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Investigation of Dynamics of Self-Similarly Evolving Magnetic Clouds

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ABSTRACT

Context. Magnetic clouds (MCs) are "magnetized plasma clouds" moving in the solar wind. MCs transport magnetic flux and helicity away from the Sun. These structures are not stationary but feature temporal evolution. Commonly, simplified MC models are considered.

Aims. The goal of the present study is to investigate the dynamics of more general, *radially expanding* MCs. They are considered as cylindrically symmetric magnetic structures with low plasma β.

Methods. The self-similar approach method and a numerical approach are used.

Results. It is shown that the forces are balanced in the considered self-similarly evolving, cylindrically symmetric magnetic structures. Explicit analytical expressions for magnetic field, plasma velocity, density and pressure within MCs are derived. These solutions are characterized by conserved values of magnetic flux and helicity. We also investigate the dynamics of self-similarly evolving MCs by means of the numerical code "Graale". In addition, their expansion in a medium with higher density and higher plasma β is studied. It is shown that the physical parameters of the MCs maintain their self-similar character throughout their evolution.

Conclusions. A comparison of the different self-similar and numerical solutions allows us to conclude that the evolving MCs are quite adequately described by our self-similar solutions - they retain their self-similar, coherent nature for quite a long time and over large distances from the Sun.

Key words. Magnetohydrodynamics (MHD) — Magnetic fields — Plasmas — Sun: solar wind

1. Introduction

It is well-known that *coronal mass ejections* (CMEs) are one of the most significant forms of solar activity. They carry enormous masses of plasma threaded by the magnetic field away into the

interplanetary medium. Further away from the Sun, these large-scale, dynamical plasma structures are commonly called *interplanetary coronal mass ejections* (ICMEs). Magnetic clouds (MCs) form a subset of ICMEs (Klein & Burlaga 1982, Burlaga 1991, Farrugia et al. 1995). Spacecrafts crossing the central parts of such MCs provide valuable information about their physical characteristics. It turns out that MCs have a strong magnetic field, low proton temperatures (low plasma β, compared to the ambient solar wind with the same speed) and they feature a substantial and smooth rotation of the magnetic field vector. These three features of MCs are selected as signatures of MCs (Nakwacki et al. 2008). The MCs are also characterized by a coherence of the magnetic field (low level of fluctuations). The radial dimension of **a MC** is typically ≈ 0.25 AU (at 1 AU).

These *in situ* observations of the physical properties of MCs are considered as important steps towards the prediction of the geophysical effectiveness of their interaction with the Earth's magnetosphere, for space weather forecasts and related issues.

Different models for the structures of magnetic clouds have been proposed. There is no general agreement about the large scale structure of MCs. Commonly, the local structure of MCs is considered in the form of cylindrically symmetric force-free configurations (Burlaga 1988, 1991, Demoulin & Dasso 2009). It is often suggested that the ends of MCs connect to the surface of the Sun while, according to other models, MCs are described as tori (Vandas et al. 2006, 2009, Romashets et al. 2006, 2007). In a number of studies, MCs are considered as force-free, static, axially symmetric flux ropes and their magnetic field is constructed on the basis of Lundquist's model (Burlaga 1988, Lepping et al. 1990, Farrugia et al. 1993). Observations show, however, that MCs do not stay static but expand while propagating in the solar wind and they keep expanding well beyond 1 AU (Burlaga 1991, Demoulin 2008, Demoulin & Dasso 2009, Bothmer & Schwenn 1998). In a large majority of the cases it is observed that the frontal parts of the MCs propagate with higher velocities than their back regions. This shows that, with respect to the MC's own cylindrical set of coordinates, the radial size of those cylindrical MCs increases (Nakwacki et al. 2008). Theoretical models including the effect of radial expansion have been proposed before (Osherovich et al. 1995, Farrugia et al. 1993, Nakwacki et al. 2008). In these models, only the radial expansion is taken into account and solutions have been found for all plasma parameters. There are other studies (Shimazu & Vandas 2002, Demoulin & Dasso 2009), however, where the axial expansion is also included.

Previous studies also showed that inside MCs the density **drops as** $d^{-2.4}$ **(Bothmer & Schwenn 1998), i.e. the** volume of MC increases as $d^{2.4}$, where d denotes the distance from the Sun. The radius of the MCs, denoted by (R), also increased and at a rate changing with the distance, viz. as: $R \sim d^{0.8}$ (Bothmer & Schwenn 1998). Since the surface of the MC's cross section perpendicular to its axis increases as $R^2 \sim d^{1.6}$, and the MC's volume increases as $d^{2.4}$, the MC's longitudinal size should increase as $d^{0.8}$. Therefore, according to Bothmer & Schwenn's data, the MCs are radially expanding and also show an extension along their axis.

In the present study, we consider self-similarly expanding cylindrical MCs that are able to expand both in the radial and longitudinal directions. We consider the problem in the frame of the MC and in cylindrical coordinates related with the MC, i.e. with a longitudinal axis Z that coincides with the MC's axis. Overall cylindrical symmetry of the MC is assumed. Based on these assumptions, we derive the appropriate full set of non-stationary MHD equations and find their analytical solutions. The logical and natural consequence of the assumptions of self-similarity and

cylindrical symmetry is that the dynamic forces acting upon the MCs are balanced. The solutions include expressions for the plasma magnetic field, velocity, mass density and thermal pressure.

An important feature of our model is that certain significant characteristics of the MCs — magnetic flux and helicity — are conserved. We separately consider also the particular case of a MC that is allowed to expand only in the radial direction. It can be shown that in this case, the MHD equations do not have any physical, self-similar solution.

2. Self-similar expanding MC models

2.1. General equations and self similar expansion

In order to perform an analytic study of the dynamics of magnetic clouds, we have to start from the full set of MHD equations:

$$\nabla \cdot \mathbf{B} = 0, \tag{1}$$

$$\partial_t \mathbf{B} = \nabla \times [\mathbf{V} \times \mathbf{B}], \tag{2}$$

$$\partial_t \varrho + \nabla \cdot (\varrho \mathbf{V}) = 0, \tag{3}$$

$$\varrho[\partial_t + (\mathbf{V} \cdot \nabla)]\mathbf{V} = (1/4\pi)(\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla \cdot p, \tag{4}$$

In these equations, p denotes the thermal plasma pressure, ϱ is the density, $\mathbf{V}$ is the velocity field and $\mathbf{B}$ denotes the magnetic field.

In a number of previous studies, the MCs were considered as cylindrical magnetic structures, characterized by axial symmetry. In the present consideration, both symmetry along the Z axis ($\partial_z = 0$) and the azimuthal symmetry ($\partial_\varphi = 0$) are assumed. The axially-symmetric magnetic field can then be expressed in the following way

$$\mathbf{B} \equiv [0, \ B_\varphi, \ B_z], \tag{5}$$

where $B_\varphi = B_\varphi(r, t)$ and $B_z = B_z(r, t)$. Note that this representation satisfies the solenoidal condition.

The self-similar approach, adopted here, implies that the temporal evolution of the physical functions is controlled by the following self-similar variable:

$$\xi = \frac{r}{\Phi(t)}, \tag{6}$$

where $\Phi(t)$ denotes a function of time. Let us search solutions of the MHD equations in the following form (in analogy with Low 1982):

$$B_\varphi = \Phi^\delta Q_\varphi(\xi), \tag{7a}$$

$$B_z = \Phi^\sigma Q_z(\xi), \tag{7b}$$

$$\varrho = \Phi^\alpha \tilde{\rho}(\xi), \tag{7c}$$

$$p = \Phi^\beta \tilde{p}(\xi), \tag{7d}$$

One can see that, the type of solutions introduced by Eqs. (7a-7d) evolve self-similarly and are characterized by a particular time scaling.

Here Q_φ, Q_z, $\tilde{\rho}$ and $\tilde{p}$ are functions of the self similar variable ξ. Φ^δ, Φ^σ, Φ^α and Φ^β show the time scaling of the azimuthal and longitudinal components of the magnetic field, the plasma density and the plasma pressure, respectively.

2.2. Solution of the induction equation

We consider both a radial and a longitudinal expansion of the MC but no motion in the azimuthal direction is considered. In this case the *Eulerian* velocity field of the plasma, **V**, can be expressed in the following way:

$$\mathbf{V} = [V_r, \ \ 0, \ \ V_z]. \tag{8}$$

Here, we assume that the radial component of the velocity $V_r = V_r(r, t)$, and the $z-$component $V_z = V_z(z, t)$, i.e. we assume that the MC maintains its cylindrical shape during its evolution.

After substitution of Eq. (8) in Eq. (2) we derive:

$$\partial_t B_z + \frac{1}{r}\partial_r(rV_r B_z) = 0, \tag{9a}$$

After taking into account relations (6), (7b) and the relations (A1) and (A2) given in the appendix, Eq. (9a) can be rewritten as follows:

$$Q_z\left[\sigma\dot{\Phi} + \frac{V_r}{\xi} + \Phi\partial_r V_r\right] + Q_z'\left[V_r - \xi\dot{\Phi}\right] = 0. \tag{9b}$$

Here Q_z' corresponds to $dQ_z(\xi)/d\xi$. Equation (9a) (therefore equation (9b)) is satisfied for arbitrary Q_z only when:

$$V_r - \xi\dot{\Phi} = 0, \tag{10a}$$

and

$$\sigma\dot{\Phi} + \frac{V_r}{\xi} + \Phi\partial_r V_r = 0. \tag{10b}$$

From Eq. (10a) and Eq. (10b) follows that radial component of the Eulerian plasma velocity is described as follows:

$$V_r = r\dot{\Phi}/\Phi, \tag{11}$$

and

$$\sigma = -2, \tag{12}$$

where Φ is the function of time mentioned in Eq. (6).

One can check that for $\sigma = -2$ the longitudinal magnetic flux ϕ_z is conserved. Nakwacki et al. (2008) analyzed different MC models and derived expressions for the magnetic flux, the magnetic helicity and the magnetic energy per unit length along the flux tube. The models which are in good agreement with observations are characterized by the conservation of ϕ_z, see also Berdichevsky et al. (2003).

Let us analyze the φ-component of the induction equation, Eq. (2):

$$\partial_t(B_\varphi) + B_\varphi\partial_z V_z + \partial_r(V_r B_\varphi) = 0. \tag{13}$$

The combination of Eqs. (11) and (13) leads to the following important relation:

$$(\delta + 1)\dot{\Phi}/\Phi + \partial_z V_z = 0. \tag{14}$$

After taking into account expressions (5) and (8) in combination with the assumption of azimuthal symmetry, one can see that radial component of the induction equation, Eq. (2), is automatically satisfied and does not lead to any additional restrictions.

2.3. Self-similar solutions

After inserting the expression for the plasma density (7c), together with the velocity from Eq. (8), with Eq. (11) for the radial component, in the mass conservation law Eq. (3), we obtain another important relation, viz.

$$(\alpha + 2)\dot{\Phi}/\Phi + \partial_z V_z = 0. \tag{15}$$

Obviously, in order to have consistency between Eqs. (14) and (15), one should have: $\alpha + 2 = \delta + 1$.

The z-component of the equation of motion, Eq. (4), helps to derive an expression for the z-component of the plasma velocity:

$$\partial_t V_z + V_z \partial_z V_z = 0. \tag{16a}$$

Let us try to solve the partial differential equation (16a) by using the variable separation technique, i.e. we assume that

$$V_z(z, t) = Z(z)T(t). \tag{16b}$$

Substitution of expression (16b) in Eq. (16a) yields:

$$\dot{T}Z + ZT^2Z' = 0, \tag{17a}$$

here $\dot{F} \equiv \partial_t F$ denotes the first order time derivative of a function F. Hereafter we will use, for indicating second order derivatives, the notation: $\ddot{F} \equiv \partial_t^2 F$. While Z' stands for dZ/dz.

It follows from Eq. (17a) that:

$$-\frac{\dot{T}}{T^2} = Z' = const, \tag{17b}$$

Equation (17b) can be decomposed into two ODEs, viz.

$$-\frac{\dot{T}}{T^2} = \lambda, \tag{17c}$$

and

$$Z' = \lambda. \tag{17d}$$

Here, λ is an arbitrary constant.

After solving the ODEs (17c-17d) with the assumption that, at the surface $z = 0$, $V_z = 0$, we derive the following expressions:

$$T = \frac{T_0}{1 + \lambda T_0 t}, \tag{18a}$$

and:

$$Z = \lambda z, \tag{18b}$$

where λ and T_0 are constants.

After inserting Eqs. (18a-18b) in Eq. (16b), we obtain the wanted expression for V_z:

$$V_z = \frac{zk}{1 + kt}. \tag{19}$$

Here, $k \equiv \lambda T_0$.

We assumed that locally the MC could be described as a cylindrical structure. Let us investigate the evolution of the length L of this cylindrical structure. For this purpose, let us describe the temporal evolution of the z-coordinate of the plasma element located at the position $z = L$ at time

t. The Lagrangian velocity of this element coincides with the Eulerian velocity of the plasma flow at time t **and** $z = L$**. If** at a certain time the coordinate of this element is L, then its Lagrangian velocity is:

$$V_L = \frac{dL}{dt}. \tag{20a}$$

From Eq. (19) we then have:

$$\frac{dL}{dt} = \frac{Lk}{1+kt}. \tag{20b}$$

The solution of this ordinary differential equation (20b) gives the following expression for the **longitudinal size of the considered cylindrical structure**:

$$L = L_0(1+kt), \tag{20c}$$

where L_0 **is the length of the cylinder at** $t = 0$**.**

The radial component of the equation of motion, in combination with the expressions for the magnetic field, the velocity and the plasma density leads to:

$$\Phi^{\alpha}\xi\ddot{\Phi}\tilde{\rho} = F_r. \tag{21}$$

Here, F_r denotes the radial component of the total force. In terms of $\tilde{p}$, $Q_{\varphi,z}$, and ξ, this force component can be expressed as (details of the derivation are given in the appendix):

$$F_r = -\frac{1}{4\pi}\left[\frac{1}{\Phi^5}Q_z'Q_z + \Phi^{\delta-3}\left(\frac{Q_\varphi^2}{\xi} + Q_\varphi'Q_\varphi\right)\right] - \Phi^{\beta-1}\tilde{p}', \tag{22}$$

where $Q_{\varphi,z}^{/} = dQ_{\varphi,z}/d\xi$, and $\tilde{p}' = d\tilde{p}/d\xi$. In order to have a self-consistent time scaling for all terms in Eq. (22), one has to require that $\delta = -2$ and $\beta = -4$. From a comparison of Eq. (14) to Eq. (15), follows that if $\delta = -2$, then $\alpha = -3$.

At the same time, from Eq. (15) and Eq. (19):

$$\frac{\dot{\Phi}}{\Phi} = \frac{k}{1+kt}. \tag{23}$$

Equation (23) is an ordinary differential equation in terms of $\Phi(t)$. After solving this ODE, we find the following explicit expression for $\Phi(t)$:

$$\Phi = \Phi_0(1+kt). \tag{24}$$

Here Φ_0 is a constant parameter.

The substitution of expression (24) in Eq. (21) leads to an important conclusion: calculating the magnetic and pressure gradient forces, we see that for the self-similarly evolving, cylindrical, axially-symmetric structure the magnetic force, $F_m \equiv 1/(4\pi)(\nabla \times \mathbf{B}) \times \mathbf{B}$ and the thermal pressure gradient force, $F_p \equiv -\nabla \cdot p$ are exactly balanced, i.e.

$$F_r = F_m + F_p = 0. \tag{25}$$

If we associate the value ξ_0 of the self-similar variable ξ with the boundary of the MC, then the expression of the MC Lagrangian velocity is given by (Low 1982):

$$V_s = \frac{dR}{dt} = \xi_0\frac{d\Phi}{dt}. \tag{26}$$

After substitution of the expression (24) for Φ in Eq. (26), we can derive a time-dependent solution for the MC radius:

$$R = R_0(1+kt). \tag{27}$$

Note that the form of this expression coincides with the one given by Nakwacki et al. (2008).

2.4. Plasma and force-free field evolution

The rest of the solutions readily follows from the derived equations, yielding:

$$V_r = \frac{rk}{1+kt}, \tag{28a}$$

$$\varrho = \frac{\tilde{\varrho}}{(1+kt)^3}, \tag{28b}$$

and

$$p = \frac{\tilde{p}}{(1+kt)^4}. \tag{28c}$$

Here, $\tilde{\varrho}$ and $\tilde{p}$ are arbitrary functions of $\xi = r/\Phi$.

After analysis of the expressions for pressure and density (28b-28c), one can check that, for systems characterized by entropy conservation, the entropy conservation law is satisfied only if the polytropic index $\gamma = 4/3$. Actually, this is a common feature of all different self-similar systems (Low 1982, Farrugia et al. 1995, Finn et al. 2004).

From various observations it is known that MCs are characterized with low plasma β's (Burlaga et al. 1981, Burlaga 1991, Bothmer & Schwenn 1998). The thermal pressure term in the total force could be neglected and this implies that we have to construct a force-free magnetic field that evolves in a self-similar way. The cylindrically symmetric force-free structure of the MC's magnetic field is indeed advocated by a number of researches (Burlaga 1988, Lepping et al. 1990, Farrugia et al. 1993, Farrugia et al. 1995, Nakwacki et al. 2008, Demoulin & Dasso 2009). A force-free magnetic field satisfies the following relation:

$$\nabla \times \mathbf{B} = \mu \mathbf{B}. \tag{29}$$

If we rewrite the vectorial equation (29) for each component of vectors, taking in to account expressions (7a), (7b) and (12), we obtain:

$$-Q_z' = \mu \Phi Q_\varphi, \tag{30a}$$

$$Q_\varphi' + \frac{Q_\varphi}{\xi} = \mu \Phi Q_z, \tag{30b}$$

here $Q_{\varphi,z}'$ stands for $dQ_{\varphi,z}/d\xi$. If we take the derivative of both terms of Eq. (30a) with respect to the variable ξ, we get:

$$-Q_z'' = \mu \Phi Q_\varphi'. \tag{30c}$$

Here it was assumed that μ does not depend on ξ. In general, however, μ could be a function of ξ.

If we take in to account expressions (30a) and (30c), we can derive from Eq. (30b) an ordinary differential equation for Q_z:

$$Q_z'' + \frac{Q_z'}{\xi} + \mu^2 \Phi^2 Q_z = 0. \tag{31a}$$

With the following transformation of variables: $x = \mu \Phi \xi$, we can rewrite Eq. (31a) as follows:

$$\frac{d^2 Q_z}{dx^2} + \frac{1}{x}\frac{dQ_z}{dx} + Q_z = 0. \tag{31b}$$

Actually Eq. (31b) is a Bessel equation of zero order, with the following solution:

$$Q_z = J_0(x) = C_0 J_0(\mu \Phi \xi), \tag{32a}$$

where $J_0(x)$ is the Bessel function of the first kind, C_0 is a constant parameter. Notice that the solution which is not characterized with a singularity at $x = 0$ has been chosen. The substitution of Eq. (32a) in relation (30a) leads to an expression for Q_φ:

$$Q_\varphi = J_1(x) = C_0 J_1(\mu\Phi\xi), \tag{32b}$$

with $J_1(x)$ the Bessel function of the first kind.

From Eqs. (32a-b) we see that Q_φ and Q_z are the functions of $\mu\Phi\xi$. Since we assumed above that μ is not a function of ξ, that Φ is only a function of t (Eq. (6)), and that $Q_{\varphi,z}$ are functions of only ξ (Eqs. (7a,7b)), it follows that $\mu\Phi = const.$ The substitution of Eqs. (32a), (32b), (24) and (6) in Eqs. (7a) and (7b) respectively, taking into account that $\delta = \sigma = -2$, leads to the following expressions for the components of the magnetic field:

$$B_r = 0, \tag{33a}$$

$$B_\varphi = \frac{B_0}{(1+kt)^2} J_1\left(\frac{r}{r_0(1+kt)}\right), \tag{33b}$$

and

$$B_z = \frac{B_0}{(1+kt)^2} J_0\left(\frac{r}{r_0(1+kt)}\right), \tag{33c}$$

where B_0 and r_0 are constants.

(C_0/Φ_0^2 is changed by B_0 and $\mu\xi/\Phi_0$ is substituted by $1/r_0$.)

From Eqs. ($33a - c$) we can calculate important expressions for the magnetic flux and the helicity (Nakwaci et al. 2008) associated with the MC:

$$\Phi_z = \frac{2\pi}{\chi} R \frac{B_0}{(1+kt)^2} J_1(\chi R), \tag{34a}$$

$$\Phi_\varphi = \frac{1}{\chi} \frac{B_0}{(1+kt)^2} L(1 - J_0(\chi R)), \tag{34b}$$

and

$$H = \frac{2\pi}{\chi} R^2 \frac{B_0^2}{(1+kt)^4} L(J_1^2(\chi R) - J_0(\chi R) J_2(\chi R) + J_0^2(\chi R)), \tag{34c}$$

where $\chi \equiv 1/(r_0(1+kt))$.

By its physical meaning Φ_z is the magnetic flux across the surface perpendicular to the axis of a MC, while Φ_φ is the magnetic flux across the surface defined by the magnetic axis and the radial direction. Moreover, R denotes the radius of the MC and L is longitudinal length of the cylindrical structure. By inserting in Eqs. ($34a - c$) the corresponding expressions for R and L we find:

$$\Phi_z = 2\pi R_0 r_0 B_0 J_1\left(\frac{R_0}{r_0}\right) = const, \tag{35a}$$

$$\Phi_\varphi = B_0 r_0 L_0 \left[1 - J_0\left(\frac{R_0}{r_0}\right)\right] = const, \tag{35b}$$

and also

$$H = 2\pi r_0 R_0^2 B_0^2 L_0 \left[J_1^2\left(\frac{R_0}{r_0}\right) - J_0\left(\frac{R_0}{r_0}\right) J_2\left(\frac{R_0}{r_0}\right) + J_0^2\left(\frac{R_0}{r_0}\right)\right] = const. \tag{35c}$$

From these results **it follows that the obtained solutions** ensure the conservation of magnetic flux and helicity inside the cylindrical MC described by our model.

3. Radially expanding MCs

The purpose of this section is to find solutions for the physical variables in the case where only the radial size of the MC increases. One can see that the solutions in this case do not remain self-similar, although initially a self-similar expansion is assumed in the radial direction.

Below we consider MCs that are expanding only radially; i.e. with $V_z = 0$. In this case Eq. (14) implies $\dot{\Phi} = 0$ or $\delta = -1$. The case with $\dot{\Phi} = 0$ corresponds to the *stationary* state, which is trivial. Let us consider the case when $\dot{\Phi} \neq 0$ but $\delta = -1$. In order to provide a consistent time-scaling of all terms in Eq. (22), we have to satisfy:

$$Q_\varphi Q'_\varphi + \frac{Q_\varphi^2}{\xi} = \frac{Q_\varphi}{\xi}\partial_\xi(\xi Q_\varphi) = 0. \tag{36a}$$

An analysis of Eq. (6) ($\xi = r/\Phi(t)$), Eq. (7a) ($B_\varphi = \Phi^\delta Q_\varphi(\xi)$) and Eq. (36a) leads to the following expression:

$$\frac{1}{r}\partial_r(rB_\varphi) = 0. \tag{36b}$$

Here, it is taken into account that $\partial_r = \partial_\xi/\Phi(t)$.

From Eq. (36b) we can conclude that:

$$B_\varphi = \frac{C}{r}, \tag{36c}$$

with $C = \text{const}$.

Note that the expression for B_φ is characterized by a singularity at the axis ($r = 0$). It seems reasonable to conclude that, if we do not consider the axial stretching of self-similarly evolving MCs, we can not obtain a physically valid solution for the $B_\varphi-$ **component on the axis of the MC.**

4. Numerical study: higher density and higher plasma β case

In this section, we investigate the evolution of MCs in a medium by means of the model described in Section 2. For this purpose, the Lagrangian numerical MHD code **"Graale"** (Finn et al. 2004) is used, which enables us to check whether the above-obtained solutions maintain their self-similar nature when they propagate in a medium. In the numerical code, azimuthal and cylindrical symmetries are implied. Furthermore, it is assumed that the magnetic structure expands uniformly in the longitudinal direction, in other words $V_z = z\dot{L}/L$; with L the length of the cylinder. An analysis of Eqs. (20a-20c) and Eq. (19) shows that the derived expression for the longitudinal velocity coincides with the one implemented in the code.

The assumptions of cylindrical and azimuthal symmetries in the code and the prescription of the character of the longitudinal motions makes the numerical simulations 1D. In the numerical runs the units of the physical parameters are chosen as follows: the unit length $L_{unit} = 0.1AU = 15 \cdot 10^6$ km is of the order of the MC's radius at 1 AU, the unit magnetic field $B_{unit} = 3$ nT, and the unit number density $n_{unit} = 10$ cm^{-3}. After taking into account that the proton mass $m_p \approx 1.7 \cdot 10^{-27}$ kg, one derives that the unit mass density is $\rho_{unit} = m_p n_0 = 0.8 \cdot 10^{-14}$ kg/m^3. The unit speed is the Alfvén speed corresponding to ρ_{unit} and B_{unit}: $V_{unit} = V_{0A} = 20.5$ km/s, which is of the order of the MC's edges expansion velocity in the frame of the MC (Vandas et al., 2009), and the unit time $t_{unit} = \frac{L_{unit}}{V_{unit}} = 200$ h. A domain with $R_{min} = 0$ and $R_{max} = 10$ is discretised with 2000 grid cells. The time step used in the simulations is $\Delta t = 5 \cdot 10^{-7}$ (R_{min}, R_{max} and Δt are given in units introduced

above-L_{unit} and t_{unit}). Open boundary conditions are applied. Inside the calculation domain, we introduce initial conditions for the physical variables in two different regions: inside the MC and outside the MC.

Evidently, the solutions inside and outside the magnetic structure should satisfy the following jump conditions across the surface of any MC:

$$[\varrho v_r] = 0 \tag{37a}$$

$$\left[\varrho v_r^2 + p + \frac{1}{8\pi}B^2\right] = 0 \tag{37b}$$

$$\left[\frac{1}{2}\varrho v_r^3 + (\frac{\gamma p}{\gamma - 1} + \frac{1}{4\pi}B^2)v_r\right] = 0 \tag{37c}$$

and, finally,

$$[\varrho v_r v_t] = 0 \tag{37d}$$

Here, $[\cdot]$ denotes the jump of the quantity between the brackets across the surface of the MC. Also, $v_r = V_r - V_s$, where V_r and V_s are the plasma and the MC's surface velocity, respectively, while v_t denotes the plasma velocity tangential to the surface of the MC. Equation (28a) and Eq. (26) show that $v_r = 0$, which is logical for ideal MHD. Equations (37a,37c,37d) are satisfied for arbitrary values of the plasma density and Eq. (37b) leads to the condition:

$$\left[p + \frac{1}{8\pi}B^2\right] = 0. \tag{38}$$

We know that the plasma mass density inside the MCs is lower than outside them and the plasma β within a MC is lower than in the ambient plasma. We therefore consider $\beta \sim 1$ in the ambient environment and $\beta \sim 0.1$ inside the MC. For the magnetic field within the MC, we use the solution expressed by Eqs. (33a-33c). For the magnetic field outside the MC, we assume that the azimuthal component of the magnetic field $B_{\varphi out} = 0$, while the longitudinal component B_{zout} is uniform. We also assume that the mass density and the thermal pressure are uniform in both regions of the computational domain.

Bearing in mind these assumptions and jump conditions (37 – 38), one can find explicit expressions for the plasma pressure and magnetic field outside the MC.

Figure 1 represents the numerical solutions for the plasma mass density and velocity, while Fig. 2 shows solutions for the magnetic field components at different moments in time. On panels a and b we plotted the dimensionless values of the density, velocity and magnetic field. On panels c and d the dependence of the modified values of the physical parameters on the self-similar variable is presented.

The dependence of the modified mass density and velocity as well as the dependence of the modified magnetic field components on the self-similar variable clearly shows that our solutions maintain their self-similarity in the course of the MC expansion.

5. Discussion and Conclusions

In this paper, we presented a detailed derivation of a class of self-similar analytic solutions of the MHD equations for both radially and axially expanding MCs and a numerical investigation of these solutions. The usage of the self-similar approach is quite common for the modeling of various kinds of solar plasma structures, flows and eruptions (Low 1982, Osherovich 1993, Farrugia et al. 1995,

Nakwacki et al. 2008, Shapakidze et al. 2010). In most of the previous studies, however, *only* the radial expansion of the MCs was considered. In the present study, we took into account also the axial stretching of the MCs, which is a common observed feature of at least some MCs. We have obtained explicit analytical expressions for the magnetic field, the plasma velocity, the density and the plasma pressure. Essentially, our solutions maintain their self-similar nature during the whole course of their evolution and propagation through the solar wind. These solutions are complete and well-defined, fully analytic and, moreover, in the particular case of the absence of the longitudinal expansion, our solutions self-consistently match the analytic solutions derived by other authors (Farrugia et al. 1995).

Note that for the class of solutions introduced by Eqs. (7a-7d), the assumptions of self-similarity and axial and azimuthal symmetry lead to the fact that $\Phi(t)$ is a linear function of time ($\Phi(t)$ is the time dependent function of the self similar variable $\xi = r/\Phi(t)$). In this case, the forces within the MCs are bound to be balanced. We can thus conclude that the case in which the magnetic structures are characterized by a low plasma-β, corresponds to the force-free magnetic field case. Remark that this result is also in agreement with the conclusion of **previous** studies. We therefore believe that this is a correct and proper time-dependent generalization of the widely used stationary Lundquist model (Lundquist 1950). Note also that in their recent papers Vandas et al. (2006, 2009) made a comparison of the generalized Lundquist model with observations and found good agreements between this classic model and the experimental data.

It must be emphasized that our study is not the only one in which the axial stretching of the MCs is taken into account together with their radial expansion. As a matter of fact, Shimazu & Vandas (2002) also considered MCs with similar properties. In this particular paper, the authors used the mathematical approach introduced by Osherovich et al. (1995). In order to separate the time-dependent parts of the solutions from multiplicative functions of the self-similar variable only, Shimazu & Vandas (2002) imply a so-called “separable magnetic” field, which was introduced in Osherovich et al. (1995). The approach introduced in Osherovich et al. (1995), turns out to be quite restrictive because it requires an *ad hoc* relation between the different components of the magnetic field. Also, in order to separate the time-dependent part from the coordinate dependent parts in the momentum equation, in addition to the polytropic law, the authors introduced a specific mathematical expression for the thermal pressure (see Eq. 17 Osherovich et al. 1995). Actually, the mentioned expression relates pressure and mass density (see Eqs. (13-17) Osherovich et al. 1995). In our study, on the contrary, we used B.C. Low’s approach (1982) and required a similar time-scaling for all parts of the Lorentz force and the force caused by the gradient of the thermal pressure. We argue that our approach is more general and puts less non-physical restrictions upon the physical parameters.

Another difference of the results presented here with those of Shimazu & Vandas (2002) is with the temporal expansion scaling. Shimazu & Vandas assume that the longitudinal and the radial expansion have the same time scaling, while in our work this is *not* assumed but it rather logically *follows* as the by-product of the accurate solution of the MHD equations. In their paper, the time-dependent function of the self-similar variable is characterized by a linear dependence on time *only* when the thermal pressure is zero, while we have derived an explicit expression of this time-dependent function of the self-similar variable and it has been shown that Φ is a linear function of the time variable and it does not depend on the character of the pressure function.

Yet another difference between the results presented here and the results of Shimazu & Vandas is related to the structure of the MC magnetic field. Shimazu & Vandas, in order to derive explicit expressions for the magnetic field, used the assumption that the magnetic structure of the MC is described by a force-free magnetic flux rope. In our study, however, we derived an explicit expression for the magnetic field. We have derived ordinary differential equations (Eqs. (30a)-(30b)) for the functions describing the components of the magnetic field, after solving the equation of motion for the case which corresponds to a low plasma β within the MC. For a particular type of parameters, we have found explicit, analytical solutions for the components of the magnetic field (Eqs. (33a)-(33c)). Note that these expressions are a time dependent generalization of the well-known Lundquist solutions (Lundquist 1950, Burlaga 1988).

Our model implies the conservation of magnetic flux and helicity by design, which is satisfactory and in good agreement with previous investigations (Nakwacki et al. 2008, Demoulin & Dasso 2009, Kumar & Rust 1996).

For further confirmation of the validity of our solutions, we investigated the dynamics of magnetic clouds numerically. In the numerical code Graale we introduced our self-similar solutions as initial conditions. The obtained numerical results showed that during the evolution and propagation of these MCs, their physical variables maintained their self-similar character. This circumstance was illustrated by Fig. 1 and Fig. 2.

Obviously, the class of solutions found in this paper is quite idealized. The assumptions about the self-similar evolution and the consideration of a cylindrical symmetric structure are quite well-justified, but real MCs show self-similar coherence and cylindrical symmetry only approximately. Hence, in a future study it would be reasonable and interesting to consider more realistic configurations. There are several issues related to the model which can be tested and generalized in a forthcoming study:

1. Our assumptions, just like in previous investigations (Low 1982, Farrugia et al. 1995, Finn et al. 2004, Shapakidze et al. 2010), for the systems where entropy is conserved, put a restriction on the value of the polytropic index $\gamma = 4/3$. We would like to develop a model that helps to avoid this restriction.
2. Our model describes the plasma dynamics only inside the MC. In the near future, we plan to investigate the interaction of an MC with its environment by constructing consistent solutions of the MHD equations outside the MC.
3. We investigated the obtained analytical solutions numerically using a 1D MHD code and a simple model for the flow outside the MC was implemented. It would be interesting and reasonable to study the MC evolution also with 3D numerical codes, where a more complicated and realistic background flow can be implemented (in preparation). For this purpose the obtained solutions could be used as initial state in the 3D numerical simulation codes. We are interested in an investigation of the different possible boundary conditions on the surface of the magnetic cloud.

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Appendix A:

In this appendix we would like to give some details of derivation **for** Eq. (21) and Eq. (22).

During the process of derivation were implied following relations:

$$\partial_r = \frac{1}{\Phi}\partial_\xi, \tag{A1}$$

$$\partial_t = -\frac{\xi}{\Phi}\dot{\Phi}\partial_\xi. \tag{A2}$$

These relations follow from Eq. (6).

Equation (21) is the radial component of the equation of motion-Eq. (4). In cylindrical coordinates the left term of Eq. (4) can be written as follows:

$$\rho\left[\partial_t + (\mathbf{V}\cdot\nabla)V_r\right] = \rho\left[\partial_t V_r + V_r\partial_r V_r\right]. \tag{A3}$$

If we take into account relation (6) and expression for V_r (Eq. 11) we can obtain following expressions:

$$\partial_t V_r = r\frac{\ddot{\Phi}}{\Phi} - r\left(\frac{\dot{\Phi}}{\Phi}\right)^2, \tag{A4}$$

$$V_r\partial_r V_r = r\left(\frac{\dot{\Phi}}{\Phi}\right)^2. \tag{A5}$$

After substitution of Eq. (A4) and Eq. (A5) into Eq. (A3) we get:

$$\rho\left[\partial_t + (\mathbf{V}\cdot\nabla)V_r\right] = \rho\xi\ddot{\Phi}. \tag{A6}$$

If we combine of Eq.(7c) and Eq (A6) we obtain:

$$\rho\left[\partial_t + (\mathbf{V}\cdot\nabla)V_r\right] = \Phi^\alpha\xi\ddot{\Phi}\tilde{\rho}, \tag{A7}$$

the left term of Eq. (21).

In order to derive the first part of the right term of Eq. (22) let us introduce the following notation:

$$\nabla\times\mathbf{B} \equiv \mathbf{J}. \tag{A8}$$

After taking into account expressions for magnetic field (7a-7b) with $\sigma = -2$ we get:

$$J_r = \frac{1}{r}\partial_\varphi B_z - \partial_z B_\varphi = 0, \tag{A9}$$

$$J_\varphi = \partial_z B_r - \partial_r B_z = -\partial_r B_z = -\frac{Q_z'}{\Phi^3}, \tag{A10}$$

and

$$J_z = \frac{1}{r}\partial_r(rB_\varphi) - \frac{1}{r}\partial_\varphi B_r = \frac{1}{r}\partial_r(rB_\varphi) = \frac{Q_\varphi\Phi^{\delta-1}}{\xi} + Q_\varphi'\Phi^{\delta-1}. \tag{A11}$$

Since we know expressions of vector **J** we can derive the vectorial product of **J** and **B** which represents the first part of the right term of Eq. (22).

Combination of relation (A1) with the expression for pressure Eq. (7d) leads to the expression of the second part of the right term of Eq. (22).

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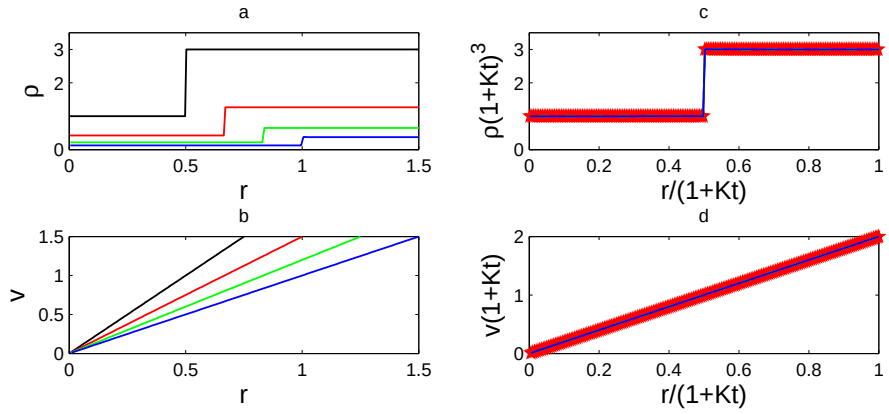

Fig. A.1. Snapshots of the evolution of the plasma density and velocity field. Panel a) represents the dependence of the plasma density on the radial coordinate for four moments in time; Panel b) shows the dependence on the plasma velocity on the radial coordinate for four time moments; Panel c) and Panel d) illustrate the dependence of the modified density and the modified velocity, respectively, on the self similar variable. Parameter values for this case are $k = 2$, $B_0 = 1$, $\varrho_{out} = 3\varrho_{in}$, $\beta_{in} = 0.1$, $\beta_{out} = 1$, and $\Phi_0 = 1$. Black line corresponds to $t = 0$, red line represents the moment $t = 0.5/3$, green line shows the time moment $1/3$, and blue line corresponds to the moment $t = 0.5$.

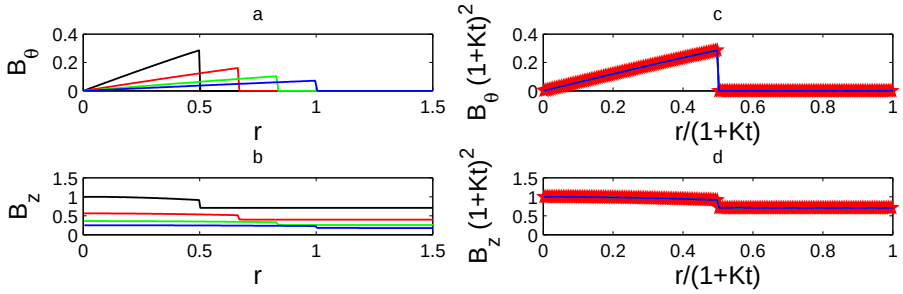

Fig. A.2. Snapshots of the evolution of the magnetic field components. Panel a) represents the dependence of the azimuthal component of the magnetic field on the radial coordinate for four time moments; Panel b) shows the dependence of the z-component of the magnetic field on the radial coordinate for four time moments; Panel c) and Panel d) illustrate the dependence of the modified azimuthal and z-components of the magnetic field, respectively, on the self similar variable. Parameter values for this case are $k = 2$, $B_0 = 1$, $\varrho_{out} = 3\varrho_{in}$, $\beta_{in} = 0.1$, $\beta_{out} = 1$, $r_0 = 1$, and $\Phi_0 = 1$. Black line corresponds to $t = 0$, red line represents the moment $t = 0.5/3$, the line shows the time moment $1/3$, and blue line corresponds to the moment $t = 0.5$.