Comparison of model predictions for elliptic flow with experiment for $Pb + Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV

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A simple kinematic model based on the superposition of $p + p$ collisions, relativistic geometry and final-state hadronic rescattering is used to predict the elliptic flow observable in $\sqrt{s_{NN}} = 2.76$ TeV $Pb + Pb$ collisions. A short proper time for hadronization is assumed. The predictions are compared with recent experimental measurements of elliptic flow which have been made for this colliding system and energy. It is found that the model predictions do a reasonable job in describing the experimental results, suggesting that the parton phase in these collisions is short-lived.

The CERN Large Hadron Collider has recently begun delivering $Pb + Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV to experiments. These are the highest energy heavy-ion collisions ever to be produced in the laboratory. The LHC ALICE experiment[1] has already posted two experimental papers based on data from these collisions, one in which the charged particle (hadron) multiplicity density is measured[2] and another in which the elliptic flow of charged hadrons is measured[3]. These are both important and basic observables to measure in heavy-ion collisions since the charged particle multiplicity density is related to the initial particle density and the elliptic flow is sensitive to the initial dynamics of the particles, both of which could be messengers of possible exotic phenomena taking place in these collisions[1].

In order to better understand the underlying physics driving these observables at the LHC, a simple kinematic model has been constructed[4, 5] with the goal of comparing predictions of this model with the experimentally measured observables. The basis of the model is that the initial state of the heavy-ion collision is determined by the superposition of proton-proton collisions followed by the mutual scattering of the hadrons produced in the collision. Besides its simplicity, the advantages of this model are 1) the model has been shown to describe the overall trends of hadronic observables in lower energy $Au + Au$ collisions at $\sqrt{s_{NN}} = 0.20$ TeV from the Relativistic Heavy Ion Collider (RHIC)[5], and 2) the model is easily scalable to LHC energies. These will be "limiting case scenario" predictions in the sense that only hadrons are used as the degrees of freedom in this model even at the early stages of the collision where parton (quark and gluon) degrees of freedom are thought to be more appropriate, i.e. a short proper time for hadronization is assumed.

A prediction of the multiplicity density from this model has already been compared with the measured multiplicity density in central $Pb + Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV from ALICE in Reference [2]. The outcome is that the model under-predicts the multiplicity density observable, $dN/d\eta$, at mid-rapidity by $\sim 20\%$ with respect to the lower bound of the measurement. Considering that the model was used to extrapolate from RHIC to LHC energy which represents a factor of 14 increase in $\sqrt{s_{NN}}$, it is somewhat encouraging that the model prediction is even this close to the measurement, and could be considered a validation to some degree of the simple approach taken in the model.

In this paper a further and more stringent test of the model is presented by comparing model predictions for elliptic flow with the recent ALICE measurements of this observable[3]. The model predictions are based on the same calculations which were made in the prediction of the multiplicity density mentioned above in order to get a consistent picture of the ability of this simple model to describe the experimental results. A brief description of the model is presented below followed by the comparison of the model predictions for elliptic flow with the ALICE experiment for $Pb + Pb$ collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

The model calculations are carried out in five main steps: 1) generate hadrons in $p + p$ collisions from the event-generator PYTHIA, 2) superpose $p + p$ collisions in the geometry of the colliding nuclei, 3) employ a simple space-time geometry picture for the hadronization of the PYTHIA-generated hadrons, 4) calculate the effects of final-state rescattering among the hadrons, and 5) calculate the hadronic observables. These steps will now be discussed in more detail.

The $p + p$ collisions were modeled with the PYTHIA code [6], version 6.409. The internal parton distribution functions "CTEQ 5L" (leading order) were used in these calculations. Events were generated in "minimum bias" mode, i.e. setting the low-p_T cutoff for parton-parton collisions to zero (or in terms of the actual PYTHIA parameter, $ckin(3) = 0$) and excluding elastic and diffractive collisions (PYTHIA parameter $msel = 1$). Runs were made at $\sqrt{s} = 2.76$ TeV to simulate LHC collisions. Information saved from a PYTHIA run for use in the next step of the procedure were the momenta and identities of the "direct" (i.e. redundancies removed) hadrons (all charge states) π, K, p, n, Δ, Λ, ρ, ω, η, η', ϕ, and K^*. These particles were chosen since they are the most common hadrons produced and thus should have the greatest effect on the hadronic observables in these calculations.

A main assumption of the model is that an adequate job of describing the heavy-ion collision can be obtained by superposing PYTHIA-generated $p+p$ collisions calculated at the beam $\sqrt{s}$ within the collision geometry of the colliding nuclei. Specifically, for a collision of impact parameter b, if $f(b)$ is the fraction of the overlap volume of the participating parts of the nuclei such that $f(b=0)=1$ and $f(b=2R)=0$, where $R=1.2A^{1/3}$ and A is the mass number of the nuclei, then the number of $p+p$ collisions to be superposed will be $f(b)A$. The positions of the superposed $p+p$ pairs are randomly distributed in the overlap volume and then projected onto the $x-y$ plane which is transverse to the beam axis defined in the z-direction. The coordinates for a particular $p+p$ pair are defined as x_{pp}, y_{pp}, and $z_{pp}=0$. The positions of the hadrons produced in one of these $p+p$ collisions are defined with respect to the position so obtained of the superposed $p+p$ collision (see below).

As was done in similar calculations for lower-energy RHIC collisions to give better agreement with experimental $dn/d\eta$ distributions[5], a lower multiplicity cut was applied to the $p+p$ collisions used in the present calculations which rejected the lowest 20% of the events. The spirit of this cut is to partially compensate for the fact that there is no re-interaction of primary nucleons from the projectile-target system in this model.

The space-time geometry picture for hadronization from a superposed $p+p$ collision located at (x_{pp}, y_{pp}) consists of the emission of a PYTHIA particle from a thin uniform disk of radius 1 fm in the $x-y$ plane followed by its hadronization which occurs in the proper time of the particle, τ. The space-time coordinates at hadronization in the lab frame (x_h, y_h, z_h, t_h) for a particle with momentum coordinates (p_x, p_y, p_z), energy E, rest mass m_0, and transverse disk coordinates (x_0, y_0), which are chosen randomly on the disk, can then be written as

$$x_h = x_{pp} + x_0 + \tau\frac{p_x}{m_0} \tag{1}$$

$$y_h = y_{pp} + y_0 + \tau\frac{p_y}{m_0} \tag{2}$$

$$z_h = \tau\frac{p_z}{m_0} \tag{3}$$

$$t_h = \tau\frac{E}{m_0} \tag{4}$$

The simplicity of this geometric picture is now clear: it is just an expression of causality with the assumption that all particles hadronize with the same proper time, τ. A similar hadronization picture (with an initial point source) has been applied to $e^+ - e^-$ collisions[7]. For all results presented in this work, τ will be set to 0.1 fm/c as was done in applying the present model to calculating predictions for RHIC $Au+Au$ collisions[5] and Tevatron $p+\bar{p}$ collisions[8].

The hadronic rescattering calculational method used is similar to that employed in previous studies [9, 10]. Rescattering is simulated with a semi-classical Monte Carlo calculation which assumes strong binary collisions between hadrons. Relativistic kinematics is used throughout. The hadrons considered in the calculation are the most common ones: pions, kaons, nucleons and lambdas (π, K, N, and Λ), and the ρ, ω, η, η', ϕ, Δ, and K^* resonances. For simplicity, the calculation is isospin averaged (e.g. no distinction is made among a π^+, π^0, and π^-).

The rescattering calculation finishes with the freeze out and decay of all particles. Starting from the initial stage ($t=0$ fm/c), the positions of all particles in each event are allowed to evolve in time in small time steps ($\Delta t = 0.5$ fm/c) according to their initial momenta. At each time step each particle is checked to see a) if it has hadronized ($t > t_h$, where t_h is given in Eq. (4)), b) if it decays, and c) if it is sufficiently close to another particle to scatter with it. Isospin-averaged s-wave and p-wave cross sections for meson scattering are obtained from Prakash et al.[11] and other cross sections are estimated from fits to hadron scattering data in the Review of Particle Physics[12]. Both elastic and inelastic collisions are included. The calculation is carried out to 400 fm/c which allows enough time for the rescattering to finish (as a test, calculations were also carried out for longer times with no changes in the results). Note that when this cutoff time is reached, all un-decayed resonances are allowed to decay with their natural lifetimes and their projected decay positions and times are recorded.

The rescattering calculation is described in more detail elsewhere [9, 10]. The validity of the numerical methods used in the rescattering code have been studied using the subdivision method, the results of which have verified that the methods used are valid [13].

Model runs are made to be “minimum bias” by having the impact parameters of collisions follow the distribution $d\sigma/db \propto b$, where $0 < b < 2R$. Observables are then calculated from the model in the appropriate centrality bin by making multiplicity cuts as normally done in experiments, as well as kinematic cuts on rapidity and p_T. For the present study, a 3200 event minimum bias run was made from the model for $\sqrt{s_{NN}} = 2.76$ TeV $Pb+Pb$ collisions which was then used to calculate hadronic observables. As mentioned earlier, these are the same events which were used in the prediction of the multiplicity density for Reference [2].

The elliptic flow variable, V_2, is defined as

$$V_2 = <\cos(2\phi)> \tag{5}$$
$$\phi = \arctan(\frac{p_y}{p_x})$$

where “<>” implies a sum over particles in an event and a sum over events and where p_x and p_y are the x and y components of the particle momentum, and x is

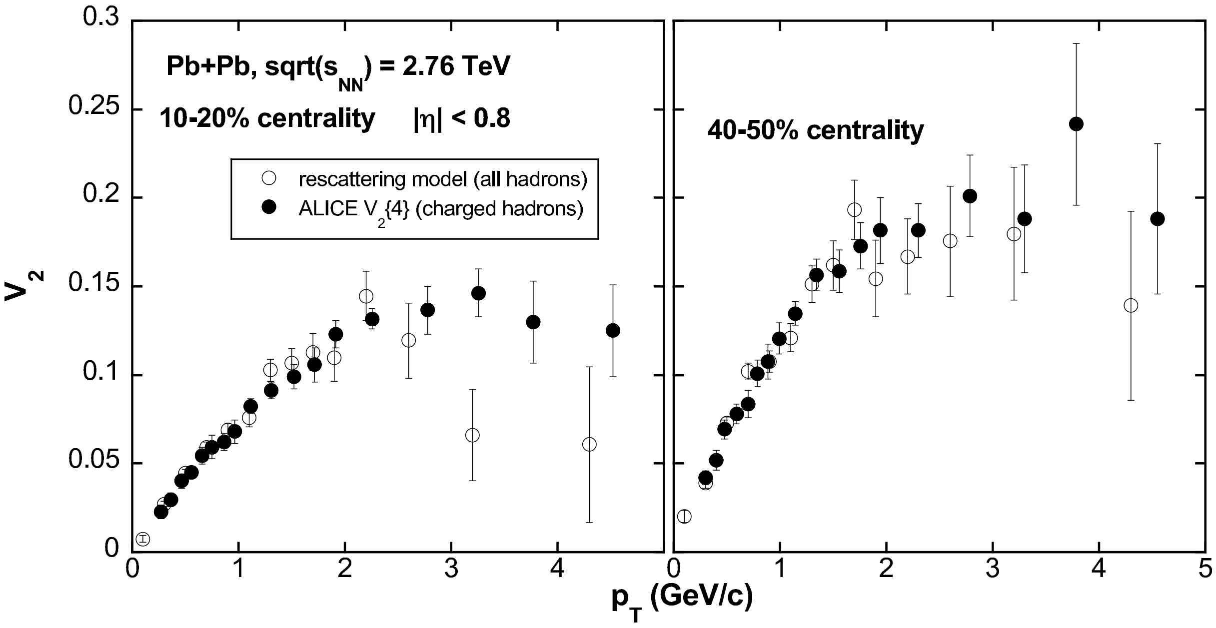

FIG. 1. Comparison of model V_2 vs. p_T plots for $\sqrt{s_{NN}} = 2.76$ TeV $Pb+Pb$ collisions with the ALICE experiment for $10-20\%$ and $40-50\%$ centrality classes.

in the impact parameter direction, i.e. reaction plane direction, and y is in the direction perpendicular to the reaction plane. The V_2 variable is calculated from the model using Eq. (5) and taking the reaction plane to be the model $x-z$ plane. As seen, if $< p_x > \sim < p_y >$, then $V_2 \sim 0$, and for $< p_x > \gg < p_y >$, then $V_2 \sim 1$.

Figure 1 shows the comparison of model V_2 vs. p_T plots for $\sqrt{s_{NN}} = 2.76$ TeV $Pb+Pb$ collisions with measurements from the ALICE experiment[3] for $10-20\%$ and $40-50\%$ centrality classes. Note that all model predictions in this paper are compared with the experimental $V_2\{4\}$ value, which is V_2 extracted using the 4-particle cumulant method[3], since this observable minimizes non-flow correlations in the elliptic flow which are not present in the model, thus making it more comparable. The model predictions are seen to follow the experimental points fairly closely within the uncertainties shown for both centrality classes. The uncertainties on the model prediction are statistical. The model accurately describes the increasing V_2 with increasing p_T for $p_T < 2$ GeV/c and then the "flattening" of the dependence of V_2 on further increase of p_T for $p_T > 2$ GeV/c, although somewhat under-predicting the measurement for $p_T > 2$ GeV/c for the $10-20\%$ centrality class. From the model picture all of these dependences, as well as the non-vanishing values of V_2, are a result of the final-state hadronic rescattering. If rescattering is turned off in the model, or equivalently $\tau \gg 0.1$ fm/c, $V_2 \rightarrow 0$ for all cases[4, 5].

Figure 2 compares V_2 integrated over $0.2 < p_T < 5.0$ GeV/c vs. centrality class with the ALICE experiment[3]. The model is seen to qualitatively describe the trend of the measurements of increasing V_2 with increasing centrality class and then reaching a maximum, but to under-predict the measurements on average by $\sim 14\%$. This under-prediction could be a reflection of the under-prediction of the model for the particle multiplicity, mentioned earlier (which could be corrected in the model by including some degree of re-interaction of the primary nucleons from the projectile-target system).

In conclusion, a kinematic model based on the superposition of PYTHIA-generated $p+p$ collisions, relativistic geometry and final-state hadronic rescattering has been used in the present work to predict the elliptic flow observable in $\sqrt{s_{NN}} = 2.76$ TeV $Pb+Pb$ collisions. A short proper time for hadronization of $\tau = 0.1$ fm/c has been assumed as in previous studies with this model which have shown qualitative agreement with experiments. It has been shown that the simple hadronic rescattering model accurately describes the features of the ALICE elliptic flow measurements, suggesting that final-state hadronic rescattering plays an important role in determining the properties of the elliptic flow observable in these collisions. These results also suggest that the parton phase in these collisions is short-lived.

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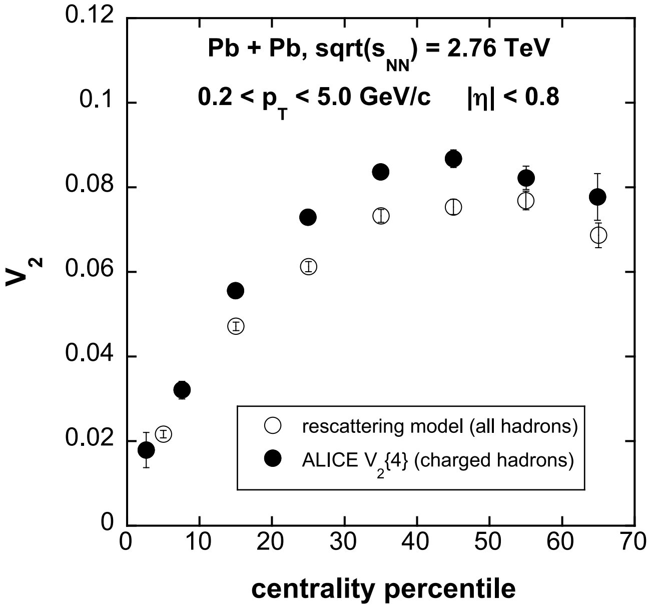

FIG. 2. Comparison of model integrated V_2 vs. centrality class for $\sqrt{s_{NN}} = 2.76$ TeV $Pb+Pb$ collisions with the ALICE experiment.

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