

# Gauge-invariant formulation of $NN \rightarrow NN\gamma$

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A complete, rigorous relativistic field-theory formulation of the nucleon-nucleon ( $NN$ ) bremsstrahlung reaction is presented. The resulting amplitude is unitary as a matter of course and it is gauge invariant, i.e., it satisfies a generalized Ward-Takahashi identity. The novel feature of this approach is the consistent microscopic implementation of local gauge invariance across all interaction mechanisms of the hadronic systems, thus serving as a constraint for all subprocesses. The formalism is quite readily adapted to approximations and thus can be applied even in cases where the microscopic dynamical structure of the underlying interacting hadronic systems is either not known in detail or too complex to be treated in detail. We point out how the interaction currents resulting from the photon being attached to nucleon-nucleon-meson vertices can be treated by phenomenological four-point contact currents that preserve gauge invariance. In an advance application of the present formalism [K. Nakayama and H. Haberzettl, Phys. Rev. C **80**, 051001 (2009)], such interaction currents were found to contribute significantly in explaining experimental data. In addition, we provide a scheme that permits — through an introduction of phenomenological five-point contact currents — the approximate treatment of current contributions resulting from pieces of the  $NN$  interaction that cannot be incorporated exactly. In each case, the approximation procedure ensures gauge invariance of the entire bremsstrahlung amplitude. We also discuss the necessary modifications when taking into account baryonic states other than the nucleon  $N$ ; in detail, we consider the  $\Delta(1232)$  resonance by incorporating the couplings of the  $NN$  to the  $N\Delta$  and  $\Delta\Delta$  systems, and the  $\gamma N \rightarrow \Delta$  transitions. We apply the formalism to the 280-MeV bremsstrahlung data from TRIUMF [Phys. Rev. D **41**, 2689 (1990)] incorporating  $\gamma N\Delta$  transition currents and find good agreement with the data.

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## I. INTRODUCTION

The two-nucleon system is one of the simplest strongly interacting systems. The study of the nucleon-nucleon ( $NN$ ) bremsstrahlung reaction, therefore, offers one of the most fundamental and direct avenues for understanding how the electromagnetic field interacts with strongly interacting hadronic systems. In the past, the  $NN$  bremsstrahlung reaction had been applied extensively mainly to learn about off-shell properties of the  $NN$  interaction. It should be clear, however, that off-shell effects are model-dependent and cannot be measured, and therefore are meaningless quantities for the purpose of comparison.

Even though the original motivation for investigating the  $NN$  bremsstrahlung reaction has fallen away, understanding the dynamics of such a fundamental process, nevertheless, is of great importance from a general theoretical perspective. This is highlighted by the fact that none of the past models of  $NN$  bremsstrahlung could describe the high-precision proton-proton bremsstrahlung data from KVI [1, 2] for coplanar geometries involving small proton scattering angles. This was generally considered all the more surprising since the irrelevance of

off-shell effects was taken as implicit proof positive that the coupling of a photon to the interacting two-nucleon system was under control. The discrepancy between the KVI data and the existing theoretical models, therefore, was quite unexpected. This longstanding discrepancy of nearly a decade was resolved recently by the present authors [3] who put forward a novel approach to the  $NN$  bremsstrahlung reaction that takes into account details of the photon coupling to interacting systems that had previously been neglected. The study, in particular, revealed the importance of accounting for the corresponding interaction currents in a manner consistent with the gauge-invariance constraint.

Another recent bremsstrahlung experiment concerns the hard bremsstrahlung process  $p + p \rightarrow pp(^1S_0) + \gamma$  measured for the first time by the COSY-ANKE Collaboration [4]. In the absence of free systems of bound diprotons, this process was considered as an alternative to the  $\gamma + pp(^1S_0) \rightarrow p + p$  process which complements the photo-disintegration of the deuteron. Here, the hardness of the bremsstrahlung is due to the fact that the invariant mass of the two protons in the final state is constrained experimentally to be less than 3 MeV above its minimum value of twice the proton mass. In this kinematic regime, the two protons in the final state are practically confined to the  $^1S_0$  state and most of the available energy is carried by the bremsstrahlung. Therefore, this kinematic regime is as far away from the soft-photon limit as pos-

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sible. The proton-proton ( $pp$ ) hard bremsstrahlung reaction has been also measured at CELSIUS-Uppsala [5]. In spite of extensive studies of the  $NN$  bremsstrahlung reaction in the past, no dedicated experiments of  $pp$  hard bremsstrahlung with the diproton in the final state had been available until these recent measurements [4, 5]. Also, apart from the very recent study of Ref. [6], the theoretical investigation of the  $p + p \rightarrow pp(^1S_0) + \gamma$  reaction is virtually inexistent so far.

Apart from the intrinsic interest in the elementary  $NN$  bremsstrahlung process, the investigation of this reaction has also an immediate impact in the area of heavy-ion physics. Indeed, di-lepton production in heavy-ion collisions is used intensively as a probe of hadron dynamics in the nuclear medium. Due to their weak interaction with hadrons, di-leptons are well suited to probe the hadron dynamics in the dense region of heavy-ion collisions. The HADES Collaboration, in particular, is currently engaged actively in the study of di-electron production in heavy-ion as well as in elementary  $NN$  collisions in the 1-2 GeV/ $u$  energy domain [7]. In order to interpret the experimental data in heavy-ion collisions, it is imperative to understand the underlying basic elementary processes. Unfortunately, these basic elementary processes are not yet fully under control. In fact, there is a number of theoretical efforts to understand these basic reaction processes [8, 9]. According to these studies, among the various competing mechanisms, the  $NN$  bremsstrahlung is one of the major mechanisms in producing di-electrons in these reactions.

On the theoretical side, the majority of the existing models of  $NN$  bremsstrahlung are potential models. They have been applied to the analyses of experimental data (mostly in coplanar geometries) obtained up until the early 2000s, before the more recent experiments mentioned above were performed. Among those models, the most recent and sophisticated ones that have been used in the analysis of the high-precision KVI data [1, 2] are the microscopic meson-exchange models of Refs. [10–12]. There are also a number of other microscopic model calculations throughout the 1990s [13–21], which are dynamically similar to Refs. [10–12], addressing a variety of issues in the  $NN$  bremsstrahlung process. All these models satisfy current conservation (at least in the soft-photon approximation),<sup>1</sup> but none of them obey the more general gauge-invariance condition in terms of the generalized Ward-Takahashi identity (WTI) employed in Ref. [3] (and explained in more detail

in the present work). Quite recently, it was shown formally [22] how to maintain gauge invariance in a theory of undressed non-relativistic nucleons if one introduces a finite cutoff in a reference theory that is presumed to be already gauge invariant. When applied to effective field theories, in particular, this implies that gauge invariance can be maintained in such theories order-by-order in the expansion.

The purpose of the present paper is to present the complete, rigorous covariant formulation of the  $NN$  bremsstrahlung reaction whose successful advance application was reported in Ref. [3]. The approach is based on a relativistic field theory in which the photon is coupled in all possible ways to the underlying two-nucleon  $T$ -matrix obtained from the corresponding covariant Bethe-Salpeter-type  $NN$  scattering equation. This formulation follows the basic procedures of the field-theoretical approach of Haberzettl [23] developed for pion photoproduction off the nucleon. The resulting bremsstrahlung amplitude satisfies unitarity and gauge invariance as a matter of course. The latter, in particular, is shown explicitly by deriving the corresponding generalized WTI.

The formalism is quite readily adapted to approximations and thus can be applied even in cases where the microscopic dynamical structure of the underlying interacting hadronic systems is either not known in detail or too complex to be treated in detail. As a case in point, we mention that the success of the advance application [3] of the present formalism to the KVI data [1, 2] was due to the incorporation of phenomenological four-point contact currents that preserve gauge invariance following the approach of Haberzettl, Nakayama, and Krewald [24] based on the original ideas of Refs. [23, 25]. In addition, we provide a scheme that permits the approximate treatment of current contributions resulting from pieces of the  $NN$  interaction that cannot be incorporated exactly. In each case, the approximation procedure ensures gauge invariance of the entire bremsstrahlung amplitude. We also discuss the necessary modifications when taking into account baryonic states other than the nucleon  $N$ ; in detail, we consider the  $\Delta(1232)$  resonance by incorporating the couplings of the  $NN$  to the  $N\Delta$  and  $\Delta\Delta$  systems, and the  $\gamma N \rightarrow \Delta$  transitions.

In Sec. II, we present the details of the full four-dimensional relativistic formulation, including a proof of the gauge invariance of the resulting bremsstrahlung amplitude. We also introduce the necessary modifications for a covariant three-dimensional reduction and discuss its implications for the description of the dynamics of the process. We point out, in particular, that if one aims for a dynamically consistent *microscopic* description of all reaction mechanisms, one must implement gauge invariance in terms of generalized Ward-Takahashi identities for each subprocess — mere global current conservation is not sufficient. We show how one can preserve the gauge invariance of the amplitude even if some interaction-current mechanisms — both for hadronic three- and four-point functions — cannot be incorporated exactly. Fur-

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<sup>1</sup> The current conservation of earlier models usually comes about because for  $pp$  bremsstrahlung, there are no exchange currents for (uncharged) mesons and the four-point contact current discussed in Ref. [3] is absent for phenomenological meson-nucleon-nucleon form factors that depend only on the momentum of the exchanged meson. For  $pn$  bremsstrahlung (see, e.g., Ref. [13]) the meson-exchange currents are taken into account via Siegert's theorem which preserves current conservation in the soft-photon limit.

thermore, we discuss what needs to be done to add additional baryonic degrees of freedom, in particular, the coupling of  $NN$ ,  $N\Delta$ , and  $\Delta\Delta$  channels. In Sec. III, we report on our results for the TRIUMF data [26]. A summarizing assessment of the present work is given in Sec. IV. Some technical details follow in the Appendix.

We emphasize that the present formalism is *completely* general and applies to proton-proton, proton-neutron as well as neutron-neutron bremsstrahlung processes. Furthermore, the photon can be either real or virtual. The former corresponds to the usual  $NN$  bremsstrahlung process while the latter case is suited for applications in dilepton production in both the elementary  $NN$  and heavy ion collisions.

## II. FORMALISM

The bremsstrahlung current  $B^\mu$  is obtained from the nucleon-nucleon  $T$ -matrix by attaching an outgoing photon to all reaction mechanisms of  $T$  in all possible ways. To this end, we use here the gauge-derivative procedure developed in Ref. [23] in the context of pion photoproduction. This procedure is formally equivalent to employing minimal substitution for the connected part of the hadronic Green's function, and then taking the functional derivative with respect to the electromagnetic four-potential  $A^\mu$ , in the limit of vanishing  $A^\mu$  (for details, see [23]). The current is then obtained by removing the propagators of all external hadron legs from this derivative in a Lehmann-Szycmanzik-Zimmermann (LSZ) reduction procedure [27, 29].

### A. Full four-dimensional formalism

The nucleon-nucleon  $T$ -matrix is determined by the corresponding four-dimensional Bethe-Salpeter scattering equations,

$$T = V + VG_0T \quad \text{or} \quad T = V + TG_0V, \quad (1)$$

where  $V$  is the  $NN$  interaction given by the set of all two-nucleon irreducible scattering mechanisms.  $G_0$  describes the intermediate propagation of two non-interacting nucleons, i.e., schematically we have

$$G_0 = [t_1 \circ t_2], \quad (2)$$

where  $t_i$  denotes the propagator of the individual nucleon  $i$  and “ $\circ$ ” stands for the convolution of the intermediate loop integration.

The bremsstrahlung current  $\tilde{B}^\mu$  is obtained by evaluating the LSZ-type equation

$$\tilde{B}^\mu = -G_0^{-1} \{G_0TG_0\}^\mu G_0^{-1} \quad (3)$$

where  $\{\dots\}^\mu$  denotes the gauge derivative [23] taken here of the connected hadronic  $NN$  Green's function  $G_0TG_0$ ,

FIG. 1. Graphical representation of Eq. (8). Solid lines depict nucleons and wavy lines indicate the outgoing bremsstrahlung photon.

with  $\mu$  being the Lorentz index of the current. Using then the product rule  $\{YX\}^\mu = Y\{X\}^\mu + \{Y\}^\mu X$  for an (ordered) product  $YX$  of a two-step sequence of hadronic reaction mechanisms described by operators  $X$  (first step) and  $Y$  (second step), we employ Eq. (1) repeatedly to find

$$\tilde{B}^\mu = (1 + TG_0)(d^\mu + V^\mu)(G_0T + 1) - d^\mu, \quad (4)$$

where  $d^\mu$  defined by

$$d^\mu = -G_0^{-1} \{G_0\}^\mu G_0^{-1} \quad (5)$$

subsumes the one-body current contributions from the individual nucleons and

$$V^\mu = -\{V\}^\mu \quad (6)$$

is the interaction current resulting from attaching the photon to any *internal* mechanisms of the  $NN$  interaction. Details of  $V^\mu$  will be discussed below.

Explicitly, the photon contributions from the two-nucleon propagator are found as

$$d^\mu = \Gamma_1^\mu (\delta_2 t_2^{-1}) + (\delta_1 t_1^{-1}) \Gamma_2^\mu, \quad (7)$$

where  $\Gamma_i^\mu$  is the electromagnetic current operator of nucleon  $i$ ;  $\delta_i$  denotes an implied  $\delta$  function that makes the incoming and outgoing momenta for the intermediate spectator nucleon  $i$  the same. We thus have

$$G_0 d^\mu G_0 = [t_1 \Gamma_1^\mu t_1 \circ t_2] + [t_1 \circ t_2 \Gamma_2^\mu t_2] \quad (8)$$

which is represented graphically in Fig. 1.

Note that Eq. (4) — apart from the subtraction by  $d^\mu$  — possesses the structure of a distorted-wave Born approximation (DWBA), with the factors  $(G_0T + 1)$  and  $(1 + TG_0)$  supplying the Møller operators producing the initial and final scattering states, respectively, distorted by the  $NN$  interaction. The  $d^\mu$  contribution by itself — without any initial-state or final-state  $NN$  interactions — is disconnected, as one sees clearly from Fig. 1. The overall subtraction of  $d^\mu$  in Eq. (4), therefore, is necessary to remove this (unphysical) disconnected structure from  $\tilde{B}^\mu$  and retain only connected physical contributions.

It is possible — and indeed desirable for the following — to equivalently rewrite Eq. (4) to provide a full DWBA structure for  $\tilde{B}^\mu$  of the form

$$\tilde{B}^\mu = (1 + TG_0) \tilde{J}^\mu (G_0T + 1), \quad (9)$$

where

$$\tilde{J}^\mu = d^\mu G_0V + VG_0d^\mu + V^\mu - VG_0d^\mu G_0V \quad (10)$$

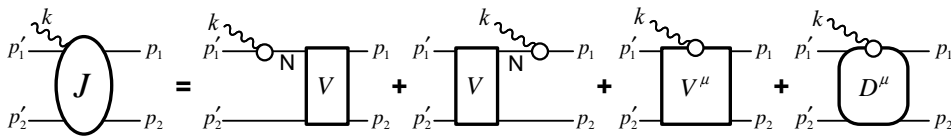


FIG. 2. Basic production current  $J^\mu$  of Eq. (11) for  $NN \rightarrow NN\gamma$ . Time proceeds from right to left. External legs are labeled by the four-momenta of the respective particles. Boxes labeled  $V$  subsume *all* two-nucleon irreducible contributions to the  $NN$  interaction that drive the Bethe-Salpeter equation (1). The interaction current  $V^\mu$  contains all mechanisms where the photon emerges from *within* the interaction  $V$  (i.e., any current mechanism not associated with an external leg of the interaction  $V$ ). The last diagram,  $D^\mu$ , as given in Eq. (12), subsumes all possible completely transverse contributions [cf. Eq. (15)], in addition to the subtraction that corrects the double counting arising from the first two contributions (see text). Diagrams where the photon emerges from the lower nucleon line are suppressed; the antisymmetrization of nucleons is implied.

is the completely connected current that describes the bremsstrahlung reaction in the absence of any hadronic initial-state or final-state interactions (with the exception of the subtraction  $VG_0d^\mu G_0V$ ; see below); we shall refer to this tree-level-type current as the basic production current. The equivalence of this form of  $\tilde{B}^\mu$  to Eq. (4) is easily seen by repeated applications of Eq. (1). The  $NN$   $T$ -matrices appearing to the right or left of the basic production current  $J^\mu$  in (9) thus provide the initial-state interaction (ISI) or final-state interaction (FSI), respectively, of the two nucleons external to the basic production current  $J^\mu$ .

The subtraction term  $VG_0d^\mu G_0V$  in  $J^\mu$  removes here the double counting of contributions  $TG_0d^\mu G_0T$  to the full current  $\tilde{B}^\mu$  that arise from the two  $d^\mu$  contributions in  $\tilde{J}^\mu$ . As such, therefore, it is not a dynamically independent contribution to  $\tilde{B}^\mu$  and appears here only because, for formal reasons, we wish to retain the DWBA form of  $\tilde{B}^\mu$  in Eq. (9). We shall see below, in Sec. II C, where we treat the covariant three-dimensional reduction of Eq. (9), that special considerations are needed for this subtraction term if one wants to maintain gauge invariance for the reduced amplitude.

Equations (9) and (10) are *not* the complete solution of the bremsstrahlung problem yet, because the gauge-derivative procedure — indeed any procedure based on minimal substitution — cannot produce current contributions that are completely transverse. Such contributions must be added to the mechanisms obtained above for  $\tilde{B}^\mu$ . Without lack of generality, we may do so by modifying the basic production current  $\tilde{J}^\mu$  according to

$$J^\mu = d^\mu G_0 V + V G_0 d^\mu + V^\mu + D^\mu \quad (11)$$

where

$$D^\mu = T^\mu - V G_0 d^\mu G_0 V \quad (12)$$

contains the sum of all explicitly *transverse* five-point currents denoted by  $T^\mu$ , in addition to the subtraction current  $VG_0d^\mu G_0V$ . In other words,

$$k_\mu T^\mu = 0 \quad (13)$$

is true irrespective of whether the external nucleons are on-shell or not. The complete bremsstrahlung current

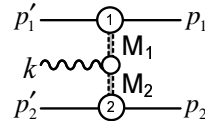


FIG. 3. Generic example for meson transition currents  $J_M^\mu$  containing a *transverse*  $\gamma M_1 M_2$  vertex, with  $M_1 \neq M_2$ , occurring in  $D^\mu$  of Eq. (12) via its transverse contribution  $T^\mu$ . The present numerical results were obtained incorporating  $\gamma\rho\pi$  and  $\gamma\omega\pi$  transition currents.

then is given by

$$B^\mu = (1 + T G_0) J^\mu (G_0 T + 1). \quad (14)$$

We emphasize that this equation and Eq. (11) provide an exact generic description of the bremsstrahlung process off the  $NN$  system. The structure of the basic production current  $J^\mu$  is depicted in Fig. 2.

The detailed nature of the transverse contribution  $T^\mu$  in (12) must be specified by the underlying interaction Lagrangians. Examples are meson-transition currents  $J_M^\mu$  as depicted in Fig. 3 and  $\gamma N\Delta$ -transition currents  $J_\Delta^\mu$ , i.e.,

$$T^\mu = J_M^\mu + J_\Delta^\mu + \dots \quad (15)$$

The latter will be discussed below, in Sec. II F, when we consider  $\Delta(1232)$  contributions in detail.

The specific details of any particular application, of course, depend on the mechanisms taken into account in the  $NN$  interaction  $V$  that drives the scattering process in the Bethe-Salpeter equation (1). For driving interactions based on single-meson exchanges, the complete structure of  $J^\mu$  is discussed below, in Sec. II D. In Sec. II F, as mentioned already, we also consider the structures arising from  $\Delta$  contributions that go beyond single-meson exchanges.

## B. Gauge invariance of the full four-dimensional amplitude

It should be clear that the procedure used for deriving the bremsstrahlung current  $B^\mu$  does produce a cur-

rent that is gauge invariant as a matter of course. Nevertheless, we will now explicitly prove gauge invariance because this will guide us later, in Sec. II C, in how to implement gauge invariance when we calculate  $B^\mu$  in a covariant three-dimensional reduction.

To prove the gauge invariance of the current (14), we first note that for an outgoing photon with four-momentum  $k$ , the four-divergence of  $d^\mu$  may be schematically written as

$$k_\mu d^\mu = \hat{Q} G_0^{-1} - G_0^{-1} \hat{Q} , \quad (16)$$

where  $\hat{Q}$  is short for

$$\hat{Q} = \hat{Q}_1 + \hat{Q}_2 . \quad (17)$$

The operator  $\hat{Q}_i$  describes the charge  $Q_i$  of nucleon  $i$  and it removes the four-momentum  $k$  carried away by the photon from any subsequent interaction of nucleon  $i$  (appearing on the left of  $\hat{Q}_i$ ). This notation allows one to keep track of the kinematics without explicit four-momentum arguments, i.e., specifying initial momenta for the two nucleons, the placement of the  $\hat{Q}_i$  immediately allows one to find the momenta along each nucleon line. Equation (16) is an immediate consequence of the Ward-Takahashi identity for the nucleon current operator  $\Gamma_i^\mu$  [28, 29], i.e.,

$$k_\mu \Gamma_i^\mu = \hat{Q}_i t_i^{-1} - t_i^{-1} \hat{Q}_i , \quad (18)$$

applied to Eq. (7). Explicitly, with  $G_0$  specified as in (2), we have

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$$\begin{aligned} \hat{Q} G_0^{-1} - G_0^{-1} \hat{Q} &= [\hat{Q}_1 t_1^{-1} - t_1^{-1} \hat{Q}_1] \circ t_2^{-1} + t_1^{-1} \circ [\hat{Q}_2 t_2^{-1} - t_2^{-1} \hat{Q}_2] \\ &= [Q_1 t_1^{-1}(p_1) - t_1^{-1}(p_1 - k) Q_1] \circ t_2^{-1}(p_2) + t_1^{-1}(p_1) \circ [Q_2 t_2^{-1}(p_2) - t_2^{-1}(p_2 - k) Q_2] \end{aligned} \quad (19)$$

for a two-nucleon system where  $p_1$  and  $p_2$  are the initial four-momenta of nucleons 1 and 2, respectively, i.e., the first term results from the photon being emitted by nucleon 1 and for the second term, it is being emitted by nucleon 2.

In the same schematic notation, the four-divergence of the interaction current  $V^\mu$  then is simply [23, 30]

$$\begin{aligned} k_\mu V^\mu &= V \hat{Q} - \hat{Q} V \\ &= V(p'_1, p'_2; p_1 - k, p_2) Q_1 + V(p'_1, p'_2; p_1, p_2 - k) Q_2 - Q_1 V(p'_1 + k, p'_2; p_1, p_2) - Q_2 V(p'_1, p'_2 + k; p_1, p_2) , \end{aligned} \quad (20)$$

where the arguments of  $V$  are nucleon momenta and the momentum dependence of the interaction current is

$$V^\mu = V^\mu(k, p'_1, p'_2; p_1, p_2) , \quad \text{with} \quad p'_1 + p'_2 + k = p_1 + p_2 , \quad (21)$$

i.e., the momenta  $p_1, p_2$  and  $p'_1, p'_2$  are those of the incoming and outgoing nucleons, respectively. Whether the charge operators  $Q_i$  in (20) pertain to incoming or outgoing nucleons is clear from where the  $Q_i$  are placed in the equation. In other words, if placed on the right of  $V$ ,  $Q_i$  describes the charge of the incoming nucleon  $i$ , and if placed on the left, it describes the charge of the outgoing nucleon  $i$ . Placing the charge operators in this manner is necessary since they interact with the isospin dependence of the interaction  $V$ .

The four-divergence of  $J^\mu$  then follows as

$$\begin{aligned} k_\mu J^\mu &= (\hat{Q} G_0^{-1} - G_0^{-1} \hat{Q}) G_0 V + V G_0 (\hat{Q} G_0^{-1} - G_0^{-1} \hat{Q}) + V \hat{Q} - \hat{Q} V - V G_0 (\hat{Q} G_0^{-1} - G_0^{-1} \hat{Q}) G_0 V \\ &= -G_0^{-1} \hat{Q} G_0 V + V G_0 \hat{Q} G_0^{-1} - V G_0 \hat{Q} V + V \hat{Q} G_0 V . \end{aligned} \quad (22)$$

For the entire current, we then find

$$k_\mu B^\mu = (1 + T G_0) (V G_0 \hat{Q} G_0^{-1} - G_0^{-1} \hat{Q} G_0 V - V G_0 \hat{Q} V + V \hat{Q} G_0 V) (G_0 T + 1) , \quad (23)$$

and thus finally, using (1),

$$k_\mu B^\mu = T G_0 \hat{Q} G_0^{-1} - G_0^{-1} \hat{Q} G_0 T . \quad (24)$$

This is the correct generalized Ward-Takahashi identity [31] for the bremsstrahlung current providing a conserved current for external on-shell nucleons. In a more explicit notation, using the same arguments for  $T$  and  $B^\mu$  as for  $V$  and  $V^\mu$ , respectively, in Eqs. (20) and (21), this reads

$$\begin{aligned} k_\mu B^\mu &= T(p'_1, p'_2; p_1 - k, p_2) t_1(p_1 - k) Q_1 t_1^{-1}(p_1) + T(p'_1, p'_2; p_1, p_2 - k) t_2(p_2 - k) Q_2 t_2^{-1}(p_2) \\ &\quad - t_1^{-1}(p'_1) Q_1 t_1(p'_1 + k) T(p'_1 + k, p'_2; p_1, p_2) - t_2^{-1}(p'_2) Q_2 t_2(p'_2 + k) T(p'_1, p'_2 + k; p_1, p_2) . \end{aligned} \quad (25)$$

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The inverse nucleon propagators  $t_i^{-1}(p)$  appearing here

ensure that this four-divergence vanishes (i.e., that the

bremsstrahlung current is conserved) if all external nucleon legs are on-shell.

### C. Covariant three-dimensional reduction

To calculate any reaction amplitude in a full four-dimensional framework is a daunting numerical task. In practical applications of relativistic reaction theories, therefore, one often employs three-dimensional reductions that eliminate the energy variable from loop integrations in a covariant manner leaving only integrations over the components of three-momenta. Many such reduction schemes can be found in the literature [32–35]. Our results presented below hold true for any reduction scheme that puts both nucleons in loops on their respective energy shells.

For hadronic reactions, the primary technical constraint to be satisfied by any three-dimensional reduction is the preservation of covariance and (relativistic) unitarity. For reactions involving electromagnetic interactions, there is the additional constraint of gauge invariance. This is a non-trivial constraint since the reduction scheme, in general, will destroy gauge invariance as a matter of course. Hence, to restore it, one must introduce additional current mechanisms as part of the reduction prescription for photoprocesses. As we shall see, this cannot be done in a unique manner because gauge invariance does not constrain transverse current contributions.

For the  $NN$  problem, three-dimensional reductions result from replacing the free two-nucleon propagator  $G_0$  by one containing a  $\delta$  function that eliminates the energy integration in loops. In the following, we make the replacement

$$G_0 \rightarrow g_0 \quad (26)$$

to indicate that the internal integration is a three-dimensional one over the three-momentum of the loop. To obtain on-the-energy-shell *integral equations* from the Bethe-Salpeter equations (1) when using this reduction, the external nucleon legs must be taken on shell as well. This provides the reduced integral equations

$$t = v + vg_0t = v + tg_0v, \quad (27)$$

where lower-case letters  $v$  and  $t$  (instead of  $V$  and  $T$ , respectively) signify that all nucleons — internal and external — are on their energy shell. However, when considering below the gauge invariance of the bremsstrahlung current that results from the reduction (26), we require fully off-shell  $T$  matrices. They can be obtained from iterated versions of (1) which is then subjected to the reduction (26), producing

$$T = V + V(g_0 + g_0tg_0)V, \quad (28)$$

where all external nucleons may be considered off-shell. This off-shell  $T$ , thus, is obtained by quadratures from

the integral-equation on-shell solution  $t$ . However, this  $T$  is *not* the same as the solution of the full four-dimensional Bethe-Salpeter scattering equation (1) even though we use the same notation to keep matters simple, even at the risk of inviting confusion. The  $T$ 's appearing in the following context *always* refer to fully off-shell or half off-shell versions of the  $T$  matrix defined by the quadrature formula (28).

The previous proof of gauge invariance of the full four-dimensional formalism given in Sec. IIB shows that gauge invariance depends on an intricate interplay of all hadronic reaction mechanisms. Any approximation, therefore, will in general destroy gauge invariance. Hence, we expect that simply subjecting the full bremsstrahlung current  $B^\mu$  of Eq. (14) to the reduction prescription (26) will *not* retain gauge invariance, and that, therefore, additional steps will be necessary to ensure gauge invariance. Since, from a formal point of view, *any* modification of a given current can, without lack of generality, always be expressed by adding an extra current, we may write the *gauge-invariant* current  $B_r^\mu$  that results from a judicious adaptation of the three-dimensional reduction procedure to  $B^\mu$  of Eq. (14) in the form

$$B^\mu \rightarrow B_r^\mu = \left[ (TG_0 + 1)J^\mu(1 + G_0T) \right]_{\text{red}} + X_{\text{GIP}}^\mu. \quad (29)$$

The first term on the right-hand side,  $[\dots]_{\text{red}}$ , schematically denotes the necessary modifications of the hadronic mechanisms in  $B^\mu$  itself and the last term,  $X_{\text{GIP}}^\mu$ , is the additional gauge-invariance preserving (GIP) current that is to be determined to make  $B_r^\mu$  gauge invariant. In other words, we *demand* that

$$k_\mu B_r^\mu = TG_0\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}G_0T, \quad (30)$$

i.e., that the four-divergence of the reduced current  $B_r^\mu$  remains identical in form to the generalized WTI of Eq. (24) and we are going to ensure this by choosing  $X_{\text{GIP}}^\mu$  accordingly after having determined  $[\dots]_{\text{red}}$  in (29). [We repeat here that — completely consistent with the covariant three-dimensional reduction — the off-shell  $T$ 's of (30), and of the subsequent equations in this section, are those defined by the off-shell extension (28) of (27), and *not* the solutions of the original Bethe-Salpeter equations (1) that appear in (24); see also the discussion surrounding Eq. (28).]

As a first step, we employ the reduction for the external  $G_0$  factors and write

$$B_r^\mu = (Tg_0 + 1)\tilde{J}_r^\mu(1 + g_0T) + X_{\text{GIP}}^\mu, \quad (31)$$

where  $\tilde{J}_r^\mu$  is the reduced form of the basic production current  $J^\mu$  of (11). For its determination, we note that one cannot simply employ the reduction (26) for every  $G_0$  appearing in (11) since this produces unphysical mechanisms. The box-graph contribution

$$b^\mu = VG_0d^\mu G_0V \quad (32)$$

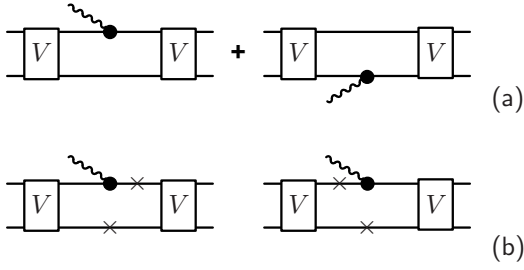


FIG. 4. (a) Sum of box graphs describing Eq. (32), with rectangles labeled  $V$  subsuming all mechanisms of the  $NN$  interaction. All internal nucleon lines are off-shell, in general. (b) The two possibilities of putting nucleons on-shell in the internal loop of the first diagram of (a), as indicated by “ $\times$ ” on the corresponding nucleon line, leaving one nucleon at the photon vertex off shell. Analogous diagrams can be drawn for the corresponding second diagram.

shown in Fig. 4(a), that appears as a subtraction in (12), *cannot* be reduced in the form

$$VG_0d^\mu G_0V \rightarrow Vg_0d^\mu g_0V \quad (\text{unphysical}) \quad (33)$$

since this would have the bremsstrahlung photon emerg-

ing from intermediate on-shell nucleons which is not possible for a physical photon. At least one of the nucleon legs at the photon vertex — either the incoming or the outgoing one — must remain off-shell. The two possibilities of doing that allowing the production of physical photons are

$$VG_0d^\mu G_0V \rightarrow \begin{cases} VG_0d^\mu g_0V, \\ Vg_0d^\mu G_0V, \end{cases} \quad (34)$$

as shown in Fig. 4(b). Since there is nothing that suggests that one choice is to be preferred over the other, we allow for both and thus make the replacement

$$b^\mu \rightarrow b_r^\mu = \lambda_i VG_0d^\mu g_0V + \lambda_f Vg_0d^\mu G_0V, \quad (35)$$

where the constant factors are constrained by

$$\lambda_i + \lambda_f = 1 \quad (36)$$

to avoid double counting. The symmetric choice would be  $\lambda_i = \lambda_f = 1/2$ , of course, but we want to allow here more flexibility for reasons given below.

In detail, the reduced bremsstrahlung current thus reads

$$B_r^\mu = (Tg_0 + 1)[d^\mu G_0V + VG_0d^\mu + V^\mu + D_r^\mu](1 + g_0T) + X_{\text{GIP}}^\mu, \quad (37)$$

where

$$D_r^\mu = T_r^\mu - b_r^\mu \quad (38)$$

is the reduced form of (12), with  $T_r^\mu$  denoting the reduced form of the transverse current  $T^\mu$  satisfying  $k_\mu T_r^\mu = 0$ . Note that the  $G_0$ 's appearing in the  $d^\mu$  terms cannot be reduced to  $g_0$ 's for the same reason that the reduction (33) is not possible. Evaluating the four-divergence of this expression provides

$$\begin{aligned} k_\mu B_r^\mu &= (Tg_0 + 1) \left[ \left( \hat{Q}G_0^{-1} - G_0^{-1}\hat{Q} \right) G_0V + VG_0 \left( \hat{Q}G_0^{-1} - G_0^{-1}\hat{Q} \right) + V\hat{Q} - \hat{Q}V - k_\mu b_r^\mu \right] (1 + g_0T) + k_\mu X_{\text{GIP}}^\mu \\ &= -(Tg_0 + 1)G_0^{-1}\hat{Q}G_0T + TG_0\hat{Q}G_0^{-1}(1 + g_0T) - (Tg_0 + 1)k_\mu b_r^\mu(1 + g_0T) + k_\mu X_{\text{GIP}}^\mu \\ &= TG_0\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}G_0T - (Tg_0 + 1)k_\mu b_r^\mu(1 + g_0T) + k_\mu X_{\text{GIP}}^\mu. \end{aligned} \quad (39)$$

It was used here that

$$TG_0\hat{Q}G_0^{-1}g_0T = 0 \quad \text{and} \quad Tg_0G_0^{-1}\hat{Q}G_0T = 0 \quad (40)$$

vanish identically. For the proof, consider

$$g_0G_0^{-1}\hat{Q}G_0 \rightarrow \frac{\Lambda_1\Lambda_2}{s - (\varepsilon_1 + \varepsilon_2)^2} \circ [t_1^{-1}(p)Q_1t_1(p+k)], \quad (41)$$

where the right-hand side here provides one generic contribution (stripped of all extraneous factors) contained in the expression on the left. The “ $\circ$ ” symbol indicates the remaining three-momentum loop integration. The variable  $s$  is the squared total energy of the system and

the  $\varepsilon_i$  are the individual on-shell energies of the two on-shell nucleons in the loop [indicated by the symbol “ $\times$ ” in Fig. 4(b)]. The  $\Lambda_i$  are the positive-energy projectors of the nucleons. The energy component of  $p = (\varepsilon_1, \mathbf{p})$  is on-shell and thus

$$\Lambda_1(\mathbf{p})t_1^{-1}(p) = \frac{p^2 - m^2}{2m}\Lambda_1(\mathbf{p}) = 0. \quad (42)$$

Because of  $p^2 = m^2$ , this term *always* vanishes for any  $\mathbf{p}$ . Therefore, even if  $s - (\varepsilon_1 + \varepsilon_2)^2$  should vanish as well for some  $\mathbf{p}$ , this cannot be compensated, i.e., the limit of the corresponding  $\frac{0}{0}$  situation is zero. This proves that  $Tg_0G_0^{-1}\hat{Q}G_0T = 0$ ; the proof for  $TG_0\hat{Q}G_0^{-1}g_0T = 0$  follows in a similar fashion.

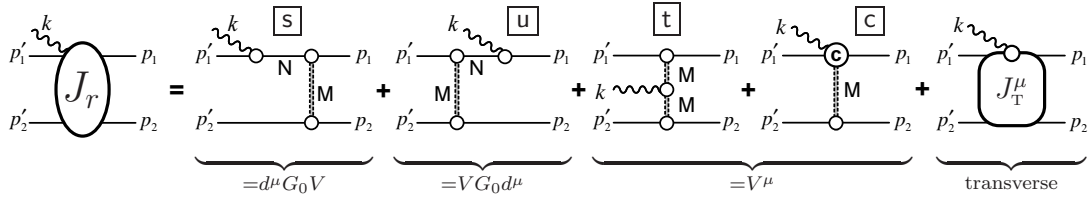


FIG. 5. Reduced basic bremsstrahlung production current  $J_r^\mu$  of Eq. (48) resulting from the covariant three-dimensional reduction (see Sec. II C) for nucleon-nucleon interactions based on single-meson exchanges. Time proceeds from right to left. The mesons  $M$  taken into account in the present work and in Ref. [3] are  $\pi$ ,  $\eta$ ,  $\rho$ ,  $\omega$ ,  $\sigma$ , and  $a_0$  (formerly  $\delta$ ). The nucleonic current corresponds to the first two diagrams on the right-hand side and the meson-exchange current is depicted by the third diagram. The first four diagrams respectively labeled s, u, t, and c correspond to the *complete* gauge-invariant description for the process  $NM \rightarrow N\gamma$  for the upper nucleon line, with the labels s, u, and t alluding to the kinematic situations described by the corresponding Mandelstam variables (see Fig. 6 and the corresponding discussion in Sec. II D). The fourth diagram contains the  $NM \rightarrow N\gamma$  four-point contact current  $M_{\text{int}}^\mu$  discussed in Sec. II E, labeled “c” in the diagram. The correct gauge-invariance treatment of this diagram was found to be crucial in reproducing the KVI data in Ref. [3]. The diagrams corresponding to s, u, and c for the lower nucleon line are suppressed. The last diagram (labeled  $J_T^\mu$ ) subsumes the transverse five-point currents of (46). Transverse transition-current contributions are also subsumed in this diagram; examples are the  $\gamma\rho\pi$ ,  $\gamma\omega\pi$ , and  $\gamma N\Delta$  transition currents depicted in Figs. 3 and 9, respectively. Antisymmetrization of identical nucleons is implied.

The first two terms in the last line of (39) already provide the complete four-divergence (30) necessary for the gauge-invariance condition to hold true. It follows then that the last two terms must vanish,

$$k_\mu X_{\text{GIP}}^\mu - (Tg_0 + 1)k_\mu b_r^\mu(1 + g_0T) \stackrel{!}{=} 0. \quad (43)$$

The four-divergence of  $X_{\text{GIP}}^\mu$  thus is constrained by

$$k_\mu X_{\text{GIP}}^\mu = (Tg_0 + 1)k_\mu b_L^\mu(1 + g_0T), \quad (44)$$

where  $b_L^\mu$  describes the longitudinal pieces of the reduced box-graph current  $b_r^\mu$ . Hence, without lack of generality, we may write

$$X_{\text{GIP}}^\mu + (Tg_0 + 1)D_r^\mu(1 + g_0T) = (Tg_0 + 1)J_T^\mu(1 + g_0T) \quad (45)$$

where  $J_T^\mu$  is the purely *transverse* current

$$J_T^\mu = T_r^\mu - \lambda_i V G_0 d_T^\mu g_0 V - \lambda_f V g_0 d_T^\mu G_0 V; \quad (46)$$

$d_T^\mu$  here only contains the transverse pieces of the nucleon currents as they appear in  $d^\mu$ . As far as gauge invariance of  $B_r^\mu$  is concerned, the current  $J_T^\mu$  is irrelevant. Therefore, the parameters  $\lambda_i$  and  $\lambda_f$  may now also be treated as independent parameters, unconstrained by (36).

To summarize the present results obtained for the three-dimensional reduction, in this approximation the gauge-invariant bremsstrahlung current reads

$$B_r^\mu = (Tg_0 + 1)J_r^\mu(1 + g_0T), \quad (47)$$

where the reduced basic production current is given as

$$J_r^\mu = d^\mu G_0 V + V G_0 d^\mu + V^\mu + J_T^\mu. \quad (48)$$

How one chooses  $J_T^\mu$  in an application is not fixed by the formalism, beyond the generic form given in (46). Note that the generic graphical structure depicted in Fig. 2 remains valid also for  $J_r^\mu$ , with the last graph labeled  $D^\mu$  on

the right-hand side of the figure depicting now the transverse five-point current  $J_T^\mu$ . For the specific case of  $NN$  interactions based on single-meson exchanges only, the reduced basic production current  $J_r^\mu$  of (48) is illustrated diagrammatically in more detail in Fig. 5.

Let us add some remarks here. Even though the procedure to preserve gauge invariance was presented here in terms of an additional *ad hoc* current  $X_{\text{GIP}}^\mu$ , the derivation shows that, rather than adding a current, the application of the three-dimensional reduction procedure to the current  $B^\mu$  really amounts to *dropping* (at least part of) the reduced contribution from  $b^\mu$  that was necessary in the full four-dimensional treatment to prevent double counting of the  $TG_0 d^\mu G_0 T$  contribution. The particular form of  $J_T^\mu$  of (46) follows from exploiting the constraint (44) to the extent to which it is possible since the gauge invariance cannot constrain transverse contributions. Our applications reported here, in Sec. III, and in [3] suggest that the choice  $\lambda_i = \lambda_f = 0$  yields by far the best numerical results. In view of the fact that the double-counted term  $TG_0 d^\mu G_0 T$  in the full four-dimensional formulation of Sec. II A now appears in (47) as two distinctly different contributions  $TG_0 d^\mu g_0 T + Tg_0 d^\mu G_0 T$ , there are now no longer any terms counted doubly and thus the corresponding subtraction is no longer necessary either. The finding that  $\lambda_i = \lambda_f = 0$  yields the best numerical results is completely consistent with this fact. In our calculations, therefore,  $J_T^\mu$  only contains transverse contributions from electromagnetic  $\gamma\rho\pi$ ,  $\gamma\omega\pi$ , and  $\gamma N\Delta$  transitions (for the latter, see discussion in Sec. II F).

#### D. Local gauge invariance: Constraining subprocesses

The gauge-invariance constraints in terms of the generalized Ward-Takahashi identities, either in the form (24) for the full amplitude or as (30) for the reduced



one, are just formal constraints, of course, since the only physically relevant — i.e., measurable — ramification is current conservation when all external nucleons of the bremsstrahlung process are on their respective energy shells. It is for this reason that the current-conservation constraint,

$$k_\mu B^\mu = 0 \quad (\text{external nucleons on-shell}) , \quad (49)$$

called *global* gauge invariance, is the only constraint that is implemented in many reaction models of photoprocesses. We would like to advocate, however, that using this as the sole constraint is not enough if one aims at providing a consistent *microscopic* description of the photoreaction at hand. It was shown in the preceding sections that the gauge invariance of the total bremsstrahlung amplitude hinges in an essential way on each subprocess providing its correct current share to ensure the gauge invariance of the entire current — and thus ultimately provide a conserved current. This is only possible if the current associated with each subprocess satisfies its own generalized Ward-Takahashi identity, as exemplified here by Eq. (16) for the propagators and by Eq. (20) for the interaction current. Thus, imposing *local* gauge invariance, i.e., imposing consistent *off-shell* constraints of this kind for all subprocesses in a microscopic description of the reaction at hand will then automatically ensure that the total amplitude will satisfy a generalized WTI of its own. This not only means that it will indeed satisfy the physical constraint of a conserved current, but beyond that it will ensure that if the process at hand will be used as a subprocess for another, larger reaction, it will *automatically* provide the correct contribution to make the larger process gauge invariant as well.

In this respect, we draw attention to the fact that the reaction dynamics depicted by the first four diagrams on the right-hand side of Fig. 5 correspond to the capture reaction  $NM \rightarrow N\gamma$  where the meson  $M$  is emitted by the spectator nucleon depicted by the lower nucleon line. The corresponding time-reversed equivalent meson-production process is shown in Fig. 6 for the example of pion production. The current amplitude for this process can always be broken down into four generic contributions (see, for example, [23]),

$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{\text{int}}^\mu , \quad (50)$$

where the indices  $s$ ,  $u$ , and  $t$  allude to the Mandelstam variables describing the kinematic situations of the corresponding diagrams in the figure. The corresponding interaction current, in particular, is obtained by attaching the photon to the inner workings of the meson-nucleon-nucleon vertex  $F$  according to

$$M_{\text{int}}^\mu = -\{F\}^\mu . \quad (51)$$

If we demand now local gauge invariance, the current  $M^\mu$  must satisfy an (off-shell) generalized WTI. In addition

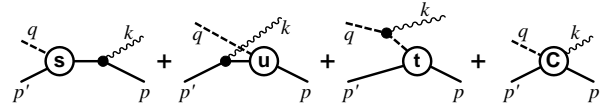


FIG. 6. Generic pion photoproduction diagrams  $\gamma + N \rightarrow \pi + N$ . Time proceeds from right to left. If read from left to right, this corresponds to the pion-capture reaction as it appears in the first four diagrams on the right-hand side of Fig. 5 along the upper nucleon line. The last diagram here labeled “c” corresponds to the interaction current arising from attaching the photon to the interior of the  $\pi NN$  vertex. The properties of this contact-type four-point current  $M_{\text{int}}^\mu$  are essential to render the entire amplitude gauge invariant, with the necessary constraint equation given by (52). At the tree-level for undressed hadrons, this term reduces to the Kroll-Ruderman term.

to the trivial contributions resulting from the propagator WTIs for the currents associated with the external legs of  $M_s^\mu$ ,  $M_u^\mu$ , and  $M_t^\mu$ , similar to (18), this means in particular, that the interaction current must satisfy [23]

$$k_\mu M_{\text{int}}^\mu = Q_N F_u + Q_M F_t - F_s Q_N , \quad (52)$$

where  $F_x$  denotes the meson-nucleon-nucleon vertices  $F$ , with the subscripts  $x = s, u, t$  corresponding to Mandelstam variables of the respective kinematical situation of the vertices in  $s$ -,  $u$ -, and  $t$ -channel contributions, as depicted in Fig. 6.  $Q_M$  and  $Q_N$  are the charge operators of the meson and the nucleon, respectively. Since the charge operators interact with the isospin dependence of the vertex  $F$ , their placement before or after  $F_x$  is significant to ensure charge conservation. Note that Eq. (52) is the exact analog of Eq. (20) for the present application, with Eq. (20) providing the four-divergence for a five-point interaction current for an outgoing photon and Eq. (52) constraining the four-point interaction current for an incoming photon. Using the  $\hat{Q}$  notation introduced in Eq. (17), this may be made more obvious by writing Eq. (52) as

$$k_\mu M_{\text{int}}^\mu = \left( \hat{Q}_N + \hat{Q}_M \right) F - F \hat{Q}_N , \quad (53)$$

where the charge operators of the final hadrons appear on the left of  $F$  and that of the initial hadron on its right, as it is appropriate for the interaction current of any meson-production process  $\gamma N \rightarrow NM$  similar to Fig. 6; for the reverse process  $NM \rightarrow N\gamma$ , one needs to change  $k \rightarrow -k$  and exploit the isospin dependence of the vertex to write  $\hat{Q}_M F = -F \hat{Q}_M$  since the meson changes from being outgoing to incoming.

Demanding consistency of the microscopic dynamics across various reactions means that the entire  $NM \rightarrow N\gamma$  subprocess in Fig. 5 must satisfy the *same* generalized Ward-Takahashi identity as the amplitude  $M^\mu$  itself, except for trivial modifications arising from the fact that this is the time-reversed process. This must be true for any one of the exchanged mesons — whether scalar, pseudoscalar, or vector — in the bremsstrahlung process, not just for the pion example shown in Fig. 6.

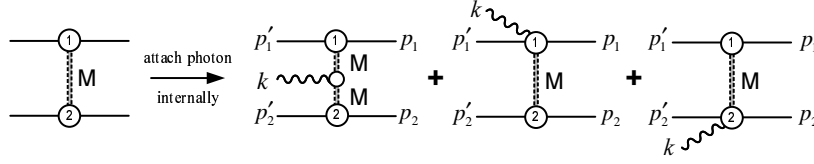


FIG. 7. Generic structure of interaction current  $V_{\text{MEC}}^\mu$ , Eq. (55), for the single-meson exchange contribution  $V_{\text{MEC}}$  of the  $NN$  interaction depicted on the left. Time proceeds from right to left. The three diagrams on the right-hand side comprise the meson-exchange current and the two interaction-current contributions  $M_i^\mu$ ,  $i = 1, 2$ , where the photon interacts with the interiors of the respective vertices for nucleons 1 and 2.

To illustrate in more detail how the requirement of local gauge invariance ties together the various current mechanisms, let us consider a single-meson-exchange contribution  $V_{\text{MEC}}$  to the full  $NN$  potential  $V$ . Generically, we may write

$$V_{\text{MEC}} = F_1 t_M F_2, \quad (54)$$

where the  $F_i$  are the meson-nucleon-nucleon vertices for nucleon  $i = 1, 2$  (including all coupling operators and isospin dependencies) and  $t_M$  describes the propagator for the exchanged meson  $M$ . Graphically, the process is given in the diagram on the left-hand side of Fig. 7. If we now attach an outgoing photon to all internal mechanism of  $V_{\text{MEC}}$ , we obtain the three diagrams on the right-hand side of the figure that comprise the corresponding  $NN$  interaction current,  $V_{\text{MEC}}^\mu = -\{V_{\text{MEC}}\}^\mu$ . This current may be written as

$$\begin{aligned} V_{\text{MEC}}^\mu &= -F_1 \{t_M\}^\mu F_2 - \{F_1\}^\mu t_M F_2 - F_1 t_M \{F_2\}^\mu \\ &= F_1 t_M \Gamma_M^\mu t_M F_2 + M_1^\mu t_M F_2 + F_1 t_M M_2^\mu, \end{aligned} \quad (55)$$

where  $M_i^\mu = -\{F_i\}^\mu$  is the four-point interaction current for the vertex  $i = 1, 2$  and

$$-\{t_M\}^\mu = t_M \Gamma_M^\mu t_M \quad (56)$$

produces the current operator  $\Gamma_M^\mu$  for the exchanged meson that satisfies the single-particle WTI [28, 29]

$$k_\mu \Gamma_M^\mu = \hat{Q}_M t_M^{-1} - t_M^{-1} \hat{Q}_M. \quad (57)$$

Now, assuming without lack of generality that four-momentum and charge flow from vertex 2 to vertex 1 in Fig. 7, the analogs of (53) read

$$k_\mu M_1^\mu = F_1 (\hat{Q}_1 + \hat{Q}_M) - \hat{Q}_1 F_1, \quad (58a)$$

$$k_\mu M_2^\mu = F_2 \hat{Q}_2 - (\hat{Q}_M + \hat{Q}_2) F_2, \quad (58b)$$

and thus

$$\begin{aligned} k_\mu V_{\text{MEC}}^\mu &= F_1 t_M \hat{Q}_M F_2 - F_1 \hat{Q}_M t_M F_2 \\ &\quad + F_1 (\hat{Q}_1 + \hat{Q}_M) t_M F_2 - \hat{Q}_1 F_1 t_M F_2 \\ &\quad + F_1 t_M F_2 \hat{Q}_2 - F_1 t_M (\hat{Q}_M + \hat{Q}_2) F_2 \\ &= F_1 t_M F_2 \hat{Q}_1 - \hat{Q}_1 F_1 t_M F_2 \\ &\quad + F_1 t_M F_2 \hat{Q}_2 - \hat{Q}_2 F_1 t_M F_2 \\ &= V_{\text{MEC}} \hat{Q} - \hat{Q} V_{\text{MEC}} \end{aligned} \quad (59)$$

which is precisely the gauge-invariance constraint (20) for this particular contribution to  $V$ . In this consistent microscopic treatment of all subprocesses, therefore, the constraints *local* gauge invariance places on the four-point interaction currents  $M_{\text{int}}^\mu$  of meson-production processes translate seamlessly into the corresponding constraints on five-point currents of the bremsstrahlung process. The essential step here is to ensure the validity of (58a) for the four-point current in the diagram labeled “c” in Fig. 5 and of (58b) for its counterpart for the lower nucleon line (not shown in Fig. 5). These relations must be true for each of the exchanged mesons.

The problem of how to ensure the validity of the constraints (52) or (58) has been studied extensively by the present authors and their collaborators for the equivalent photoproduction processes, and based on the original ideas presented in Refs. [23, 25], a general prescription was given in Ref. [24] that is applicable just as well for the most general case of explicit final-state interactions with completely dressed hadrons as it is for phenomenological vertex functions. The necessary extension in the context of bremsstrahlung — to account for the virtual nature of the incoming and outgoing nucleons and the exchanged meson at the four-point vertex — is accomplished following the work of Ref. [36].

For our present applications, and those of Ref. [3], based on single-meson exchanges containing phenomenological meson-nucleon-nucleon form factors, the details of the four-point currents  $M_1^\mu$  and  $M_2^\mu$  and of the complete descriptions of the corresponding diagrams depicted in Fig. 7 were already given in Ref. [3]; we will not repeat them here. We emphasize, however, that the inclusion of this interaction-type current, with the correct gauge-invariance dynamics that ensure the validity of Eqs. (58), is essential to bringing about the good quality of our bremsstrahlung results, in particular, the good description of the high-precision KVI data [1, 2] reported in Ref. [3]. This is a novel feature of our approach that resolves a longstanding discrepancy between the data and their theoretical description.

### E. Interaction current $V^\mu$ for phenomenological $NN$ interactions: A systematic gauge-invariance-preserving approximation

The previous discussions show that the gauge-invariance conditions (20) and (58) for the respective interaction currents are of crucial importance to ensure an overall bremsstrahlung amplitude that satisfies gauge invariance. The understanding here is that the detailed reaction mechanisms that go into providing the details of these interaction currents are completely known and that therefore it would be straightforward to make sure that the corresponding gauge-invariance conditions are indeed satisfied. This is generally the case only for  $NN$  interactions based on single-meson exchanges between nucleons *without* any phenomenological form factors. And if one employs phenomenological form factors, one must resort to the prescriptions given in Refs. [23–25, 36] to ensure Eqs. (58), as discussed in the preceding section. However, beyond problems associated with phenomenological form factors, it is conceivable that some details would not be available in some instances either because the microscopic coupling of the photon to the internal dynamics of the interaction would not be feasible or sensible or because it would be too complicated for practical applications. Examples of the former would be  $NN$  interactions based on position-space methods where there are no (momentum-dependent) exchange mechanisms that permit the coupling of a photon in a dynamically meaningful way. Examples of the latter might be baryon contributions beyond the nucleon since they require box-type  $NN$  contributions with intermediate non-nucleonic baryonic contributions that might be too cumbersome to be treated with explicit photon couplings (see Sec. II F for  $N\Delta$  and/or  $\Delta\Delta$  systems and the resulting graphical

structures depicted in Fig. 8).

Nevertheless, each of the corresponding interaction-current contributions *must* satisfy its appropriate gauge-invariance condition otherwise the gauge invariance of the entire amplitude breaks down. We will show here how one can ensure that gauge invariance is preserved for any phenomenological contribution to the  $NN$  interaction by constructing a phenomenological interaction current that satisfies appropriate constraints.

To this end let us assume the total  $NN$  interaction  $V$  can be split up into  $n$  independent contributions,

$$V = V_1 + V_2 + \dots + V_n . \quad (60)$$

Formally coupling a photon to each contribution according to  $V_i^\mu = -\{V_i\}$ , the total interaction current  $V^\mu$  then breaks down accordingly into  $n$  independent contributions,

$$V^\mu = V_1^\mu + V_2^\mu + \dots + V_n^\mu \quad (61)$$

which, because of their independence, must each *separately* satisfy a gauge-invariance condition similar to (20), i.e.,

$$k_\mu V_i^\mu = V_i \hat{Q} - \hat{Q} V_i , \quad i = 1, 2, \dots, n , \quad (62)$$

otherwise the total current  $V^\mu$  could not satisfy (20).

Let us assume now that one of the  $V_i$  is such that the construction  $V_i^\mu = -\{V\}^\mu$  is *not* readily available. In this case, we may devise an auxiliary phenomenological current  $\tilde{V}_i^\mu$  instead that satisfies exactly the same four-divergence relation (62) as it would have to be satisfied by  $V_i^\mu$  if it were available. To this end, we adapt the procedure used successfully in Refs. [23–25] for the four-point interaction currents  $M_{\text{int}}^\mu$  of meson production to the present five-point-current case and make the ansatz

$$\begin{aligned} V_i^\mu \rightarrow \tilde{V}_i^\mu = & - \left[ V_i(p'_1, p'_2; p_1 - k, p_2) - W_i(k, p'_1, p'_2; p_1, p_2) \right] \frac{Q_1(2p_1 - k)^\mu}{(p_1 - k)^2 - p_1^2} \\ & - \left[ V_i(p'_1, p'_2; p_1, p_2 - k) - W_i(k, p'_1, p'_2; p_1, p_2) \right] \frac{Q_2(2p_2 - k)^\mu}{(p_2 - k)^2 - p_2^2} \\ & - \frac{Q_1(2p'_1 + k)^\mu}{(p'_1 + k)^2 - p_1'^2} \left[ V_i(p'_1 + k, p'_2; p_1, p_2) - W_i(k, p'_1, p'_2; p_1, p_2) \right] \\ & - \frac{Q_2(2p'_2 + k)^\mu}{(p'_2 + k)^2 - p_2'^2} \left[ V_i(p'_1, p'_2 + k; p_1, p_2) - W_i(k, p'_1, p'_2; p_1, p_2) \right] , \end{aligned} \quad (63)$$

where  $W_i$  is a function to be chosen to ensure that each term here is free of propagator singularities. The four-divergence of this auxiliary current is then readily seen to produce indeed

$$k_\mu \tilde{V}_i^\mu = V_i \hat{Q} - \hat{Q} V_i \quad (64)$$

since

$$W_i(k, p'_1, p'_2; p_1, p_2) (Q_1 + Q_2)_{\text{initial}} - (Q_1 + Q_2)_{\text{final}} W_i(k, p'_1, p'_2; p_1, p_2) = 0 \quad (65)$$

vanishes because of charge conservation. The ansatz (63) thus preserves the gauge-invariance condition for the entire interaction. As to how to choose  $W_i$ , one of the simplest possibilities is

$$W_i(k, p'_1, p'_2; p_1, p_2) = V_i(p'_1, p'_2; p_1, p_2) + [(p_1 - k)^2 - p_1^2] [(p_2 - k)^2 - p_2^2] [(p'_1 + k)^2 - p_1'^2] [(p'_2 + k)^2 - p_2'^2] R_i(k, p'_1, p'_2; p_1, p_2), \quad (66)$$

where  $R_i(k, p'_1, p'_2; p_1, p_2)$ , except for symmetry constraints, is largely arbitrary (and may be equal to zero). This choice ensures that the resulting GIP current is free of kinematic singularities, as it must be. In detail, one thus has

$$\begin{aligned} \tilde{V}_i^\mu = & -\frac{V_i(p'_1, p'_2; p_1 - k, p_2) - V_i(p'_1, p'_2; p_1, p_2)}{(p_1 - k)^2 - p_1^2} Q_1(2p_1 - k)^\mu - \frac{V_i(p'_1, p'_2; p_1, p_2 - k) - V_i(p'_1, p'_2; p_1, p_2)}{(p_2 - k)^2 - p_2^2} Q_2(2p_2 - k)^\mu \\ & - Q_1(2p'_1 + k)^\mu \frac{V_i(p'_1 + k, p'_2; p_1, p_2) - V_i(p'_1, p'_2; p_1, p_2)}{(p'_1 + k)^2 - p_1'^2} - Q_2(2p'_2 + k)^\mu \frac{V_i(p'_1, p'_2 + k; p_1, p_2) - V_i(p'_1, p'_2; p_1, p_2)}{(p'_2 + k)^2 - p_2'^2} \\ & + R_i(k, p'_1, p'_2; p_1, p_2) [(p'_1 + k)^2 - p_1'^2] [(p'_2 + k)^2 - p_2'^2] \\ & \times \left\{ [(p_2 - k)^2 - p_2^2] Q_1(2p_1 - k)^\mu + [(p_1 - k)^2 - p_1^2] Q_2(2p_2 - k)^\mu \right\} \\ & + \left\{ Q_1(2p'_1 + k)^\mu [(p'_2 - k)^2 - p_2'^2] + Q_2(2p'_2 + k)^\mu [(p'_1 - k)^2 - p_1'^2] \right\} \\ & \times [(p_1 - k)^2 - p_1^2] [(p_2 - k)^2 - p_2^2] R_i(k, p'_1, p'_2; p_1, p_2). \end{aligned} \quad (67)$$

Note that the subtracted potential contribution  $V_i(p'_1, p'_2; p_1, p_2)$  is unphysical since  $p'_1 + p'_2 \neq p_1 + p_2$ .

Hence, the procedure just outlined allows a systematic hybrid treatment of all independent contributions  $V_i$  to the full interaction  $V$  where some five-point currents  $V_i^\mu$  can be treated explicitly and some may be replaced by auxiliary currents  $\tilde{V}_i^\mu$  according to (63). The total interaction current  $V^\mu$  will satisfy the condition (20) as a matter of course and gauge invariance is not at issue.

#### F. Extension to coupled channels: $NN$ , $N\Delta$ , and $\Delta\Delta$

The derivation of the bremsstrahlung current in Sec. II A is completely generic and will remain true regardless of the actual mechanisms taken into account in the nucleon-nucleon interaction  $V$  that drives the Bethe-Salpeter equation (1). The current mechanisms depicted in Fig. 5 assume an interaction based on single-meson exchanges between nucleons only. Here we briefly discuss the necessary modifications if such exchanges involve transitions between different baryonic states. We limit the discussion to transitions between the nucleon  $N$  and the  $\Delta(1232)$  mediated by single-meson exchanges, i.e., we consider the effect of coupling the channels  $NN$ ,  $N\Delta$ , and  $\Delta\Delta$ ; transitions into other (resonant) baryonic states can be treated along the same lines.

It is a very simple and straightforward exercise to de-couple the corresponding set of Bethe-Salpeter equations

that couple the  $NN$ ,  $N\Delta$ , and  $\Delta\Delta$  channels and write the  $NN$  interaction appropriate for the *single*-channel Bethe-Salpeter equation (1) as

$$V = V_{\text{MEC}} + V_\Delta. \quad (68)$$

The first term,  $V_{\text{MEC}}$ , describes single-meson exchanges between nucleons, as shown on the left-hand side of Fig. 7, that provide the current mechanisms depicted on the right-hand side of the figure. The second term,  $V_\Delta$ , contains all intermediate  $N\Delta$  or  $\Delta\Delta$  contributions and their transitions. Using the notations

$$U_N: \text{ meson-exchange transition } NN \rightarrow N\Delta \quad (69a)$$

and

$$U_\Delta: \text{ meson-exchange transition } NN \rightarrow \Delta\Delta \quad (69b)$$

for the transition interactions that mediate the coupling to the primary  $NN$  channel, we obtain

$$\begin{aligned} V_\Delta = & U_N^\dagger (G_N^\Delta + G_N^\Delta T_{N,N}^\Delta G_N^\Delta) U_N + U_\Delta^\dagger G_\Delta^\Delta T_{\Delta,N}^\Delta G_N^\Delta U_N \\ & + U_\Delta^\dagger (G_\Delta^\Delta + G_\Delta^\Delta T_{\Delta,\Delta}^\Delta G_\Delta^\Delta) U_\Delta + U_N^\dagger G_N^\Delta T_{N,\Delta}^\Delta G_\Delta^\Delta U_\Delta \end{aligned} \quad (70)$$

where  $G_N^\Delta$  and  $G_\Delta^\Delta$  describe the intermediate propagation of the  $\Delta N$  and the  $\Delta\Delta$  systems, respectively. Intermediate transitions  $\Delta N \rightarrow \Delta N$ ,  $\Delta N \rightarrow \Delta\Delta$ ,  $\Delta\Delta \rightarrow \Delta N$  and  $\Delta\Delta \rightarrow \Delta\Delta$  are subsumed in the respective  $T$  matrices  $T_{N,N}^\Delta$ ,  $T_{\Delta,N}^\Delta$ ,  $T_{N,\Delta}^\Delta$ , and  $T_{\Delta,\Delta}^\Delta$ .

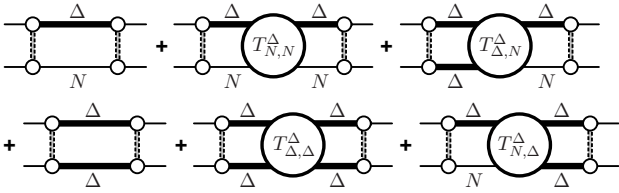


FIG. 8. Contributions to the  $NN$  interaction involving intermediate  $N\Delta$  and  $\Delta\Delta$  systems and intermediate transitions  $T_{fi}^\Delta$  between such systems as given by  $V_\Delta$  in Eq. (70). Summations over all possible exchanged mesons that mediate transitions to the  $\Delta$  state are implied. Here, we take into account  $\pi$  and  $\rho$  exchanges. Antisymmetrization of external nucleons is implied.

Figure 8 provides a graphical representation of  $V_\Delta$ . Coupling a photon to each of the mechanisms depicted here results in the interaction current

$$V_\Delta^\mu = -\{V_\Delta\}^\mu \quad (71)$$

which, together with the interaction current  $V_{\text{MEC}}^\mu$  depicted generically in Fig. 7, constitutes the total interaction current

$$V^\mu = V_{\text{MEC}}^\mu + V_\Delta^\mu \quad (72)$$

(if nucleons and  $\Delta$ 's are the only baryon degrees of freedom considered). Of course, calculating  $V_\Delta^\mu$  explicitly is a formidable task. There are eight current contributions for each of the simple box graphs (where the photon can couple to each of the four intermediate hadrons and the four vertices) and eleven for each of the graphs involving an intermediate  $T$  matrix. Considering only  $\pi$  and  $\rho$  exchanges for these graphs involving the  $\Delta$  and, allowing for the four possibilities of exchanging these mesons, altogether, therefore, there are 240 current terms. This is *without* separately accounting for the internal mechanisms resulting from coupling the photon to the  $T$  matrices with  $\Delta$  degrees of freedom which is similar in complexity to the  $NN$  bremsstrahlung current itself. In view of this complexity, we forego drawing the corresponding diagrams for  $V_\Delta^\mu$ .

Each set of currents resulting from one graph in Fig. 8 corresponds to an *independent* interaction current, as discussed in conjunction with (61), and therefore must satisfy the gauge-invariance condition (62) independently. If an exact treatment of the corresponding interaction current is not feasible in view of the complexity of the problem, from the point of view of gauge invariance this may also be done in an approximate manner by constructing an auxiliary current for each of the graphs in Fig. 8 along the lines discussed in the preceding Sec. II E.

In addition to the hadronic transitions into intermediate  $\Delta$  states, there are also direct electromagnetic transitions  $\gamma N \rightarrow \Delta$ . Such contributions *cannot* be obtained by applying the gauge-derivative method used in Sec. II A, or by any other method based on minimal substitution. They must be added by hand, in terms of their

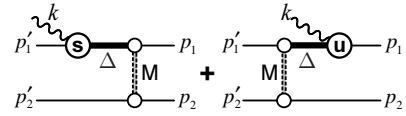


FIG. 9. Lowest-order electromagnetic  $\gamma N\Delta$ -transition contribution to  $J_\Delta^\mu$  of Eq. (73); the exchanged mesons are  $M = \pi, \rho$ . The full contribution involves replacing the single-meson exchanges in the two graphs by the  $T$ -matrix representing the sum of all two-nucleon irreducible transitions  $NN \rightarrow \Delta N$  and  $\Delta N \rightarrow NN$ , respectively. The transition currents are transverse and thus have no bearing on gauge invariance. Antisymmetrization of nucleons is implied.

own Lagrangian [see Appendix, Eq. (A.1c)]. Their full contribution to the basic production current  $J^\mu$  of (11) may be written as

$$J_\Delta^\mu = d_{N\Delta}^\mu G_N^\Delta T_{\Delta N, NN} + T_{NN, \Delta N} G_N^\Delta d_{\Delta N}^\mu, \quad (73)$$

where  $T_{\Delta N, NN}$  and  $T_{NN, \Delta N}$  are the  $T$ -matrices resulting from summing all two-nucleon irreducible transitions  $NN \rightarrow \Delta N$  and  $\Delta N \rightarrow NN$ , respectively;  $d_{N\Delta}^\mu$  and  $d_{\Delta N}^\mu$ , in an obvious schematic notation borrowed from the  $NN$  contributions, contains the electromagnetic transition current  $\Delta \rightarrow N$  and  $N \rightarrow \Delta$ , respectively, along the baryon lines that emit the photon, as depicted in Fig. 9 for the lowest-order single-meson exchange contributions to the respective  $T$ -matrices. These contributions are manifestly transverse and hence have no impact on gauge invariance. Graphically, they may be subsumed in the right-most diagrams of Figs. 2 or 5, depending on whether one considers the full formalism or its three-dimensional reduction, respectively. In other words,  $J_\Delta^\mu$  given above is part of the transverse current  $T^\mu$ , as anticipated already in Eq. (15).

In summary, the full basic interaction current for  $NN$  bremsstrahlung, including  $\Delta$  degrees of freedom, becomes

$$J^\mu = d^\mu G_0 V + V G_0 d^\mu + V^\mu + [J_M^\mu + J_\Delta^\mu - V G_0 d^\mu G_0 V], \quad (74)$$

where the groupings of the four terms corresponds to the four graphs on the right-hand side of Fig. 2. Here,  $V$  and  $V^\mu$  contain  $\Delta$  degrees of freedom according to Eqs. (68) and (72), respectively, and the transverse current is taken as  $T^\mu = J_M^\mu + J_\Delta^\mu$ . In the reduced case, we have

$$J_r^\mu = d^\mu G_0 V + V G_0 d^\mu + V^\mu + [J_M^\mu + J_{r\Delta}^\mu], \quad (75)$$

where the last grouping is the explicit transverse current  $J_r^\mu = J_M^\mu + J_{r\Delta}^\mu$ , with  $\lambda_i$  and  $\lambda_f$  of Eq. (46) put to zero; the reduced  $N\Delta$ -transition current  $J_{r\Delta}^\mu$  is obtained by the corresponding three-dimensional reductions of the loop integrations within  $J_\Delta^\mu$ .

In our present results (discussed in the next section), in view of their complexity, we do not take into account any currents that account for the box-graph mechanisms of Fig. 8, i.e.,  $V_\Delta^\mu$  is set to zero. However, we do take into account the transition-current contributions (73) in lowest order, as depicted in Fig. 9.

### III. APPLICATIONS

The present formalism was applied successfully [3] in the description of the high-precision KVI data [1] at a proton incident energy of 190 MeV. In this case, in view of the relatively low energy, the dominant current contribution is expected to be essentially nucleonic. We felt justified, therefore, to restrict ourselves to the  $NN$  channel only and neglect the effects of coupling to other baryon-baryon channels. Here, we apply the formalism to some selected data sets from TRIUMF [26] at a higher incident energy of 280 MeV. This energy is just above the pion-production threshold energy and still well below the  $\Delta(1232)$  resonance peak energy region of about 650 MeV. Nevertheless, an earlier analysis of Ref. [18] within a coupled-channel approach (employing  $NN$  and  $N\Delta$  channels) indicated that incorporating  $\Delta$  contributions leads to better agreement with the data at certain geometries. For the TRIUMF experiment at 280 MeV, in particular, the dominant  $\Delta$  contribution was found to be arising from the  $\gamma N\Delta$  transition current,  $J_{r\Delta}^\mu$ , in lowest order [18] (a similar result was found in the single-channel approach of Ref. [16]). We point out, however, that the approach reported in Ref. [18] is not gauge invariant beyond the soft photon approximation. Since here we wish to preserve full gauge invariance, we need to follow the procedure outlined in Sec. IIF for the incorporation of  $\Delta$  degrees of freedom. In general, however, this is too challenging technically at present, *except* for the lowest order of the transition-current contributions of Eq. (73) depicted in Fig. 9 which we do take into account in the results presented below. Incorporation of the interaction current  $V_\Delta^\mu$  arising from the  $\Delta$ -box-diagram contributions of Fig. 8 is beyond the scope of the present paper and we will leave this for future studies.

The details of the present model for constructing the production current  $J^\mu$  as well as the  $NN$  interaction are given in the Appendix.

Figure 10 shows a comparison of the present results for the cross sections with the TRIUMF data in the coplanar geometry at small proton scattering angles of  $\theta_1 = 12^\circ$  and  $\theta_2 = 14^\circ$  and at larger proton scattering angles of  $\theta_1 = 28^\circ$  and  $\theta_2 = 27.8^\circ$ . We mention that in the present work, we follow the convention of Ref. [1, 2] for assigning the nucleon angles  $\theta_1$  and  $\theta_2$  which is just opposite to the one adopted in other studies [12, 16, 18]. Also, no arbitrary normalization factor of  $2/3$  to the data points [26] is applied here. The parameters  $h$  and  $a$  of the phenomenological four-point contact current determined in Ref. [3] in the description of the KVI data are kept at the same values,  $h = 2.5$  and  $a = 1000$ , despite the fact that they had been determined at the much lower incident energy of 190 MeV. In principle, of course, these parameters may be functions of the incident energy. We see that the agreement is very reasonable. The influence of the  $\Delta$ -resonance contribution is relatively small but significant and definitely helps to improve the agreement with the data. This is in line with the earlier findings

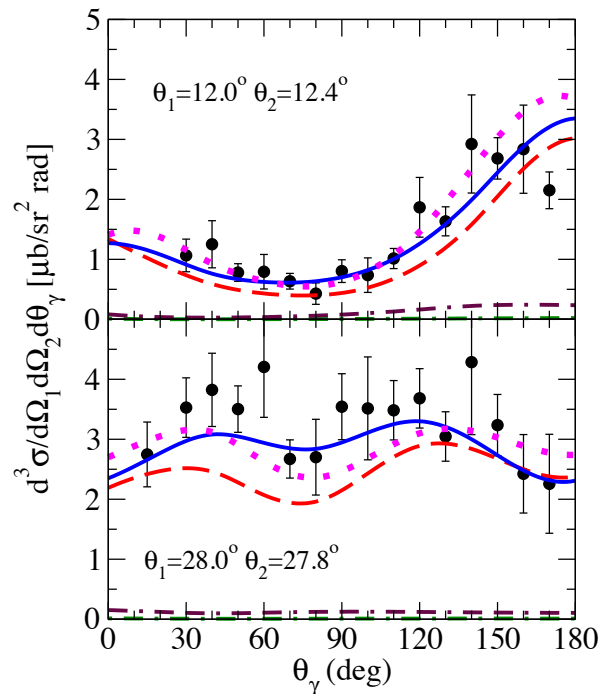


FIG. 10. (Color online) Comparison of the present prediction for the cross sections with the TRIUMF data [26] in coplanar geometry at 280-MeV proton incident energy.  $\theta_1$  and  $\theta_2$  denote the fixed scattering angles of the two protons in the final state. The cross sections are shown as functions of the emitted photon angle,  $\theta_\gamma$ , in the laboratory frame. The (blue) solid curves show the results with the total current  $J_r^\mu$  of Eq. (75). The (red) dashed curves correspond to the results when the transition current,  $T_r^\mu = J_M^\mu + J_{r\Delta}^\mu$ , is switched off in its entirety. The transition-current contributions are shown separately as (green) dash-dotted curves (which are hardly visible in the graphs) for the meson transitions ( $\gamma\pi\rho$  and  $\gamma\pi\omega$ ) subsumed in  $J_M^\mu$  and as (brown) dot-double-dashed curves for the  $\gamma N\Delta$  transitions that provide the lowest-order contributions of  $J_{r\Delta}^\mu$ . The (magenta) dotted curves correspond to the results when the generalized four-point contact current (cf. Fig. 5) is switched off.

of Refs. [16, 18]. Furthermore, the present results are of comparable quality to the earlier results [16, 18] in reproducing the TRIUMF data, indicating that, unlike for the high-precision KVI data at small proton scattering angles [1, 2], the generalized four-point contact current — required to maintain the gauge invariance of the full bremsstrahlung amplitude — is apparently not that critical. The dotted curves in Fig. 10 correspond to the results when the generalized four-point contact current is switched off. As one can see, its effect is visible but somewhat smaller than the effect of the  $\Delta$  contribution. Overall, however, apart from the differences in reaction energies and scattering-angle configurations, we note that the TRIUMF data are much less precise than the KVI data, thus making the assessment of how relevant various theoretical contributions are in reproducing the data much less conclusive.

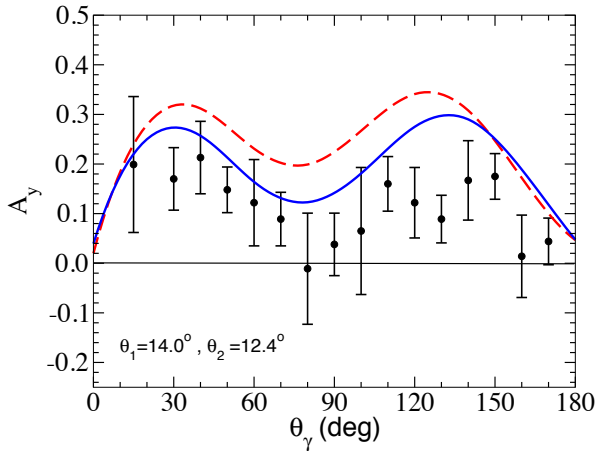


FIG. 11. (Color online) Comparison of the present prediction for the analyzing power with the TRIUMF data [26] in coplanar geometry at 280-MeV proton incident energy.  $\theta_1$  and  $\theta_2$  denote the fixed scattering angles of the two protons in the final state. The analyzing power is shown as a function of the emitted photon angle,  $\theta_\gamma$ , in the laboratory frame. The (blue) solid line is obtained with the total current  $J_r^\mu$  of Eq. (75). The (red) dashed curve corresponds to the result when the  $\gamma N\Delta$  transition current,  $J_{r,\Delta}^\mu$ , is switched off.

In Fig. 11, we show our results for the analyzing power together with the TRIUMF data at  $\theta_1 = 12^\circ$  and  $\theta_2 = 12.4^\circ$ . The agreement is reasonable given the uncertainties in the data. Again, the  $\Delta$ -resonance current helps improve the agreement with the data.

In summary, therefore, our results for the TRIUMF data at a proton incident energy of 280 MeV are in agreement with the findings of the previous studies [16, 18]. We point out, however, that our results are obtained within a fully gauge-invariant framework. Moreover, one may hope that inclusion of the  $\Delta$ -box contributions may further improve the agreement.

#### IV. SUMMARY

We have presented a complete, rigorous formulation of the  $NN$  bremsstrahlung reaction based on a relativistic field-theory approach in which the photon is coupled in all possible ways to the underlying two-nucleon  $T$ -matrix obtained from the corresponding covariant Bethe-Salpeter-type  $NN$  scattering equation using the gauge-derivative procedure of Haberzettl [23]. The resulting bremsstrahlung amplitude is unitary as a matter of course and it satisfies full local gauge invariance as dictated by the generalized Ward-Takahashi identity. The novel feature of this approach is the consistent — i.e., gauge-invariant — incorporation of interaction currents resulting from the photon coupling internally to interacting hadronic systems.

The formalism is quite readily adapted to approximations and thus can be applied even in cases where the

microscopic dynamical structure of the underlying interacting hadronic systems is either not known in detail or too complex to be treated in detail. We have pointed out how the interaction currents resulting from the photon being attached to nucleon-nucleon-meson vertices can be treated by phenomenological four-point contact currents that preserve gauge invariance following the approach of Haberzettl, Nakayama, and Krewald [24]. In an advance application of the present formalism [3], such interaction currents had been shown to contribute significantly to reproducing the high-precision proton-proton bremsstrahlung data at 190 MeV obtained at KVI [1], thus removing a longstanding discrepancy between theory and experiment. In addition, we have provided a scheme that permits the approximate treatment of current contributions resulting from pieces of the  $NN$  interaction that cannot be incorporated exactly. In each case, the approximation procedure ensures gauge invariance of the entire bremsstrahlung amplitude.

We have also discussed the necessary modifications when taking into account baryonic states other than the nucleon  $N$ ; in detail, we consider the  $\Delta(1232)$  resonance by incorporating the couplings of the  $NN$  to the  $N\Delta$  and  $\Delta\Delta$  systems, and the  $\gamma \rightarrow N\Delta$  transitions.

The formalism has been applied to the 280-MeV bremsstrahlung data from TRIUMF [26] and we find good agreement with the data similar to what was obtained by other authors. We emphasize, however, that our results obey full gauge invariance, whereas previous approaches provide a conserved current only in the soft-photon approximation. Since the gauge-invariant incorporation of intermediate  $N\Delta$  and  $\Delta\Delta$  configurations is too demanding technically at present, the only  $\Delta$  degrees of freedom employed in this calculation have been the  $\gamma N\Delta$ -transition currents.

Finally, we emphasize that despite its completeness and generality, the present approach is quite flexible and amenable to approximations, as discussed in Secs. IID and IIE. We expect, therefore, that it will also be useful in the on-going investigation of hard bremsstrahlung [4, 5], as well as in di-lepton production processes [7] where the photon is virtual.

#### Appendix

For the present application, we construct the basic production current  $J^\mu$  in the framework of a covariant three-dimensional Blankenbecler-Sugar reduction [32] (see also Ref. [33]), as described in Ref. [3], consistent with the  $NN$  FSI and ISI based on the OBEP-B version of the Bonn potential [37]. In addition to the resulting production current  $J_r^\mu$  used already in Ref. [3], the present work includes the  $\Delta$ -resonance current interaction calculated in terms of the following Lagrangians:

$$\mathcal{L}_{\Delta N\pi} = \frac{g_{\Delta N\pi}}{M_\pi} \bar{\Delta}^\mu \vec{T}^\dagger \cdot (\partial_\mu \vec{\pi}) N + \text{H. c.}, \quad (\text{A.1a})$$

$$\begin{aligned} \mathcal{L}_{\Delta N\rho} = & -i \frac{g_{\Delta N\rho}^{(1)}}{2M_N} \bar{\Delta}^\mu \gamma^\nu \gamma_5 \vec{T}^\dagger \cdot \vec{\rho}_{\mu\nu} N \\ & + \frac{g_{\Delta N\rho}^{(2)}}{4M_N^2} \bar{\Delta}^\mu \gamma_5 \vec{T}^\dagger \cdot \vec{\rho}_{\mu\nu} \partial^\nu N \\ & - \frac{g_{\Delta N\rho}^{(3)}}{4M_N^2} \bar{\Delta}^\mu \gamma_5 \vec{T}^\dagger \cdot (\partial^\nu \vec{\rho}_{\mu\nu}) N + \text{H. c.}, \quad (\text{A.1b}) \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\Delta N\gamma} = & -ie \frac{g_{\Delta N\gamma}^{(1)}}{2M_N} \bar{\Delta}^\mu \gamma^\nu \gamma_5 T_3^\dagger F_{\mu\nu} N \\ & + e \frac{g_{\Delta N\gamma}^{(2)}}{4M_N^2} \bar{\Delta}^\mu \gamma_5 T_3^\dagger F_{\mu\nu} \partial^\nu N + \text{H. c.}, \quad (\text{A.1c}) \end{aligned}$$

where  $\vec{\rho}_{\mu\nu} \equiv \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$  and  $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ . The coupling constants are taken to be  $g_{\Delta N\pi} = 2.12$  and  $g_{\Delta N\rho}^{(1)} = 40.5$  [38]; for simplicity,  $g_{\Delta N\rho}^{(2)}$  and  $g_{\Delta N\rho}^{(3)}$  are set to zero. The electromagnetic coupling constants  $g_{\Delta N\gamma}^{(1)} = 5.00$  and  $g_{\Delta N\gamma}^{(2)} = -4.73$  are extracted from the corresponding helicity amplitudes [39]. The  $NN\pi$  and  $NN\rho$  vertices are derived from the corresponding Lagrangians used for constructing the nucleonic current in

Ref. [3].

Also for simplicity, the  $\Delta N\pi$  and  $\Delta N\rho$  vertices are provided with the same form factors as in the  $NN\pi$  and  $NN\rho$  vertices [3] for the off-shell meson and baryon.

For the  $\Delta$  propagator, we use

$$\begin{aligned} S^{\mu\nu}(p) = & \frac{\not{p} + M_\Delta}{p^2 - M_\Delta + i\frac{\Gamma_\Delta}{2}} \\ & \times \left( -g^{\mu\nu} - \frac{1}{3M_\Delta} (p^\mu \gamma^\nu - p^\nu \gamma^\mu) \right. \\ & \left. + \frac{1}{3} \gamma^\mu \gamma^\nu + \frac{2}{3M_\Delta^2} p^\mu p^\nu \right), \quad (\text{A.2}) \end{aligned}$$

where  $M_\Delta = 1232$  MeV and  $\Gamma_\Delta = 120$  MeV.

## ACKNOWLEDGMENTS

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