An One Dimensional Adiabatic Model for Fusion Involving Loosely Bound and Halo Nuclei with Heavy Targets

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An one dimensional adiabatic model has been proposed for fusion of loosely bound and halo light nuclei with various heavy targets. It is shown that fusion cross sections at near and sub-barrier energies can be explained using a simple WKB tunneling through an adiabatic barrier without invoking breakup coupling explicitly. The model has been applied successfully to explain fusion cross sections for several systems including recently measured $^6Li+^{198}Pt$ system (Phy. Rev. Lett. 103, 232702, 2009) where data exists well above and below the barrier and $^8He+^{197}Au$ system (Phy. Rev. Lett. 103, 232701, 2009) where ^8He is highly neutron rich. Interestingly, the fusion of stable $^4He+^{197}Au$ system can not be explained on the basis of this adiabatic model as it requires normal tunneling through the sudden barrier. The requirement of adiabatic potential for the loosely bound and halo nuclei is linked to the fact that for such systems neutron flow leading to neck formation is initiated at a larger distance which modifies the sudden potential.

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Fusion cross section enhancement at sub-barrier energy over the prediction of a simple barrier penetration model (BPM) is a wellknown phenomena for stable nuclei which occurs due to the coupling of relative motion with the intrinsic degrees of freedoms of the projectile and target nuclei [1, 2]. Similar enhancement has also been found for fusion of loosely bound and halo light nuclei with various heavy targets [3]. However, unlike stable nuclei, the fusion cross sections at above Coulomb barrier energy shows about 10% to 30% suppression with respect to the BPM predictions [4]. This could be due to the presence of breakup components as the loosely bound projectiles are more prone to breakup due to their low binding energy as compared to the stable counterparts. Two theoretical models having different perspectives have been proposed to understand the role of breakup reaction on fusion process [5–7]. Intuitively, it can be told that increase of breakup process may hinder fusion [5, 6]. On the otherhand, considering breakup process like any other reaction channel, the coupled channel approach would lead to fusion enhancement at below barrier energy and a suppression above it [3, 7]. During last few years, high precision fusion cross section measurements have been carried out for several systems including both loosely bound and halo projectiles like ^6Li, ^7Li, ^9Be and ^6He with medium and heavy targets like ^{144}Sm, ^{198}Pt, ^{208}Pb, ^{209}Bi and ^{238}U [8–19]. Fusion enhancement over the BPM predictions below the Coulomb barrier and suppression above it appears to be a generic phenomena [4] with a few exceptions [11, 12] where fusion cross sections might have contained a large component coming from direct processes (incomplete fusion components). While improved CDCC type coupled channel calculations with inclusion of breakup coupling have been used to explain the fusion enhancement at below barrier energy, they all fail to explain the above barrier data unless 10% to 30% suppression factor (almost energy independent) is used [3]. In this letter, we propose a simple one dimensional model where tunneling probability is estimated through an adiabatic barrier which has a shape thiner than the sudden potential particularly in the nuclear interior region. It is shown that this adiabatic BPM can explain sub-barrier fusion enhancement for several systems involving loosely bound light nuclei without invoking breakup coupling explicitly. Recently, a new type of fusion hindrance has been observed at deep sub-barrier energies for fusion involving stable nuclei [20]. At sub-barrier energy, fusion cross section is enhanced (over BPM calculations) as expected which can be explained by coupled channel calculations. However, when measurements are extended to deep sub-barrier energies, fusion cross sections are suppressed with respect to the same coupled channel predictions which explains the data at sub and above barrier energies. Interestingly, the recent fusion cross section measurements of $^6Li+^{198}Pt$ systems where data exists well above and below the barrier energies shows no such deep sub-barrier hindrance [18]. Similarly, there is another recent measurement of fusion cross section of $^8He+^{197}Au$ system which shows unusual behavior of the tunneling of neutron rich ^8He nuclei as compared to normal α particle [19]. We have shown here that the present adiabatic BPM can also explain fusion cross sections of the above two systems without invoking any channel coupling mechanism explicitly.

We construct an adiabatic potential from the commonly used sudden potential by adding an extra correction term which arises due to the neck formation and is given by,

$$V(r,n) = V_N(r) + \frac{Z_1 Z_2 e^2}{r} + V_{neck}(r,n) \qquad (1)$$

where V_N is the sudden ion-ion nuclear potential, r is

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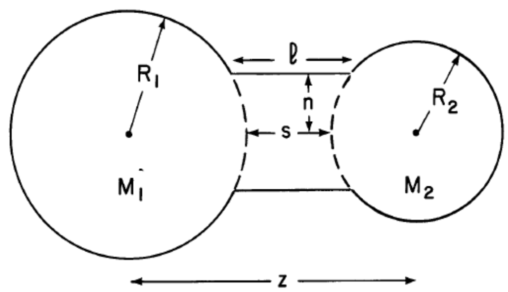

FIG. 1: A macroscopic representation of two nuclei of mass M_1 and M_2 and connected by a cylindrical neck of length l, radius n and surface to surface distance s. Note that the centre to centre distance z and the radial distance r which is used in the text have the similar meaning.

the centre to centre distance between the two nuclei and n is the neck parameter (defined below) which is meaningfull only at the nuclear overlap region being characterized by an adiabatic distance $r = R_a$. The strong nucleon exchange (nucleon flow) between two nuclei (involving loosely bound and halo nuclei) at $r > R_1 + R_2$ is well manifested, for example, by the sub-barrier fusion enhancement of $^6He+^{208}Pb$ system with sequential neutron transfer from ^6He to the Pb nucleus with positive Q value [16, 21]. An important aspect of the nucleon exchange is that it provides the formation of an intermediate di-nuclear state in the fusion reaction before the two nuclei actually fuse. We model this aspect using a macroscopic approach where two nuclei are connected by a cylindrical neck as shown in Fig. 1. The extra energy due to neck formation is proportional to the surface area of the cylinder of length l and radius n and can be written as [22],

$$V_{neck}(r,n) \approx 2\pi\gamma(ns - n^2 + \frac{n^3}{2\bar{R}}) \tag{2}$$

where $s = r-(R_a-\bar{R}/2)$, $\bar{R} = (R_1R_2)/(R_1+R_2)$ and γ is the surface tension co-efficient ~ 1.0 MeVfm^{-1}. The distance R_a is called the adiabatic parameter that decides at which point neck opening becomes favorable [23]. Using the dimensionless variables $\rho = s/(2\bar{R})$ and $\nu = n/(2\bar{R})$, Eq. 2 can be written as,

$$V_{neck}(\rho,\nu) = 8\pi\gamma\bar{R}^2(\rho\nu - \nu^2 + \nu^3) \tag{3}$$

The above expression is a simple cubic order polynomial that vanishes at $\nu = 0$ and has a minimum at $\bar{\nu} = (1+\sqrt{1-3\rho})/3$ as long as $\rho \leq 1/3$. For neck relaxation, it is required for the di-nuclear system to move from $\nu = 0$ to a state characterized by $\nu = \bar{\nu}$. It can be seen that for $1/4 \leq \rho \leq 1/3$, the minimum at $\nu = \bar{\nu}$ is always higher than the value at $\nu = 0$. Since this

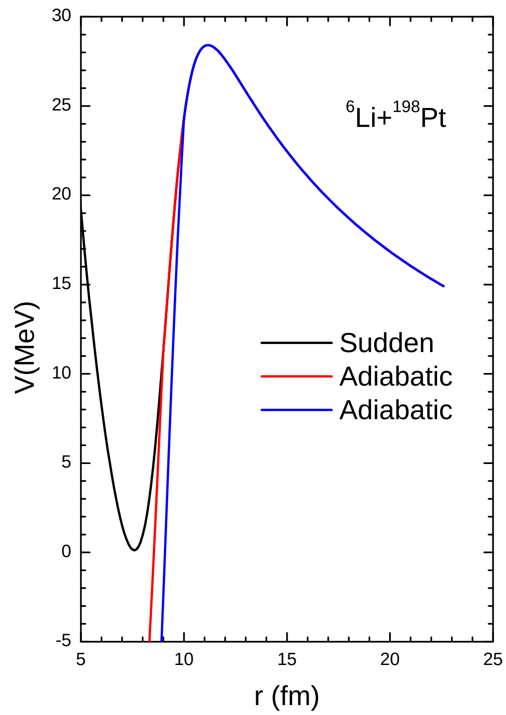

FIG. 2: The potential V as a function of inter nuclear distance r for a typical $^6Li+^{198}Pt$ system plotted using the parameters as listed in table I. The black curve represents the sudden potential where as the red and blue curves are adiabatic potentials for adiabatic parameter R_a =9.0 fm and 10.0 fm respectively. For detail see the text.

minima corresponds to a metastable sate, neck relaxation is not favored. For $\rho = 1/4$, $V_{neck}(\nu = 0)$ and $V_{neck}(\nu = \bar{\nu})$ are degenerate (equal to zero) and $V_{neck}(\bar{\nu})$ becomes less than $V_{neck}(\nu = 0)$ (becomes negative) for $\rho \leq 1/4$. Hence, neck relaxation is possible for $\rho \leq 1/4$ corresponding to $r \leq R_a$. Therefore, we set $V_{neck}(\bar{\nu}) = 0$ for $r > R_a$ and evaluate it at $\nu = \bar{\nu}$ using Eq. 3 for $r \leq R_a$. Finally, we estimate the adiabatic potential from Eq. 1 by adding the above neck potential at $\nu = \bar{\nu}$. As expected, the potential given in Eq. 1 is of sudden nature for $r > R_a$ and becomes adiabatic for $r \leq R_a$. Although, we treat R_a as parameter, we expect it to lie in between $R_1 + R_2$ and R_b where R_b is the radius of the Coulomb barrier. For the nuclear part V_N, we use the Akuyz-Winther(AW) parameterization given by [24]

$$V_N(r) = \frac{-16\gamma\,\bar{R}\,a}{1 + exp\left\{(r - R_1 - R_2 - \Delta R)/a\right\}}, \tag{4}$$

where ΔR is an adjustable parameter used to reproduce the Coulomb barrier. Here, $\gamma = 0.95$ MeV/fm^2 is the nuclear surface tension co-efficient , $R_i = 1.2A_i^{1/3} - 0.09$ fm, the diffuseness parameter $a = 0.63$ fm, and $\bar{R} = R_1R_2/(R_1 + R_2)$.

Fig. 1 shows the plot of total nuclear potential as a

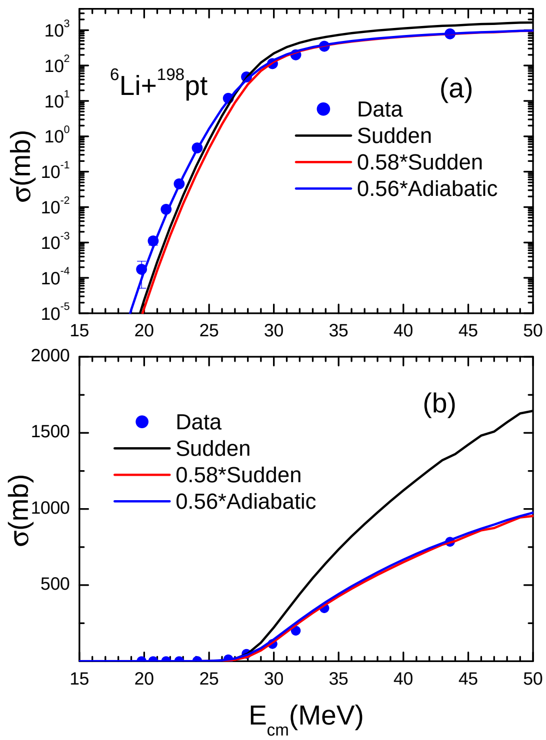

FIG. 3: (a) Fusion cross section for $^6Li +^{198} Pt$ system as a function of energy. The black curve is obtained using only the sudden potential while the red curve is obtained with a suppression factor 0f of 0.58. The blue curve is obtained using adiabatic potential with a suppression factor of 0.56. (b) The plots are shown in the linear scale. The data points are taken from [18].

function of r for $^6Li +^{198} Pt$ system as an example. The black curve is the sudden potential without any neck correction. The red and blue curves are the adiabatic potentials for two different values of R_a. As discussed before, the adiabaticity begins for $r \leq R_a$ and the adiabatic potential becomes thiner as compared to its sudden counter part. Using the above potential, we now estimate the fusion cross section using

$$\sigma_f = \sum_l \sigma_l = \frac{\pi}{k^2} \sum_l (2l+1) T_l(E), \tag{5}$$

where k is the relative wave number and $T_l(E)$ is the tunneling probability which can be estimated using the WKB approximation,

$$T_l(E) = \frac{1}{1 + exp(2S_l)}, \tag{6}$$

where S_l is the classical action given by,

$$S_l = \frac{2\mu}{\hbar^2} \int_{r_1}^{r_2} \sqrt{(V_l(r) - E)} dr, \tag{7}$$

and

$$V_l(r) = V_C(r) + V_N(r) + \frac{l(l+1)\hbar^2}{2\mu r^2}. \tag{8}$$

Under the parabolic approximation, Eq.(6) can also be estimated using Hill-Wheeler expression [25],

$$T_l(E) = \left[1 + exp\left(\frac{2\pi}{\hbar\omega}(V_b^l - E)\right)\right]^{-1}. \tag{9}$$

where

$$V_b^l = V_b + \frac{l(l+1)\hbar^2}{2\mu R_b^2}. \tag{10}$$

and V_b being the s-wave barrier. Finally, we estimate tunneling probability using WKB approximation (Eq. 6) for sub-barrier energy and Eq. 9 for energy above the Coulomb barrier. Although R_a is a variable, it is noticed that best result is obtained when R_a is close to R_b (with a few exception as listed in table I). The second parameter ΔR of our model is fixed to reproduce the Coulomb barrier for various systems which are taken from the literatures. In many cases, the Coulomb barrier have been determined in a model independent way by estimating the centroid of the experimental barrier distributions [9, 14, 15].

We estimate fusion cross section using barrier penetration model both for sudden and adiabatic potential. The black curve in Fig. 3 shows the BPM calculations using sudden potential for $^6Li +^{198} Pt$ system. As expected, the experimental data are suppressed at above barrier energies and enhanced at sub-barrier energies as compare to the predictions of the BPM calculations. The linear scale in Fig.3b shows the above barrier suppression more prominently. The red curve is obtained when the BPM result is scaled down by a factor of 0.58 to explain the above barrier suppression where as the results are not affected much at sub-barrier energies. Next, we carry out BPM calculations using adiabatic potential with $R_a \sim 11.0$ fm which is quite close to $R_b \sim 11.2$ fm . The resulting blue curve with a similar suppression factor explains the experimental data quite well both at above and below the barrier energies. It may be mentioned here that the coupled channel calculations (with inclusion of breakup channel) which has been used in [18] to explain $^6Li +^{198} Pt$ data also requires a similar suppression factor to explain the above barrier data. Whether it is BPM or coupled channel calculations, the above barrier suppression seems to be a generic feature and the fusion cross section needs to be scaled down by almost a constant factor whose magnitude may depend on the specific model used. Fig.4 shows the plot of logarithmic derivative $L(E) = d[log(\sigma E)]/dE$ and average angular momentum $< l >$ as a function of energy for the same $^6Li +^{198} Pt$ system. The BPM results with adiabatic potential explains the data quite well (blue curves). It is interesting to note that the BPM calculations with sudden

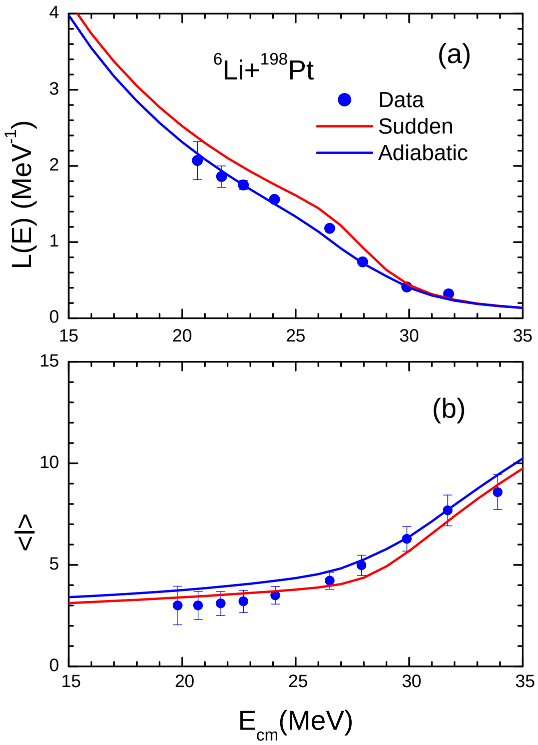

FIG. 4: The $(L(E)$ and $< l >$ as a function of energy E_{cm} for $^6Li+^{198}Pt$ system both for sudden (red curve) and adiabatic (blue curve) potentials. Data points are taken from [18].

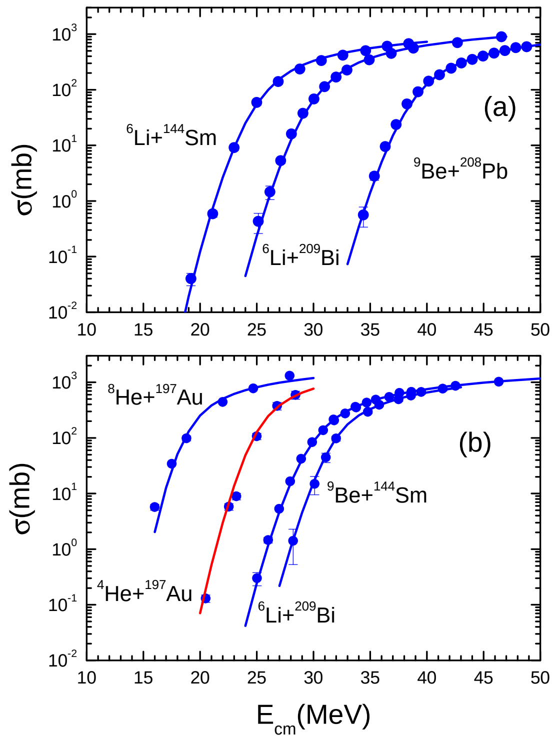

FIG. 5: Fusion cross section versus energy for different projectile and target combinations as shown in figure captions using adiabatic model except for $^4He+^{197}Au$ system which is obtained using only sudden potential (red curve). The X-axis for $^4He+^{197}Au$ system has been shifted by +5 MeV for clarity. Experimental data points are taken from [15, 17, 19, 26].

potential also explains $< l >$ measurements although it fails to explain sub-barrier fusion enhancement and $L(E)$ behavior (see red curves).

We have applied this adiabatic model to few other systems. Fig.5a shows the fusion cross sections for $^6Li+^{144}Sm$, $^6Li+^{209}Bi$, $^9Be+^{208}Pb$ systems where as Fig.5b shows the results for $^8He+^{197}Au$, $^7Li+^{209}Bi$ and $^9Be+^{208}Pb$ systems obtained using barrier penetration model with adiabatic potential except for $^4He+^{197}Au$ system which does not need any adiabatic correction. The experimental data for the last system can be explained using a BPM model with sudden potential and also without any suppression factor. The potential parameters used in the calculations are listed in table I. As mentioned before, we first adjust the ΔR parameter to reproduce the Coulomb barrier V_b (second column) which are taken from the literatures. The resulting R_b and $\hbar\omega$ values are listed in column 4 and 5. The bracketed values in column 4 shows the R_a parameter which has been used to estimate the adiabatic potential. The last column shows the suppression factor f which is required to explain the above barrier data. Again the values in the bracket shows the suppression factors which are required to explain the above barrier data using BPM with only sudden potential. Since the adiabatic potential is thiner than the sudden potential, the adiabatic model slightly overpredicts the fusion cross sections even at above barrier energies as compared to predictions of the BPM with sudden potential. Therefore, the f factors systematically turns out to be slightly lower than BPM predictions with sudden potential. Note that, fusion with ^4He nuclei does not require any suppression factor ($f = 1$) nor requires any adiabatic correction.

In conclusion, it is shown that a simple barrier penetration formalism with adiabatic potential (a potential which is thiner than the sudden potential in the nuclear interior region) can explain sub-barrier fusion enhancement of loosely bound and halo nuclei with heavy targets without invoking any breakup coupling explictly. The excellent agreement between the BPM calculations and the experimental measurements suggests that the breakup effect which is more important at above barrier energy, can be simulated by a suppression factor which is practically energy independent. Apart from breakup process, presence of other rotational and vibrational states may affect the fusion process which has not been considered in the present BPM formalism. It will be ideal to use normal couple channel formalism (without any breakup effect) with a bare potential which is adiabatic in nature. Although such a model will explain sub-barrier fusion

TABLE I: The potential parameters for various systems used in the calculations. The only parameter ΔR has been adjusted to reproduce the Coulomb barrier which are taken from the literatures.

System	ΔR	V_b	$R_b(R_a)$	$\hbar\omega$	f
$^6Li +^{198} Pt$	0.11	28.4	11.2 (11.0)	5.0	0.56(0.58)
$^6Li +^{144} Sm$	-.41	25.2	9.9(9.9)	5.0	0.60(0.64)
$^6Li +^{209} Bi$	0.07	30.1	11.2(11.2)	5.1	0.60(.64)
$^7Li +^{209} Bi$	0.06	29.7	11.4(11.3)	4.7	0.68(0.72)
$^9Be +^{208} Pb$	0.18	38.5	11.6(11.5)	4.7	0.64(0.67)
$^9Be +^{144} Sm$	0.12	31.2	10.7(10.4)	4.4	0.80(0.85)
$^8He +^{197} Au$	-.10	18.7	11.5(11.5)	3.5	0.71(0.80)
$^4He +^{197} Au$	0.10	19.8	10.8	5.2	1.0

enhancement, the above barrier measurements will still require a suppression factor f which may turn out to be slightly higher than what is shown in table I as coupled channel calculation has some amount of inherent suppression built in. It may be mentioned here that the large value of R_a indicates that neck formation becomes effective around the Coulomb barrier ($R_a \sim R_b$) due to neutron flow which may be a meaningfull proposition for halo and loosely bound nuclei as contrast to the stable system like $^4He +^{197} Au$ which does not require any adiabatic correction. Although the model used here is based on a simple BPM picture which works well for light nuclei, the message which we want to convey is that the basic ion-ion potential may become adiabatic in nature for halo and loosely bound nuclei. This aspect should not be neglected while carrying out a proper couple channel calculation.

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