Application of the Extended Pairing Model to Heavy Isotopes

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Abstract. Relative binding energies (RBEs) within three isotopic chains ($^{100-130}$Sn, $^{152-181}$Yb, and $^{181-202}$Pb) have been studied using the exactly solvable extended pairing model (EPM) [1]. The unique pairing strength G, which reproduces the experimental RBEs, has been determined. Within EPM, $\log(G)$ is a smooth function of the model space dimension $\dim(A)$, as expected for an effective coupling strength. In particular, for the Pb and Sn isotopes G can be described by a two parameter expression that is inversely proportional to the dimensionality of the model space, $G = \alpha \dim(A)^{-\beta}$ with $\beta \approx 1$.

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In many applications the infinite dimensionality of the quantum mechanical Hilbert space is an obstacle; to overcome it, one has to restrict the model space to a finite dimensional subspace and construct an appropriate effective Hamiltonian. This in turn leads from a two-body to a many-body interaction terms. Nonetheless, the effective Hamiltonian approach has been very successful and even pointed to the importance of three-body nuclear interactions [2]. The recently introduced exactly solvable extended pairing model [1] provides a framework for study of Hamiltonians with many-body interaction terms:

$$\hat{H} = \sum_{j=1}^{p} \epsilon_j n_j - G \sum_{i,j=1}^{p} B_i^+ B_j - G \sum_{\mu=2}^{p} \frac{1}{(\mu!)^2} \times \qquad (1)$$
$$\times \sum_{i_1 \neq \cdots \neq i_{2\mu}} B_{i_1}^+ \cdots B_{i_\mu}^+ B_{i_{\mu+1}} \cdots B_{i_{2\mu}}.$$

Ideally, one should be able to calculate binding energies and other observables *ab-initio* using the exact nucleon interaction. However, we are still lacking this capability. Instead, we use different models for binding energies and excitation energies. Conventionally, the liquid-drop model is the zeroth order approximation to the binding energies while the two-body pairing interaction gives the shell model corrections. The extended pairing model (EPM) (1) has terms beyond the standard Nilsson plus pairing Hamiltonian; these terms provide an alternative description of the relative binding energies (RBEs) of neighboring nuclei within the same valence space. As we will discus below, EPM is well suited to provide description of the RBEs only within the shell-model since the equations are insensitive to the binding energy of the core nucleus.

Beside the first two terms, Nilsson plus standard pairing intgeraction, the Hamiltonian in (1) contains many-pair interactions which connect configurations that differ by more than a single pair. Here p is the total number of single-particle levels considered, ϵ_j are single-particle energies, G is the overall pairing strength ($G > 0$), $n_j = c_{j\uparrow}^\dagger c_{j\uparrow} + c_{j\downarrow}^\dagger c_{j\downarrow}$ is the number operator for the j-th single-particle level, $B_i^+ = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$ are pair creation operators where $c_j^\dagger$ creates a fermion in the j-th single-particle level. The up and down arrows refer to time-reversed states. Since each Nilsson level can only be occupied by one pair due to the Pauli Exclusion Principle, the operators B_i^+, B_i, and n_i form a hard-core boson algebra: $[B_i, B_j^+] = \delta_{ij}(1 - n_i)$, $[B_i^+, B_j^+] = 0 = (B_i^+)^2$.

The pairing vacuum state $|j_1, \cdots, j_m\rangle$ is defined so that: $B_i|j_1, \cdots, j_m\rangle = 0$ for $1 \leq i \leq p$ and $i \neq j_s$, where $j_1, \cdots, j_m$ indicate those m levels that are occupied by unpaired nucleons. Any state that is occupied by a single nucleon is blocked to the hard-core bosons due to the Pauli principle. The k-pair eigenstates of (1) has the form:

$$|k; \zeta; j_1 \cdots j_m\rangle = \sum_{i_1 < \cdots < i_k} C_{i_1 \cdots i_k}^{(\zeta)} B_{i_1}^+ \cdots B_{i_k}^+ |j_1 \cdots j_m\rangle, \qquad (2)$$

where $C_{i_1 i_2 \cdots i_k}^{(\zeta)}$ are expansion coefficients to be determined. It is assumed that the level indices $j_1, \cdots, j_m$ should be excluded from the summation in (2). For simplicity, we focus only on the seniority zero case ($m = 0$).

Although Hamiltonian (1) contains many-body interaction terms that are non-perturbative, the contribution of the higher and higher energy configurations is more and more suppressed due to the structure of the equation that needs to be solved to determine the eigensystem of

the Hamiltonian (1). The eigensystem $E_k^{(\zeta)}$ and $C_{i_1 i_2 \cdots i_k}^{(\zeta)}$ depend on only one parameter $z^{(\zeta)}$, where the quantum number ζ [1] is understood as the ζ-th solution of (5):

$$E_k^{(\zeta)} = z^{(\zeta)} - G(k-1), \qquad (3)$$

$$C_{i_1 i_2 \cdots i_k}^{(\zeta)} = \frac{1}{z^{(\zeta)} - E_{i_1 \ldots i_k}}, \quad E_{i_1 \ldots i_k} = \sum_{\mu=1}^{k} 2\epsilon_{i_\mu}, \quad (4)$$

$$1 = \sum_{i_1 < i_2 < \cdots < i_k} \frac{G}{E_{i_1 \ldots i_k} - z^{(\zeta)}}. \qquad (5)$$

Due to the space limitations many details and results of the current application of this exactly solvable model are omitted, however, a more detailed paper is available [3]. For the current application the single-particle energies are calculated using the Nilsson deformed shell model with parameters from [4]. Experimental BEs are taken from [5]. Theoretical RBE are calculated relative to a specific core, ^{152}Yb, ^{100}Sn, and ^{208}Pb for the cases considered. The RBE of the nucleus next to the core is used to determine an energy scale for the Nilsson single-particle energies. For an even number of neutrons, we considered only pairs of particles (hard bosons). For an odd number of neutrons, we apply Pauli blocking to the Fermi level of the last unpaired fermion and considered the remaining fermions as if they were an even fermion system. The valence model space consists of the neutron single-particle levels between two closed shells with magic numbers 50-82 and 82-126. By using (3) and (5), values of G are determined so that the experimental and theoretical RBE match exactly.

Figure 1 shows results for the $^{181-202}$Pb isotopes. The RBEs are relative to ^{208}Pb which is set to zero, and the core nucleus is chosen to be ^{164}Pb. For the Yb and Sn isotopes the core nucleus is also the zero RBE reference nucleus (^{100}Sn and ^{152}Yb). In this respect, the calculations for the Pb-isotopes are different because the core nucleus (^{164}Pb) and the zero binding energy reference nucleus (^{208}Pb) are not the same. One can see from Figure 1 that a quadratic fit to $\ln(G)$ as function of A fits the data well. In this particular case, the pairing strength $G(A)$ for all 21 nuclei in the range A=181 − 202) was also fit to a simple two-parameter function that is inversely proportional to the dimensionality of the model space $\dim(A)$, namely, by $G(A) = \alpha \dim(A)^{-\beta}$. Similar results have been obtained for the Sn-isotopes as well using ^{132}Sn as zero.

In conclusion, we studied RBEs of nuclei in three isotopic chains, $^{100-130}$Sn, $^{152-181}$Yb, and $^{181-202}$Pb, within the recently proposed EPM [1] by using Nilsson single-particle energies as the input mean-field energies. Overall, the results suggest that the model is applicable to neighboring heavy nuclei and provides, within a pure shell-model approach, an alternative mean of calculating a RBE. In order to achieve that, the pairing strength is allowed to change as a smooth function of the model space dimension. It is important to understand that the A-dependence of G is indirect, since G only depends on the model space dimension, which by itself is different for different nuclei. In particular, in all the cases studied $\ln(G)$ has a smooth quadratic behavior for even and odd A with a minimum
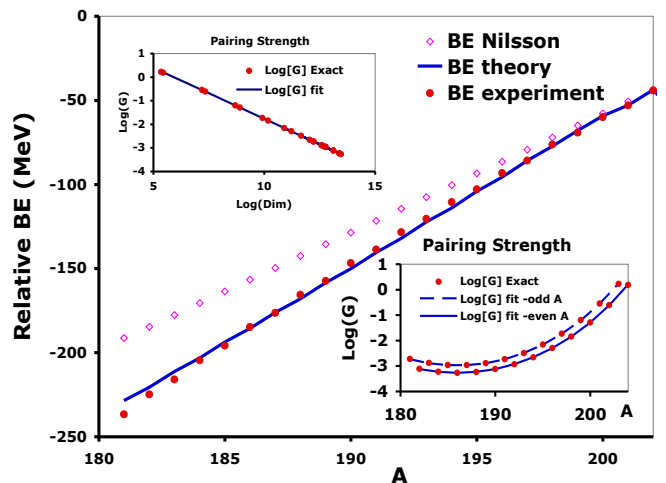
Fig. 1. The solid line gives the theoretical RBE for the Pb isotopes relative to the ^{208}Pb nucleus. The insets show the fit to the values of G that reproduce exactly the experimental data using ^{164}Pb core. The lower inset shows the two fitting functions: $\log(G(A)) = 382.3502 - 4.1375A + 0.0111A^2$ for even values of A and $\log(G(A)) = 391.6113 - 4.2374A + 0.0114A^2$ for odd values of A. The upper inset shows a fit to $G(A)$ that is inversely proportional to the size of the model space, $(\dim(A))$, that is valid for even as well as odd values of A: $G(A) = 366.7702 \dim(A)^{-0.9972}$. The Nilsson BE energy is the lowest energy of the non-interacting system.

in the middle of the model space where the dimensionality of the space is a maximal; $\ln(G)$ for even A and odd A are very similar which suggests that further detailed analyses may result in the same functional form for even A and odd A isotopes as found in the case of the Pb-isotopes and Sn-isotopes. It is a non-trivial result that G is inversely proportional to the space dimension dim in the two cases found (Pb-isotopes and Sn-isotopes) which requires further studies.

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