Scaling, decoupling and transversality of the gluon propagator

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Abstract. In this note we discuss a couple of technical issues relevant to solving the Dyson-Schwinger equation for the gluon propagator in Landau gauge Yang-Mills theory. In the deep infrared functional methods extract a one-parameter family of solutions generically showing a massive behavior referred to as 'decoupling' but also including the so-called 'scaling' solution with a conformal infrared behavior as a limiting case. We emphasize that the latter cannot be ruled out by technical arguments related to the removal of quadratic divergencies and transversality.

Introduction

The infrared behavior of the Green's functions of Landau gauge QCD has been focus of a number of discussions over the last years. In general, this interest is motivated by the desire to understand global properties of the theory such as the realization of the global gauge charges which is closely related to the description of confinement, as formulated by Kugo and Ojima many years ago [1], within the local field theory framework of covariantly gauge-fixed Yang-Mills theory. Clearly, non-perturbative approaches such as lattice QCD or functional methods are necessary to test these ideas.

From the functional methods, be it Dyson-Schwinger equations (DSEs) or the Functional Renormalization Group (FRG), it is now known that two types of solutions exist which differ in their asymptotic infrared behavior. One is a 'massive' or 'decoupling' type of solution, which is characterized by an infrared finite gluon propagator and a ghost propagator with an infrared finite dressing function [2, 3, 4, 5], and the other is the so-called 'scaling' solution with unique infrared power laws for both propagators [6, 7, 8, 9] and all other Green's functions of Landau gauge Yang-Mills theory [10, 11].

In terms of the dressing functions $G(p^2)$ and $Z(p^2)$ of the ghost and gluon propagators in Landau gauge

$$D_G(p) = -\frac{G(p^2)}{p^2}, \ D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2},$$ (1)

decoupling is characterized by the infrared behavior

$$Z(p^2) \sim p^2/M^2, \ \ G(p^2) \to \text{const.}, \ \text{for} \ p^2 \to 0, \quad (2)$$

whereas with scaling in 4 dimensions one has,

$$Z(p^2) \sim (p^2)^{2\kappa}, \ \ G(p^2) \sim (p^2)^{-\kappa}, \quad (3)$$

with a positive exponent $\kappa < 1$ which under a certain regularity assumption on the ghost-gluon vertex [7] results to be $\kappa = \kappa_c = (93 - \sqrt{1201})/98 \approx 0.6$.

Both, scaling and the decoupling type of solutions together form a one-parameter family, which has been obtained analytically and as infrared limits of numerical solutions to functional equations. With fixed gluon input this family has been found in the ghost DSE in Ref. [3]; full numerical solutions for the coupled ghost and gluon system of DSEs and FRGEs have been given for the whole one-parameter family in Ref. [5]. The decoupling results agree quantitatively well with lattice data [12]. Scaling has not been observed unambiguously on the lattice so far. There is an ongoing effort, however, to understand why that is the case and what the differences are between the functional continuum methods and those commonly used in what is called lattice Landau gauge [13, 14, 15].

Meanwhile, we would like to reply in this note to a claim made by Mike Pennington concerning an apparent problem with scaling from continuum arguments alone. In his plenary talk at this conference he purported that infrared scaling was observed only as a result of an inconsistency between the way in which quadratic divergences are being removed and the transversality of the gluon DSE in Landau gauge. In the following we explain why this conclusion is itself incorrect. There is no problem with quadratic divergences and transversality in the scaling results from the functional continuum methods.

Transversality of the gluon propagator

In order to understand the matter let us first recall, that in covariant gauges there is a Slavnov-Taylor identity for

the gluon propagator stating that its longitudinal part is not modified by interactions,

$$- \partial_\mu \partial_\nu D^{ab}_{\mu\nu}(x-y) = \xi \delta^{ab} \delta^4(x-y) \,. \qquad (4)$$

In the Landau gauge limit $\xi \to 0$ it then follows that the fully dressed gluon propagator remains transverse which implies that its momentum space Dyson-Schwinger equation is of the form,

$$\mathscr{P}^T_{\mu\nu} \frac{p^2}{Z(p^2)} = \mathscr{P}^T_{\mu\nu} p^2 Z_3 + \Pi_{\mu\nu}(p^2) \qquad (5)$$

where Z_3 is the gluon renormalization constant, $\mathscr{P}^T_{\mu\nu} = \left(\delta_{\mu\nu} - p_\mu p_\nu / p^2 \right)$ the transverse projector, and $\Pi_{\mu\nu}(p^2)$ the self-energy which is then necessarily transverse also. In practical calculations, however, the transversality of $\Pi_{\mu\nu}$ is often difficult to maintain in specific truncations. The main sources of transversality violations are thereby the following: (i) Numerical solutions of dimensionally regularized integral equations are extremely cumbersome [16]. In practice one therefore uses different schemes such as momentum subtractions with a hard cutoff. While these schemes in general preserve multiplicative renormalizability of the theory, they violate Eq. (4). (ii) Ansaetze for the different vertices in $\Pi_{\mu\nu}(p^2)$, necessary to close the gluon DSE, may be inadequate to preserve transversality. In both cases, (i) and (ii), artificial longitudinal contributions $\mathscr{P}^L_{\mu\nu} L(p^2)$ arise on the right hand side of the gluon DSE,

$$\Pi_{\mu\nu}(p^2) = \mathscr{P}^T_{\mu\nu} \Pi(p^2) + \mathscr{P}^L_{\mu\nu} L(p^2) \qquad (6)$$

with $\mathscr{P}^L_{\mu\nu} = p_\mu p_\nu / p^2$. A different but closely related problem is the appearance of quadratic divergences in $\Pi_{\mu\nu}$. Again, these are absent in dimensional regularization, but they occur with the cutoff procedure typically used in numerical studies of the gluon DSE [2, 5, 17].

When discussing the non-perturbative infrared properties of the gluon, care must be taken that artifacts like non-transversality or the appearance of quadratic divergencies do not affect the results. This may be particularly important when it comes to the discussion of scaling vs decoupling, Eqs.(2), (3). In his talk, Mike Pennington has addressed this problem, claiming that *the very appearance of the scaling solution (3) is such an artifact*. His argument relies on the assumption that the ghost-gluon vertex is essentially bare in the infrared. If one then uses a particular method to remove quadratic divergencies (see Eq. (8) below) the scaling solution disappears.

This problem has been addressed already in Ref. [7]: Scaling goes hand-in-hand with the infrared dominance of ghosts, i.e. the ghost-loop dominates $\Pi_{\mu\nu}(p^2)$, *i.e.*, for $p^2 \to 0$. Therefore, the ghost-loop must itself be transverse in the infrared. In a truncation where the full ghost-gluon vertex is replaced by the tree-level vertex of standard Faddeev-Popov theory this would require $\kappa = 3/4$, which is incompatible with the self-consistently obtained value $\kappa_c \approx 0.6$ mentioned above. Thus the non-perturbative ghost-gluon vertex can not be identical to the tree-level one.

Instead, infrared dominance of ghosts and with that the scaling solution essentially requires that the fully dressed ghost-gluon vertex becomes itself transverse with respect to the gluon momentum in the infrared [7]. Under the mild additional regularity assumption one then obtains the self-consistent value $\kappa = \kappa_c \approx 0.6$. Other values $1/2 < \kappa < 1$ are possible when this condition is relaxed [7]. Note, that all these analytic results were obtained in dimensional regularisation. They are thus unaffected by any technical difficulties that might arise with quadratic divergences in numerical investigations.

One method to remove quadratic divergences in the gluon DSE goes back to Brown and Pennington [18]. They decomposed the gluon self-energy into

$$\Pi_{\mu\nu}(p^2) = \delta_{\mu\nu} F_1(p^2) - p_\mu p_\nu F_2(p^2) \,, \qquad (7)$$

and noted that quadratic divergences can only occur in the term proportional to $\delta_{\mu\nu}$. Consequently, they suggested to project out this contribution. This is done by contracting the gluon-DSE with a general projector [17]

$$\mathscr{P}^\zeta_{\mu\nu} = \left(\delta_{\mu\nu} - \zeta \frac{p_\mu p_\nu}{p^2} \right) \,, \qquad (8)$$

and setting $\zeta = 4$ in $d = 4$ dimensions. This removes quadratic divergencies and leads to a dependence of $Z(p^2)$ on $F_2(p^2)$ alone. Alas, let us see what one obtains for general ζ upon contracting the gluon-DSE in the form of Eqs. (5) and (6) with $\mathscr{P}^\zeta_{\mu\nu}$:

$$\frac{p^2}{Z(p^2)} = p^2 Z_3 + \Pi(p^2) + \frac{1-\zeta}{3} L(p^2) \,. \qquad (9)$$

Clearly, if the gluon DSE is transverse, $L(p^2) = 0$, and the resulting Eq. (9) is independent of the parameter ζ and therefore also free from quadratic divergences. Conversely, the required ζ-independence provides a valuable test of specific truncations in numerical studies.

Two transverse truncations passing this test have been constructed explicitly in the literature, and both of them rely on a nontrivial ghost-gluon vertex thus dismissing Mike's criticism of the inconsistency that arises with the bare one. One is based on the Pinch-Technique/Background Field Method, see [19], for which the truncation of Ref. [2] has also been analyzed analytically in dimensional regularization.[1] The other

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[1] Note, however, that the numerical solutions presented in Ref. [2] have been obtained using a hard UV-cutoff and therefore need to be checked separately for transversality.

approach is the one used in Ref. [5]. There, Ansaetze for the ghost-gluon and three-gluon vertices have been constructed explicitly such that the gluon-DSE is free from quadratic divergencies and the residual contributions to $L(p^2)$ are minimized. The ghost-gluon vertex used there reduces in the infrared to the minimally dressed transverse one proposed in [7]. It contains a bare part and a nontrivial longitudinal part that serves to cancel all contributions to $L(p^2)$, thus explicitly establishing *exact* transversality of the ghost-loop in the infrared. Numerically, the remaining contributions to $L(p^2)$ for all momenta p^2 stay well below one percent of $\Pi(p^2)$. In particular $L(p^2) \to 0$ for $p^2 \to 0$. This therefore establishes unambiguously that the DSE results of [5] are *not* affected by transversality violating artifacts: there *is* a one-parameter family of solutions in the infrared including decoupling *and* scaling solutions.

The earlier truncation scheme from Ref. [17], which was particularly criticized by Mike Pennington in his plenary talk, was indeed less far developed and not manifestly transverse. The artificial longitudinal term $L(p^2)$ in Eq. (9) was removed by transverse projection, using $\zeta = 1$ rather than $\zeta = 4$ as suggested by Brown and Pennington before, which meant that quadratic divergences had to be dealt with in a different manner. Even though this procedure might seem less elegant, there were good reasons at the time for that given in [7]. Before we briefly explain those, it can now be verified a posteriori, by comparing the earlier results from [17] to those of Ref. [5], to verify that they are almost identical (for $\zeta = 1$).

The reason why transverse projection onto $\Pi(p^2)$ in Eq. (6) must be done [7] rather than $\zeta = 4$ projection onto $F_2(q^2)$ in (7), in a not manifestly transverse truncation where this can make a difference, relates to an ambiguity in the Landau gauge: the existence of a second gauge parameter which interpolates between standard Faddeev-Popov theory and the ghost/anti-ghost symmetric Curci-Ferrari gauges. In Landau gauge there is no distinction, however, and any quantity depending on this parameter is thus ambiguous. In [7] it was shown that $F_2(p^2)$ is such a quantity while $\Pi(p^2)$ is not. So we have understood over the years that it is the original proposal from [18] to avoid quadratic divergences which is ambiguous, unfortunately. Luckily, however, the problem is now completely solved with the manifestly transverse truncations that were developed since then.

Finally, we emphasize that the problem with quadratic divergences is completely unheard of in the FRGEs which are finite by construction. Yet, one still finds the full one-parameter family of scaling and decoupling solutions [9, 5]. Apart from the extreme infrared, where this parameter matters, these solutions all agree and are furthermore in almost perfect agreement with lattice data.

Conclusions

In this note we have pointed out that a claim made by Mike Pennington in his plenary talk is not correct. He purported that the infrared scaling solution for the ghost and gluon propagators of Landau gauge Yang-Mills theory is a mere artifact of a technical inaccuracy in the treatment of the gluon DSE. This is based an the overly simplistic truncation which cannot be combined with an earlier proposal to remove artificial quadratic divergences in an untlraviolet cutoff regularization. This issue has been completely solved over the years, however. The infrared-scaling solution for the Yang-Mills sector of QCD does not have fundamental problems related to quaderatic divergences or transversality.

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