Heavy Ion Initial Conditions and Correlations Between Higher Moments in the Spatial Anisotropy

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Fluctuations in the initial conditions for relativistic heavy ion collisions are proving to be crucial to understanding final state flow and jet quenching observables. The initial geometry has been parametrized in terms of moments in the spatial anisotropy (i.e. $\epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5 ...$), and it has been stated in multiple published articles that the vector directions of odd moments are uncorrelated with the even moments and the reaction place angle. In this article, we demonstrate that this is incorrect and that a substantial correlation exists between the even and odd moments in peripheral $Au+Au$ collisions. These correlations persist for all centralities, though at a very small level for the 0-55% most central collisions.

One proposal for modeling the initial geometry fluctuations in relativistic heavy ion collisions is utilizing a Monte Carlo Glauber calculation [1]. Using this model and the initial transverse positions of the struck nucleons, referred to as participants, one can calculate the participant eccentricity ϵ_2 and higher moments [2]. In [2], these moments are defined by:

$$\epsilon_n = \frac{\sqrt{\langle r^2 cos(n\phi_{part})\rangle^2 + \langle r^2 sin(n\phi_{part})\rangle^2}}{\langle r^2 \rangle} \quad (1)$$

where n is the nth moment of the spatial anisotropy calculated relative to the mean position. The axis associated with the nth moment is defined by:

$$\psi_n = \frac{atan2(\langle r^2 sin(n\phi_{part})\rangle , \langle r^2 cos(3\phi_{part})\rangle) + \pi}{n} \quad (2)$$

We show an example event display in Figure 1 that includes the positions of the participant nucleons, and a visualization of the ϵ_2, ϵ_3, ϵ_4, and ϵ_5 moments. We have drawn the vector direction of each along the long-axis of the associated moment. The nth moment has an n-muliplet of directions that are equally valid, separated by $2\pi/n$.

In [2], the authors state that the minor axis of triangularity (i.e. ϵ_3) is found to be uncorrelated with the minor axis of eccentricity (i.e. ϵ_2) in Monte Carlo Glauber calculations. This conclusion is repeated in [3] where it is stated that the orientation of triangular overlap shape due to fluctuations is random relative to the event-plane direction as determined by the elliptic anisotropy. This conclusion is then generalized in [4] where these authors state that the angles for the odd and even harmonics are uncorrelated.

There have been many initial studies for how this spatial anisotropy translates into the final momentum space distribution of particles (e.g. [2, 5–9]). The fact that the angular orientations are uncorrelated between odd and even moments has an important impact on the methodology for experiments to determine these momentum anisotropy moments v_n and for two-particle correlation measurements, with relevance for jet quenching observables.

We set out to confirm the findings of the above papers using the Monte Carlo Glauber framework. We have utilized the standard PHOBOS Monte Carlo Glauber code [10] with Woods-Saxon parameters and settings (R_0 = 6.38 fm, a = 0.535 fm, d_{min} = 0 fm). We show the angular distribution between ϵ_2 and ϵ_3 for a set of $Au+Au$ number of participant (N_{part}) selections in Figure 2. Because of the two (three) fold symmetry for the

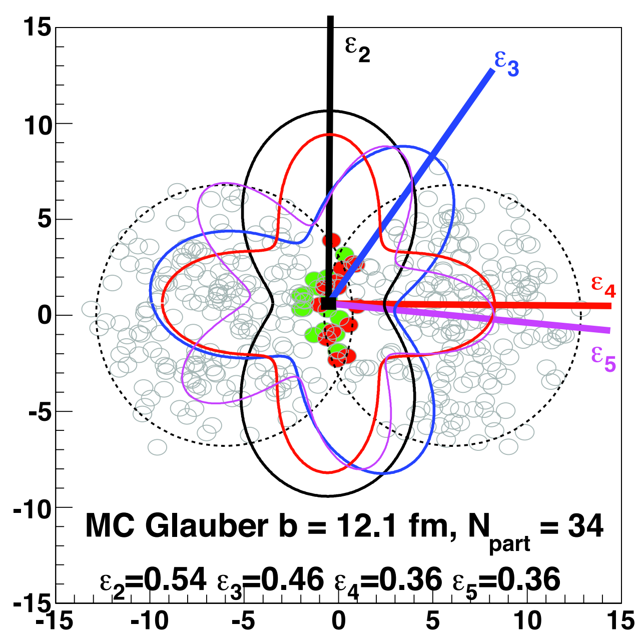

FIG. 1. Monte Carlo Glauber event display for a sample $Au+Au$ collision. The grey circles represent the positions of all nucleons. The green (red) circles are participant nucleons from the left (right) nucleus. The vector directions for the $n = 2-5$ and the spatial anisotropy pattern they represent are overlaid. These are centered on the mean position as indicated by a black square.

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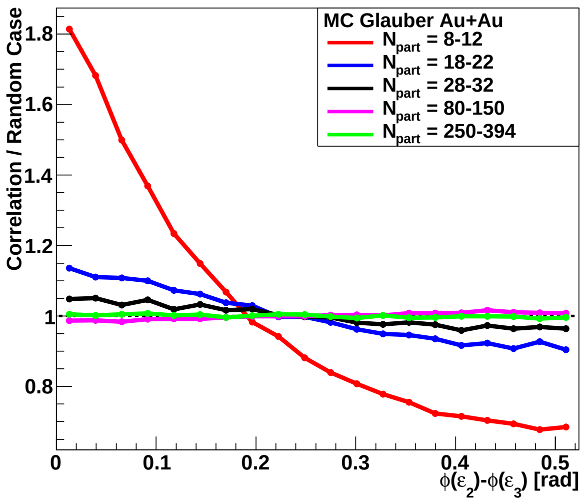

FIG. 2. Monte Carlo Glauber $Au+Au$ distribution for relative angular difference $(\epsilon_2 - \epsilon_3)$ [radians] for different selections in N_{part}. A flat distribution at one from angular separations 0.0-$\pi/6$ indicates totally uncorrelated quantities.

ϵ_2 (ϵ_3) moments, if the two angles are uncorrelated, the distribution should be flat at one from 0.0 - $\pi/6$ radians. A clear correlation is found between the two moments in peripheral $Au+Au$ events. The correlation strength then decreases for more central events [11].

We quantify the degree of correlation by taking the root-mean-square (RMS) of the distribution. If there were no correlation between these moments, the RMS of the angular separation would be 0.302 radians. The results as a function of N_{part} are shown in Figure 3. For our initial view, we have zoomed in for $N_{part} < 100$. The results confirm the strong correlation between the two angles (i.e. the large downward deviation in the RMS from the flat case) for $N_{part} \approx 10$, the deviation then weakens for larger N_{part} values. In this figure, the correlation appears to disappear for $N_{part} > 50$. However, in the lower panel of Figure 3, we show the full N_{part} range zooming the vertical axis around the default value. We observe a very small remaining anti-correlation for $N_{part} \approx 80 - 150$, which translates into an approximate 1% lower probability of having the two moments within 0.0-$\pi/12$ and a correspondingly higher probability of having the two moments separated by $\pi/12$-$\pi/6$. There is a similar magnitude positive correlation for $N_{part} > 200$.

For the lower N_{part} region where the alignment is strongest, the correlation may be due to small number fluctuations of the particular geometry in that impact parameter range for $Au+Au$. Thus, also shown in Figure 3 are calculations from $Si+Si$ and $Cu+Cu$ collisions. One can see that the angular correlation largely tracks with N_{part} and thus it is the number fluctuations and not the average geometry that dominate the correlation. However, the Si, Cu, Au results do not scale perfectly versus

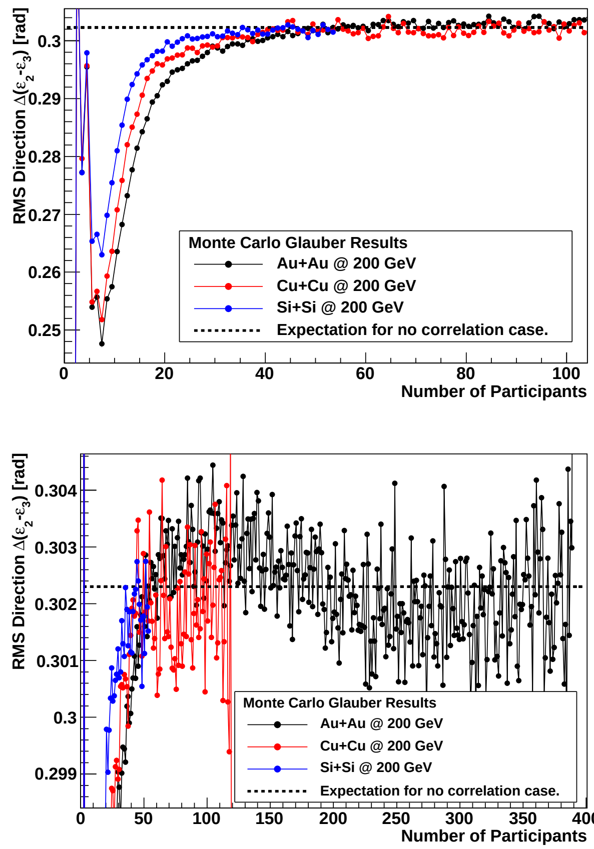

FIG. 3. The root-mean-square (RMS) of the angular difference between ϵ_2 and ϵ_3 as a function of number of participant nucleons, N_{part}, for $Au+Au$, $Cu+Cu$, and $Si+Si$ collisions, as well as the expectation of no correlation. The upper panel highlights the significant correlations that are present at small N_{part}. The lower panel covers all centralities, but is zoomed to highlight the small deviations from no correlation that remain at large N_{part}.

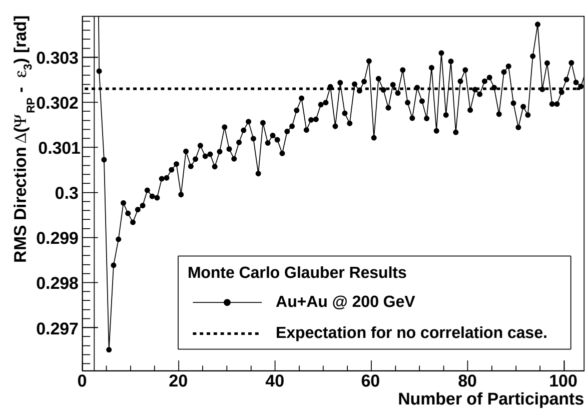

FIG. 4. The RMS angular difference between the reaction plane, Ψ_{RP}, and ϵ_3 as a function of N_{part} for $Au+Au$ collisions. A weak correlation is found between the 3rd-order moment and the reaction plane direction for small N_{part}. Note the zoomed vertical scale.

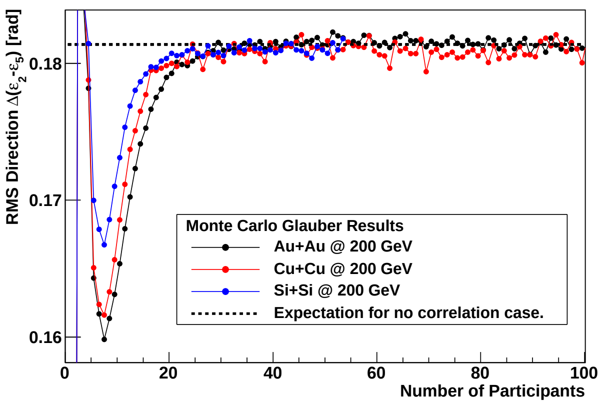

FIG. 5. The root-mean-square (RMS) of the angular difference between ϵ_2 and ϵ_5 as a function of number of participant nucleons, N_{part}, for Au+Au, Cu+Cu, and Si+Si collisions, as well as the expectation of no correlation. The result demonstrates that a similar behavior holds for correlation between ϵ_2 and higher order odd moments.

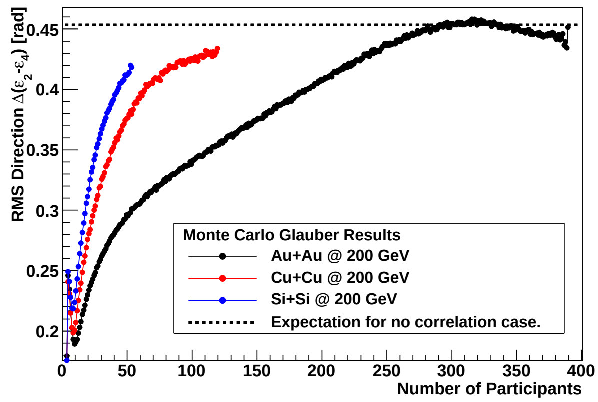

FIG. 6. The root-mean-square (RMS) of the angular difference between ϵ_2 and ϵ_4 as a function of number of participant nucleons, N_{part}, for Au+Au, Cu+Cu, and Si+Si collisions, as well as the expectation of no correlation. The result shows the same procedure applied to the difference between the lowest even-order moments where a more significant correlation results from the average geometry.

N_{part}, and thus the particular geometric configuration plays some role, as one might expect since the geometry correlates with the magnitude of these moments.

Furthermore, the authors of [12] state that the fluctuations are random with respect to the reaction plane (the plane defined the line between the centers of the nuclei and the longitudinal axis). We have tested this as well and show the resulting RMS angular separation between the ϵ_3 and the reaction plane in Figure 4. We find a similar trend as shown above. The correlation between ϵ_3 and the reaction plane is also restricted to small numbers of participating nucleons. The correlation here is much weaker than what was found between ϵ_2 and ϵ_3.

We also show the angular correlation between ϵ_2 and ϵ_5 in Figure 5. One again sees a strong correlation in peripheral collisions and then smaller correlations for larger numbers of participants. Finally, in Figure 6, we show the angular correlation between ϵ_2 and ϵ_4. These two even moments are expected to be highly correlated because the initial overlap in mid-central collisions has an approximately elliptical shape, which can be well described by a combination of aligned ϵ_2 and ϵ_4 moments if it has a large enough eccentricity. This correlation tracks with the geometry (i.e. impact parameter), and so the resulting RMS values for Au+Au, Cu+Cu, and Si+Si do not track each other when plotted as a function of N_{part}. The relative degree of correlation is of interest since if these spatial moments were to translate perfectly into momentum space, this indicates the difference one might measure between v_4 measured with respect to the ϵ_4 participant plane, as opposed to previous measurements of v_4 with respect to the ϵ_2 plane.

In summary, we find that contrary to previous publications, there is a significant correlation within the Monte Carlo Glauber calculation between the angular directions of ϵ_2 and ϵ_3 (and more generally between even and odd moments). A much weaker correlation between the reaction plane and ϵ_3 is also found. The effects are strongest for peripheral Au+Au events, and the importance in accounting for this correlation in mid-central and central events will depend on the details of the analysis in question. The multi-dimensional correlation between the magnitude of the eccentricity orders and their angular orientations may also prove important and the direct integration of Monte Carlo calculations of initial state geometries may be warranted in many studies to account for the full set of correlations.

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[1] M. L. Miller, K. Reygers, S. J. Sanders, and P. Steinberg, Ann. Rev. Nucl. Part. Sci. **57**, 205 (2007).
[2] B. Alver and G. Roland, Phys.Rev. **C81**, 054905 (2010), arXiv:1003.0194.
[3] H. Agakishiev, M. Aggarwal, Z. Ahammed, A. Alakhverdyants, I. Alekseev, et al. (2010),

arXiv:1010.0690.
[4] R. A. Lacey, R. Wei, N. Ajitanand, and A. Taranenko (2010), arXiv:1009.5230.
[5] G.-Y. Qin, H. Petersen, S. A. Bass, and B. Muller (2010), arXiv:1009.1847.
[6] B. H. Alver, C. Gombeaud, M. Luzum, and J.-Y. Ollitrault, Phys.Rev. **C82**, 034913 (2010), arXiv:1007.5469.
[7] H. Holopainen, H. Niemi, and K. J. Eskola (2010), arXiv:1007.0368.
[8] D. Teaney and L. Yan (2010), arXiv:1010.1876.
[9] B. Schenke, S. Jeon, and C. Gale (2010), arXiv:1009.3244.
[10] B. Alver, M. Baker, C. Loizides, and P. Steinberg (2008), arXiv:0805.4411.
[11] B. Alver, private communication, further calculation confirms the ϵ_2-ϵ_3 correlation in peripheral events. The original publication [2] only checked for $N_{part} > 60$.
[12] H. Petersen, G.-Y. Qin, S. A. Bass, and B. Muller (2010), arXiv:1008.0625.