Short range correlations and wave function factorization in light and finite nuclei

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Abstract Recent BNL and Jlab data provided new evidence on two nucleon correlations (2NC) in nuclei. The data confirm the validity of the convolution model, describing the spectral function (SF) of a correlated pair moving in the mean field with high and low relative and center-of-mass (cm) momenta, respectively. The model is built assuming that the wave function (WF) of a nucleus A, describing a configuration where the cm momentum of a correlated pair is low and its relative momentum is high, factorizes into the product of the two-body WF and that of the A-2 system. Such a factorization has been shown to occur in nuclear matter (NM). Here it is shown that few-body systems exhibit factorization, which seems to be therefore a general property, to be reproduced also in studies of the WF of finite nuclei.

A new generation of semi-inclusive and exclusive electron scattering experiments off nuclei is accounting for 2NC with unprecedented accuracy (see, e.g., Ref. [1]). This activity could have important implications for studies of cold dense nuclear matter. The nuclear SF is an important quantity for studies of 2NC. For a nucleon with momentum **k** and removal energy E in a nucleus A, the SF is defined as follows

$$P(k,E) \equiv \frac{1}{2J_0+1} \sum_{M_0\sigma} \langle \Psi_A^0 | a_{\mathbf{k},\sigma}^{\dagger} \, \delta[E-(H-E_A)] \, a_{\mathbf{k},\sigma} | \Psi_A^0 \rangle$$

$$\equiv \frac{1}{2J_0+1} \sum_{M_0\sigma} \sum_f \left| \langle \Psi_{A-1}^f | a_{\mathbf{k},\sigma} \left| \Psi_A^0 \rangle \right|^2 \, \delta[E-(E_{A-1}^f - E_A)] \quad (1)$$

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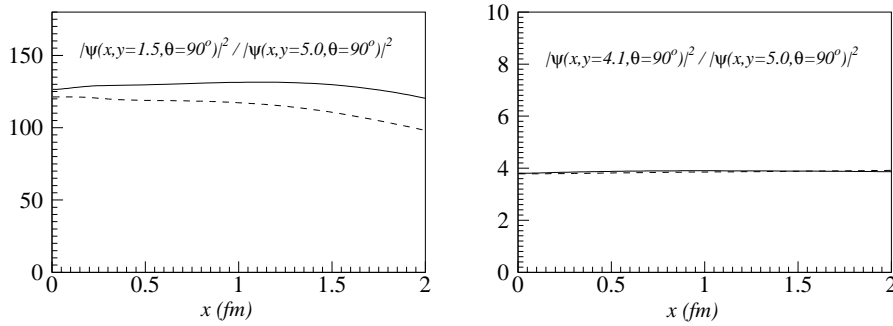Sapporo Gakuin University, Japan E-mail: hiko@earth.sgu.ac.jp

Fig. 1 The ratio Eq. (5) is shown for small values of x, for $\theta_{\widehat{\mathbf{xy}}} = 90$ ^{o} and for $y' = 5$ fm. Left panel: $y = 1.5$ fm (far from the 2NC region); right panel: $y = 4$ fm (in the 2NC region) dashed line: the same ratio evaluated assuming that the cm motion is in S wave.

It is clear from this definition that both the exact wave function (WF) of the ground state and the continuum WF of the $A - 1$ system are necessary to evaluate the SF. These WF are very difficult to calulate, and models of the SF can be very helpful. 2NC are 2-body properties, while $P(k, E)$ is a 1-body quantity. Anyway, in the so called 2NC model [2], it is argued that, at high values of k and E, $P(k, E)$ is dominated by ground-state configurations where the high momentum of one nucleun, $\mathbf{k}_1 \equiv \mathbf{k}$, is entirely balanced by that of another one, $\mathbf{k}_2 \simeq -\mathbf{k}$, while the remaining $(A - 2)$ system has a momentum $\mathbf{k}_{A-2} \simeq 0$ [2]. This yields

$$P(k, E) \simeq P_{2NC}(k, E) = \frac{1}{4\pi} n(k) \; \delta[E - E^{(2NC)}(k)] \tag{2}$$

where $n(k)$ is the nucleon momentum distribution and energy conservation, $E^*_{A-1} + \frac{k^2}{2M_{A-1}} = \frac{k^2}{2M}$, fixes $E^{(2NC)}(k) = |E_A| - |E_{A-2}| + \frac{A-2}{A-1} \frac{k^2}{2M}$. For these values of E, at high values of k, the exact nuclear SF of NM and ^{3}He show indeed a maximum, supporting the idea of the realization of a 2NC-dominated configuration. The convolution model of Ref. [3] is a refined version of the 2NC one. At high values of k and E, the two correlated particles move in the mean field created by the slow $(A - 2)$ system. To model the SF, approximations are needed for the WF of the ground state and that of the $(A - 1)$ system. As for the ground state WF, taking the motion of the cm of the pair in S wave $\chi_0(\mathbf{y})$, and the $A - 2$ system in a low excitation state, $\Psi^0_{A-2}(\{\mathbf{r}_i\}_{A-2})$, yields the factorized expression

$$\Psi^0_A(\{\mathbf{r}_i\}_A) \simeq \hat{A} \left\{ \chi_0(\mathbf{y}) \; \left[\Phi(\mathbf{x}) \otimes \Psi^{\bar{0}}_{A-2}(\{\mathbf{r}_i\}_{A-2}) \right] \right\} \tag{3}$$

As for the $A - 1$ states, the interaction between the $A - 2$ system and the correlated particles is neglected. Using these arguments, the spectral function turns out to be given by a convolution

$$P_{FNC}(k, E) = \int d\mathbf{k}_{cm} \; \delta \left[E - |E_A| + |E_{A-2}| - \frac{(A-2)}{2M(A-1)} \left(\mathbf{k} + \frac{(A-1)\mathbf{K}_{cm}}{(A-2)} \right)^2 \right]$$
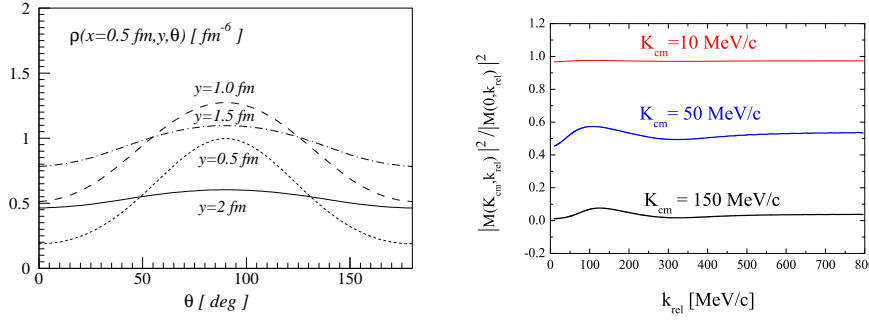$$\cdot \; n_{rel} \left(\left| \mathbf{k} + \frac{\mathbf{K}_{cm}}{2} \right| \right) \; n_{cm}(|\mathbf{K}_{cm}|) \; , \tag{4}$$

Fig. 2 Left panel: the quantity $\rho(x, y, \cos\theta_{\widehat{\mathbf{xy}}})$ (see text), for $x = 0.5$ fm, for four different values of y, as a function of $\theta_{\widehat{\mathbf{xy}}}$. Right panel: the ratio Eq. (9), as a function of k_{rel}, for three different values of $\mathbf{K}_{cm}$.

where the argument of the δ function has been obtained from energy conservation, and $\mathbf{k}_{rel} = (\mathbf{k}_1 - \mathbf{k}_2)/2$, $\mathbf{K}_{cm} = \mathbf{k}_1 + \mathbf{k}_2 = -\mathbf{k}_3$. One should notice that the simple $2NC$ model is recovered placing $n_{cm}(\mathbf{k}_{cm}) = \delta(\mathbf{k}_{cm})$, and that $P_{FNC}(k, E)$ is governed by the behaviour of the cm and relative momentum distributions, n_{cm} and n_{rel}, respectively, whose calculation requires the knowledge of the ground-state wave function only. One should remember that, to evaluate the exact SF, the exact WF of the ground state, as well as the continuum WF of the $A - 1$ system, are necessary. In the 2NC region, good agreement between $P_{FNC}(k, E)$ and the exact calculation for NM and ^{3}He is achieved [3]. Experimentally, a recent confirmation of the model has been obtained at BNL for ^{12}C [4]. Formally, the model has been justified for NM [5]. As for Few Body Systems, the case of ^{3}He and ^{4}He is discussed here. More details will be shown in a forthcoming paper [6]. For the convolution model to hold, in the 2NC region, the WF has to factorize. For ^{3}He, this means that, when $x = |\mathbf{r}_1 - \mathbf{r}_2| << y = |\mathbf{r}_1 + \mathbf{r}_2|/2$, $\Psi_{\frac{1}{2}\mathcal{M}}(\mathbf{x}, \mathbf{y}) \simeq \psi_x(\mathbf{x}) \cdot \psi_y(\mathbf{y})$ In the following, the occurrence of this property will be studied. Use will be made of the "Pisa" wave function [7], corresponding to the AV18 interaction [8]. If the WF factorizes in the 2NC region, also the quantity $\rho(x, y, \cos\theta_{\widehat{\mathbf{xy}}}) = \frac{1}{2}\sum_{\mathcal{M}} \left| \Psi_{\frac{1}{2}\mathcal{M}}(\mathbf{x}, \mathbf{y}) \right|^2$ does. This means that this quantity has to depend weakly upon the angle $\theta_{\widehat{\mathbf{xy}}}$, and the dependence upon $|\mathbf{x}|$ and $|\mathbf{y}|$ can be separated. The formal expresion is cumbersom and will be shown elsewhere [6]. If the factorization holds in the 2NC region, one should have, for $x << y, y'$,

$$R_1(x, y, y', \theta_{\widehat{\mathbf{xy}}}) = \frac{\rho(x, y, \cos\theta_{\widehat{\mathbf{xy}}})}{\rho(x, y', \cos\theta_{\widehat{\mathbf{xy}}})} \simeq \frac{f(y)}{f(y')} = constant \qquad (5)$$

This behavior, a clear signature of factorization, is found indeed around the 2NC region (see Fig. 1). It is also found that the cm motion is mainly in S wave (see Fig. 1) and that the relative motion is mainly deuteron-like [6]. Moreover, approaching the 2NC region, where $x << y$, the dependence upon the angle $\theta_{\widehat{\mathbf{xy}}}$ gets weaker and weaker, and the independence on the relative directions of $\mathbf{x}$ and $\mathbf{y}$ is obtained (see Fig.2, left

panel). Let us discuss now the SF. In the convolution model, the SF for ^{3}He reads

$$P_{FNC}(k_1, E) = \int d\mathbf{K_{cm}} \, \delta\left[E - \mathcal{E}(\mathbf{k_1}, \mathbf{K_{cm}})\right] n_{rel}\left(\left|\mathbf{k_1} + \frac{\mathbf{K_{cm}}}{2}\right|\right) n_{cm}(\mathbf{K}_{cm}) , \quad (6)$$

with $\mathcal{E} = |E_3| + \frac{1}{4M}(\mathbf{k_1} + 2\mathbf{K_{cm}})$, while the exact expression is found to be

$$P(k_1, E) = \int d\mathbf{K_{cm}} \, \delta\left[E - \mathcal{E}(\mathbf{k_1}, \mathbf{K_{cm}})\right] \frac{1}{2} \sum_{\mathcal{S}=\mathcal{M},S_{12},\Sigma_{12},\sigma_3} |\Phi_{\mathcal{S}}(\mathbf{K}_{cm}, \mathbf{k}_{rel})|^2 , \quad (7)$$

where $\Phi_{\mathcal{S}}(\mathbf{K}_{cm}, \mathbf{k}_{rel})$ is the intrinsic ^{3}He wave function in momentum space. Comparing Eqs. (5) and (6) it is clear that, in the 2NC region, for the convolution model to hold, one has to find

$$\frac{1}{2} \sum_{\mathcal{S}=\mathcal{M},S_{12},\Sigma_{12},\sigma_3} |\Phi_{\mathcal{S}}(\mathbf{K}_{cm}, \mathbf{k}_{rel})|^2 \simeq n_{cm}(|\mathbf{K}_{cm}|) \, n_{rel}(|\mathbf{k}_{rel}|) . \quad (8)$$

The quantity $|\Phi_{\mathcal{S}}(\mathbf{K}_{cm}, \mathbf{k}_{rel})|^2$ has been studied. Approaching the 2NC region, where $|\mathbf{K}_{cm}| << |\mathbf{k}_{rel}|$, the dependence upon the angle $\theta_{\widehat{\mathbf{K}_{cm}\mathbf{k}_{rel}}}$ gets weaker and weaker. This behavior is the one to be studied in forth-coming experiments, measuring $n(|\mathbf{K}_{cm}|)$ and $n(|\mathbf{k}_{rel}|)$. Around the 2NC region, where $|\mathbf{K}_{cm}| << |\mathbf{k}_{rel}|$ and the convolution model holds, one finds

$$R = \frac{|\Phi(\mathbf{K}_{cm}, \mathbf{k}_{rel})|^2}{|\Phi(0, \mathbf{k}_{rel})|^2} \simeq \frac{n_{cm}(|\mathbf{K}_{cm}|)}{n_{cm}(|\mathbf{K}_{cm}| = 0)} = constant \cdot n_{cm}(|\mathbf{K}_{cm}|) \quad (9)$$

This allows one to see, for any $|\mathbf{K}_{cm}|$, at which value of $|\mathbf{k}_{rel}|$ the convolution model starts to hold (see Fig. 2, right panel). Using this analysis, $n(|\mathbf{K}_{cm}|)$ and $n(|\mathbf{k}_{rel}|)$ can be obtained in the 2NC region [6].

As for the factorization of the ^{4}He WF, encouraging preliminary results have been obtained using an ATMS wave function corresponding to the RSC interaction [9]. A result qualitatively similar to that of ^{3}He is found.

Summarizing, workable models of the SF, even if valid only in a peculiar region, can be very useful for phenomenological studies. The convolution model is a good approximation to the exact spectral function in the 2NC region. For ^{3}He, it has been shown that, in the 2NC region, the exact wave function exhibits the factorization properties which justify the convolution model. A corresponding analysis is going on for ^{4}He, and preliminary results are encouraging. These findings are useful also for finite nuclei: the factorization into cm and relative momentum distributions seems to be a general property in the 2NC region, and many-body calculations should reproduce it.

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