in Table 1. Then, by integrating each PDF from $-\infty$ to λ , we obtain the corresponding cumulative distribution function (CDF).

Figure 1 displays the results by plotting the prior and posterior probability of λ . The top panel presents the PDF, and the bottom panel the CDF, for uniform, logarithmic, and inverse-uniform priors, for model `Optimistic`, which sets Δt_1 (the time it took life to emerge once Earth became habitable) to 0.2 Gyr, and Δt_2 (the time life had available to emerge in order that sentient creatures would have a chance to evolve) to 3.0 Gyr. The dashed and solid curves represent, respectively, prior and posterior probability functions. In this figure, the priors on λ have $\lambda_{\min} = 10^{-3} \text{Gyr}^{-1}$ and $\lambda_{\max} = 10^3 \text{Gyr}^{-1}$. The green, blue, and red curves are calculated for uniform, logarithmic, and inverse-uniform priors, respectively. The results of the corresponding calculations for the other models and bounds on the assumed priors are presented in the Supporting Information, but the cases shown in Fig. 1 suffice to demonstrate all of the important qualitative behaviors of the posterior.

In the plot of differential probability (PDF; top panel), it appears that the inferred posterior probabilities of different values of λ are conditioned similarly by the data (leading to a jump in the posterior PDF of roughly an order of magnitude in the vicinity of $\lambda \sim 0.5 \text{Gyr}^{-1}$). The plot of cumulative probability, however, immediately shows that the uniform and the inverse priors produce posterior CDFs that are completely insensitive to the data. Namely, small values of λ are strongly excluded in the uniform in λ prior case and large values are equally strongly excluded by the uniform in λ^{-1} prior, but these strong conclusions are not a consequence of the data, only of the assumed prior. This point is particularly salient, given that a Bayesian interpretation of [8] indicates an implicit uniform prior. In other words, their conclusion that q cannot be too small and thus that life should not be too rare in the Universe is *not* a consequence of the evidence of the early emergence of life on the Earth but almost only of their particular parameterization of the problem.

For the `Optimistic` parameters, the posterior CDF computed with the uninformative logarithmic prior does reflect the influence of the data, leading to making greater values of λ more probable in accordance with one's intuitive expectations. However, with this relatively uninformative prior, there is a significant probability that λ is very small (12% chance that $\lambda < 1 \text{Gyr}^{-1}$). Moreover, if we adopted smaller λ_{\min} , smaller λ_{\max} , and/or a larger $\Delta t_1/\Delta t_2$ ratio, the posterior probability of an arbitrarily low λ value can be made acceptably high (see Fig. 3 and the Supporting Information).

The Case of Mars. We have no strong evidence that life ever arose on Mars (although no strong evidence to the contrary either). Recent observations have tentatively suggested the presence of methane at the level of ~ 20 parts per billion (ppb) [20], which could potentially be indicative of biological activity. The case is not entirely clear, however, as alternative analysis of the same data suggests that an upper limit to the methane abundance is in the vicinity of ~ 3 ppb [21]. For the purposes of the present discussion, we consider two alternative data: (*i*) that Mars is (and always has been) lifeless (and we evaluate the posterior probability density of λ for a Mars-like planet); (*ii*) that life arose at least once on Mars (and we evaluate the posterior probability density of λ assuming that the same λ holds for both Earth and Mars).

If Mars and Earth have different λ s and life never arose on Mars, then the likelihood of Mars' λ (case *i*) is simply taken from Eq. (2), with n (the number of times life arises) set to

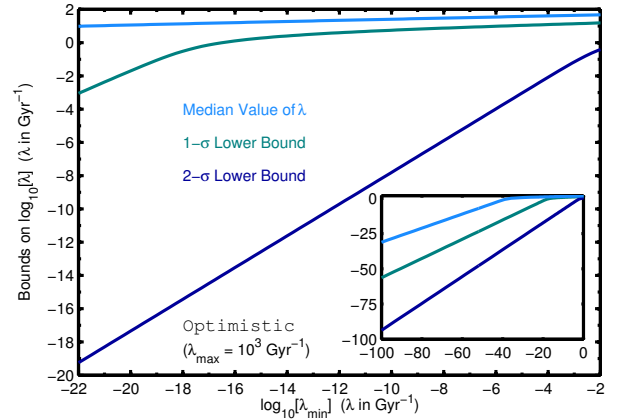


Fig. 3. Lower bound on λ for logarithmic prior, `Hypothetical` model. The three curves depict median (50%), 1- σ (68.3%), and 2- σ (95.4%) lower bounds on λ , as a function of λ_{\min} .

0; i.e., the probability is $P_{\text{Poisson}}[\lambda, 0, t]$, for appropriate t :

$$P_i[\mathcal{D}|\mathcal{M}] = \exp[-\lambda(t_{\max}^{\text{Mars}} - t_{\min}^{\text{Mars}})]. \quad [13]$$

If Mars and Earth share a single λ and life did arise on Mars, then the likelihood of Mars' λ (case *ii*) is the joint probability of our data on Earth and of life arising on Mars. Assuming no panspermia in either direction, these events are independent:

$$P_{ii}[\mathcal{D}|\mathcal{M}] = \left(1 - \exp[-\lambda(t_{\text{emerge}}^{\text{Mars}} - t_{\min}^{\text{Mars}})]\right) \times \frac{1 - \exp[-\lambda(t_{\text{emerge}}^{\text{Earth}} - t_{\min}^{\text{Earth}})]}{1 - \exp[-\lambda(t_{\text{required}}^{\text{Earth}} - t_{\min}^{\text{Earth}})]}. \quad [14]$$

For Mars, we take $t_{\max}^{\text{Mars}} = t_{\text{emerge}}^{\text{Mars}} = 1 \text{Gyr}$ and $t_{\min}^{\text{Mars}} = 0.5 \text{Gyr}$. The posterior probability distribution of λ , given a logarithmic prior between 0.001Gyr^{-1} and 1000Gyr^{-1} , is as represented in Fig. 2 for three different cases: (*i*); (*ii*); and, for comparison, the `Optimistic` case for Earth. For case (*i*), in contrast to Fig. 1, the influence of the datum (no life) is to shift the posterior in the direction of making lower values of λ more probable. That the observation of lifelessness would have the opposite type of effect on the posterior inference to the observation of life is entirely to be expected. For case (*ii*), we see that, should future researchers find that life arose independently on Mars, this would dramatically reduce the posterior probability of very low λ relative to our current inferences.

Arbitrarily Low Posterior Probability of λ . We do not actually know what the appropriate lower (or upper) bounds on λ are. Figure 3 portrays the influence of changing λ_{\min} on the median posterior estimate of λ , and on 1- σ and 2- σ confidence lower bounds on posterior estimates of λ . Although the median estimate is relatively insensitive to λ_{\min} , a 2- σ lower bound on λ becomes arbitrarily low as λ_{\min} decreases.

Conclusions

Within a few hundred million years, and perhaps far more quickly, of the time that Earth became a hospitable location

²We note that the comparatively very *late* emergence of radio technology on Earth could, analogously, be taken as an indication (albeit a weak one because of our single datum) that radio technology might be rare in our galaxy.

for life, it transitioned from being merely habitable to being inhabited. Recent rapid progress in exoplanet science suggests that habitable worlds might be extremely common in our galaxy [22, 23, 24, 25], which invites the question of how often life arises, given habitable conditions. Although this question ultimately must be answered empirically, via searches for biomarkers [26] or for signs of extraterrestrial technology [27], the early emergence of life on Earth gives us some information about the probability that abiogenesis will result from early-Earth-like conditions.²

A Bayesian approach to estimating the probability of abiogenesis clarifies the relative influence of data and of our prior beliefs. Although a “best guess” of the probability of abiogenesis suggests that life should be common in the galaxy if early-Earth-like conditions are, still, the data are consistent (under plausible priors) with life being extremely rare, as shown in Figure 3. Thus, a Bayesian enthusiast of extraterrestrial life should be significantly encouraged by the rapid appearance of life on the early Earth but cannot be highly confident on that basis.

Our conclusion that the early emergence of life on Earth is consistent with life being very rare in the Universe for plausible priors is robust against two of the more fundamental simplifications in our formal analysis. First, we have assumed that there is a single value of λ that applies for all Earth-like planets (without specifying exactly what we mean by “Earth-like”). If λ actually varies from planet to planet, as seems far more plausible, anthropic-like considerations imply planets with particularly large λ values will have a greater chance of producing (intelligent) life and of it appearing relatively rapidly, *i.e.*, of the circumstances in which we find ourselves. Thus, the information we derive about λ from the existence and early appearance of life on Earth will tend to be biased towards large values and may not be representative of the value of λ for, say, an “average” terrestrial planet orbiting within the habitable zone of a main sequence star. Second,

our formulation of the problem analyzed in this paper implicitly assumes that there is no increase in the probability of intelligent life appearing once δt_{evolve} has elapsed following the abiogenesis event on a planet. A more reasonable model in which this probability continues to increase as additional time passes would have the same qualitative effect on the calculation as increasing δt_{evolve} . In other words, it would make the resulting posterior distribution of λ even less sensitive to the data and more highly dependent on the prior because it would make our presence on Earth a selection bias favoring planets on which abiogenesis occurred quickly.

We had to find ourselves on a planet that has life on it, but we did not have to find ourselves (*i*) in a galaxy that has life on a planet besides Earth nor (*ii*) on a planet on which life arose multiple, independent times. Learning that either (*i*) or (*ii*) describes our world would constitute data that are not subject to the selection effect described above. In short, if we should find evidence of life that arose wholly independently of us – either via astronomical searches that reveal life on another planet or via geological and biological studies that find evidence of life on Earth with a different origin from us – we would have considerably stronger grounds to conclude that life is probably common in our galaxy. With this in mind, research in the fields of astrobiology and origin of life studies might, in the near future, help us to significantly refine our estimate of the probability (per unit time, per Earth-like planet) of abiogenesis.

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Supporting Information

Formal Derivation of the Posterior Probability of Abiogenesis Let t_a be the time of abiogenesis and t_i be the time of the emergence of intelligence (t_0 , t_{emerge} , and t_{required} are as defined in the text: t_0 is the current age of the Earth; t_{emerge} is the upper limit on the age of the Earth when life first arose; and t_{required} is the maximum age the Earth could have had when life arose in order for it to be possible for sentient beings to later arise by t_0). By “intelligence”, we mean organisms that think about abiogenesis. Furthermore, let

$$\begin{aligned} E &= t_{\min} < t_a < t_{\text{emerge}} \\ R &= t_{\min} < t_a < t_{\text{required}} \\ I &= t_{\min} < t_i \leq t_0 \end{aligned}$$

\mathcal{M} = “The Poisson rate parameter has value λ ”

We assert (perhaps somewhat unreasonably) that the probability of intelligence arising (I) is independent of the actual time of abiogenesis (t_a), so long as life shows up within t_{required} (R).

$$P[I|R, \mathcal{M}, t_a] = P[I|R, \mathcal{M}]. \quad [15]$$

Although the probability of intelligence arising could very well be greater if abiogenesis occurs earlier on a world, the consequence of relaxing this assertion (discussed in the Conclusion and elsewhere in the text) is to increase the posterior probability of arbitrarily low λ .

Using the conditional version of Bayes’s theorem,

$$P[t_a|R, \mathcal{M}, I] = \frac{P[I|R, \mathcal{M}, t_a] \times P[t_a|R, \mathcal{M}]}{P[I|R, \mathcal{M}]}, \quad [16]$$

and Eq. (15) then implies that $P[t_a|R, \mathcal{M}, I] = P[t_a|R, \mathcal{M}]$. An immediate result of this is that

$$P[E|R, \mathcal{M}, I] = P[E|R, \mathcal{M}]. \quad [17]$$

We now apply Bayes’s theorem again to get the posterior probability of λ , given our circumstances and our observations:

$$P[\mathcal{M}|E, R, I] = \frac{P[E|R, \mathcal{M}, I] \times P[\mathcal{M}|R, I]}{P[E, R, I]} \quad [18]$$

Note that, since $t_e < t_r$, $E \Rightarrow R$. And, as discussed in the main text, $P[E|R, \mathcal{M}] = P[E|\mathcal{M}]/P[R|\mathcal{M}]$. Finally, since we had to find ourselves on a planet on which R and I hold, these conditions tell us nothing about the value of λ . In other words, $P[\mathcal{M}|R, I] = P[\mathcal{M}]$. We therefore use Eq. (17) to rewrite Eq. (18) as the posterior probability implied in the text:

$$P[\mathcal{M}|E, I] = \frac{\frac{P[E|\mathcal{M}]}{P[R|\mathcal{M}]} \times P[\mathcal{M}]}{P[E, I]}. \quad [19]$$

Model-Dependence of Posterior Probability of Abiogenesis In the main text, we demonstrated the strong dependence of the posterior probability of life on the form of the prior for λ . Here, we present a suite of additional calculations, for different bounds to λ and for different values of Δt_1 and Δt_2 .

Figure 4 displays the results of analogous calculations to those of Fig. 1, for three sets model of parameters (**Hypothetical**, **Optimistic**, **Conservative**) and for three values of λ_{\min} (10^{-22}Gyr^{-1} , 10^{-11}Gyr^{-1} , 10^{-3}Gyr^{-1}). For all three models, the posterior CDFs for the uniform and the inverse-uniform priors almost exactly match the prior CDFs, and, hence, are almost completely insensitive to the data. For the **Conservative** model (in which $\Delta t_1 = 0.8$ Gyr and $\Delta t_2 = 0.9$ Gyr – certainly not ruled out by available data), even the logarithmic prior’s CDF is barely sensitive to the observation that there is life on Earth.

Finally, the effect of δt_{evolve} – the minimum timescale required for sentience to evolve – is to impose a selection effect that becomes progressively more severe as δt_{evolve} approaches $t_0 - t_{\text{emerge}}$. Figure 5 makes this point vividly. For the **Optimistic** model, posterior probabilities are shown as color maps as functions of λ (abscissa) and δt_{evolve} (ordinate). At each horizontal cut across the PDF plots (left column), the values integrate to unity, as expected for a proper probability density function. For short values of δt_{evolve} , the selection effect (that intelligent creatures take some time to evolve) is unimportant, and the data might be somewhat informative about the true distribution of λ . For larger values of δt_{evolve} , the selection effect becomes more important, to the point that the probability of the data given λ approaches 1, and the posterior probability approaches the prior.

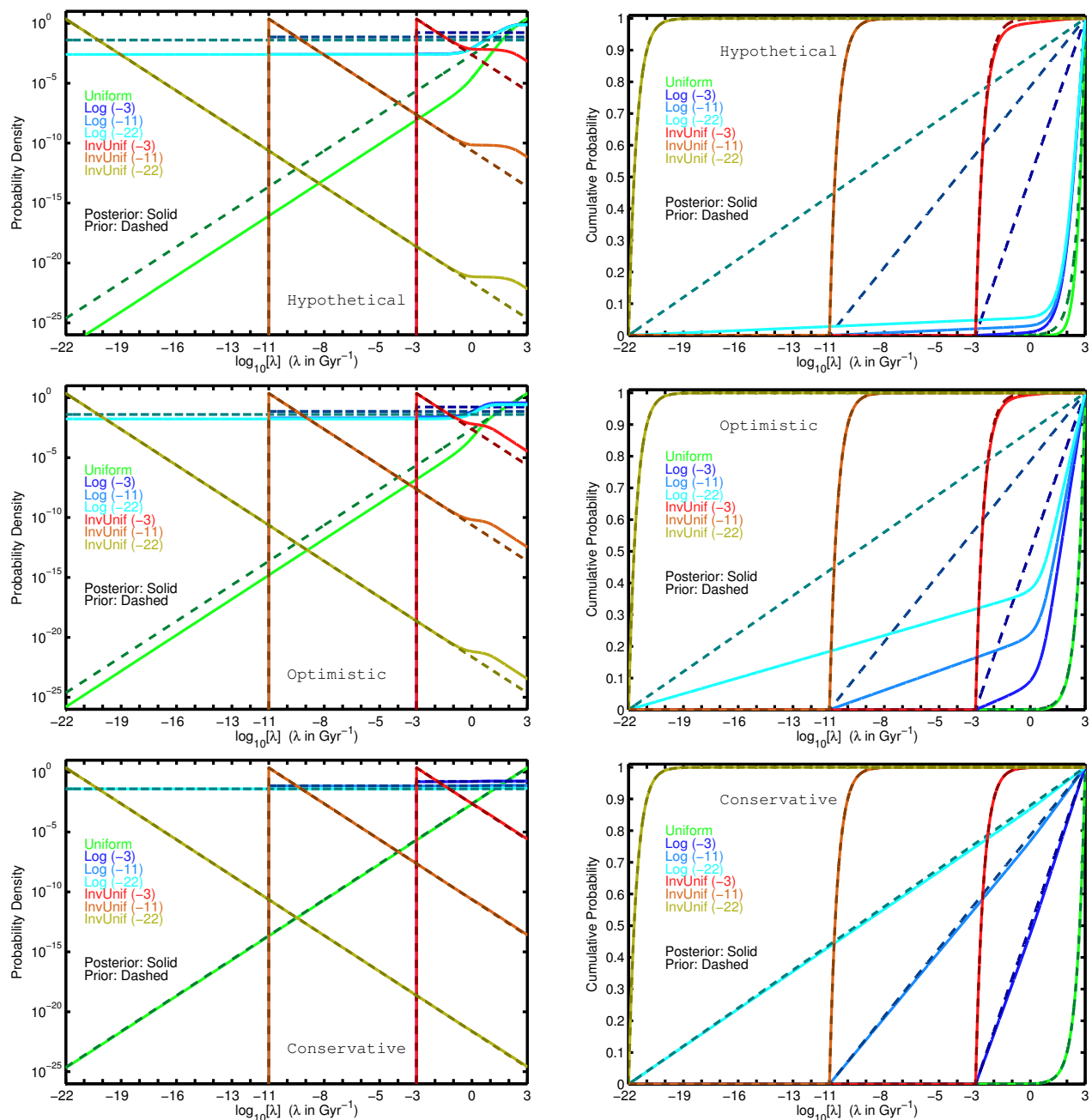


Fig. 4. PDF (left) and CDF (right) of λ for uniform, logarithmic, and inverse-uniform priors, for models Hypothetical (top), Optimistic (middle), Conservative (bottom). Curves are shown for priors with $\lambda_{\min} = 10^{-22}\text{Gyr}^{-1}$, $\lambda_{\min} = 10^{-11}\text{Gyr}^{-1}$, and $\lambda_{\min} = 10^{-3}\text{Gyr}^{-1}$. The uniform and inverse-uniform priors lead to CDFs that are completely insensitive to the data for all three models; for the Conservative model, even the logarithmic prior is insensitive to the data.

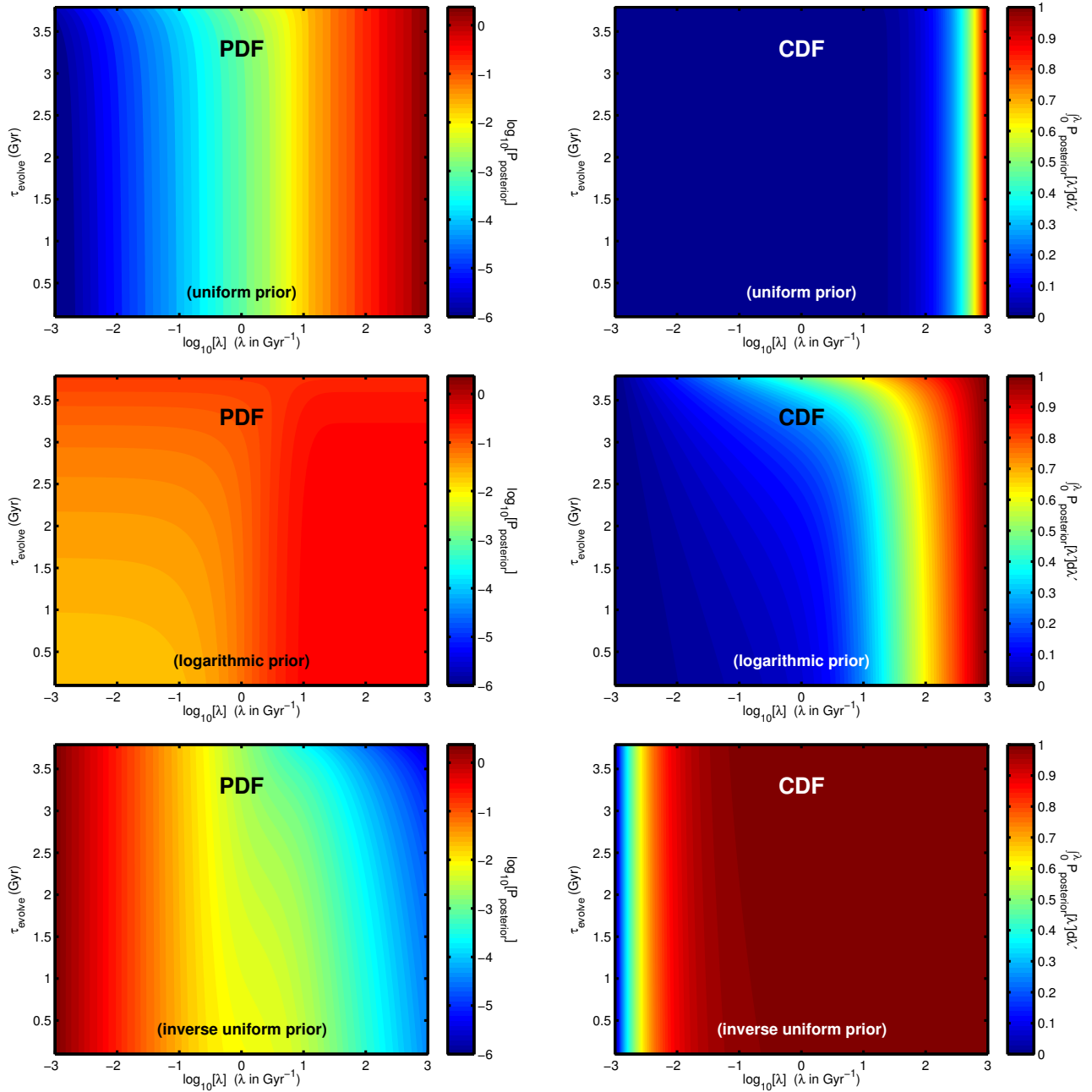


Fig. 5. The influence of δt_{evolve} . Uniform prior (*top*), logarithmic prior (*middle*), and inverse-uniform prior (*bottom*). PDF (*left*) and CDF (*right*). Aside from δt_{evolve} , parameters are set to the **Optimistic** model with $\lambda_{\text{min}} = 10^{-3} \text{Gyr}^{-1}$ in the logarithmic and inverse-uniform cases.