Deciphering Azimuthal Correlations
in Relativistic Heavy-Ion Collisions

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We discuss various sources of azimuthal correlations in relativistic heavy-ion collisions. The integral measure Φ is applied to quantify the correlations. We first consider separately the correlations caused by the elliptic flow, resonance decays, jets and transverse momentum conservation. An effect of randomly lost particles is also discussed. Using the PYTHIA and HIJING event generators we produce a sample of events which mimic experimental data. By means of kinematic cuts and particle's selection criteria, the data are analyzed to identify a dominant source of correlations.

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I. INTRODUCTION

Particles produced in relativistic heavy-ion collisions are correlated in azimuthal angle due to various mechanisms. One mentions here extensively studied jets and minijets resulting from (semi-)hard parton-parton scattering and collective flow due to the cylindrically asymmetric pressure gradients, see the review articles [1] and [2], respectively. More exotic sources of correlations are also possible. As argued in [3], the plasma instabilities, which occur at an early stage of collisions, can generate the azimuthal fluctuations. Except the dynamically interesting mechanisms, there are also rather trivial effects caused by decays of hadronic resonances or by energy-momentum conservation.

There is a variety of methods designed to study fluctuations on event-by-event basis. In particular, the so-called measure Φ proposed in [4] was used to measure the transverse momentum [5, 6] and electric charge fluctuations [7]. The measure proved to be very sensitive to dynamical correlations and it was suggested to apply it to study azimuthal ones [8]. Such an analysis is underway using experimental data accumulated by the NA49 and NA61 Collaborations and some preliminary results are already published [9]. The aim of this paper is to present model simulations to be used in interpretation of the experimental data. The fact that the measure Φ is sensitive to correlations of various origin is advantage and disadvantage at the same time, as it is difficult to disentangle different contributions. Therefore, we model the azimuthal correlations driven by several processes and we look how the correlations show up when quantified by the measure Φ. We first consider separately in terms of toy models, the elliptic flow, resonance decays, jets and transverse momentum conservation. An effect of randomly lost particles is also examined. Then, we analyze the data provided by the PYTHIA and HIJING event generators showing how to identify the main sources of correlations by applying kinematic cuts and particle's selection criteria.

II. MEASURE Φ

Let us first introduce the correlation measure Φ. One defines the variable $z \stackrel{\text{def}}{=} x - \overline{x}$, where x is a single particle's characteristics such as the particle transverse momentum, electric charge or azimuthal angle. The overline denotes averaging over a single particle inclusive distribution. In the subsequent sections, x will be identified with the particle azimuthal angle ϕ and the fluctuation measure will be denoted as Φ_ϕ. The event variable Z, which is a multiparticle analog of z, is defined as $Z \stackrel{\text{def}}{=} \sum_{i=1}^{N}(x_i - \overline{x})$, where the summation runs over particles from a given event. By construction, $\langle Z \rangle = 0$, where $\langle ... \rangle$ represents averaging over events (collisions). The measure Φ is finally defined as

$$\Phi \stackrel{\text{def}}{=} \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}} \,. \tag{1}$$

It is evident that $\Phi = 0$, when no inter-particle correlations are present. The measure also possesses a less trivial property - it is *independent* of the distribution of the number of particle sources if the sources are identical and independent from each other. Thus, the measure Φ is 'blind' to the impact parameter variation as long as the 'physics' does not change with the collision centrality. In particular, Φ is independent of the impact parameter if the nucleus-nucleus collision is a simple superposition of nucleon-nucleon interactions. In the following sections we discuss how various mechanisms responsible for azimuthal correlations contribute to Φ_ϕ. Then, using the event generators we show how the dominant contributions can be identified.
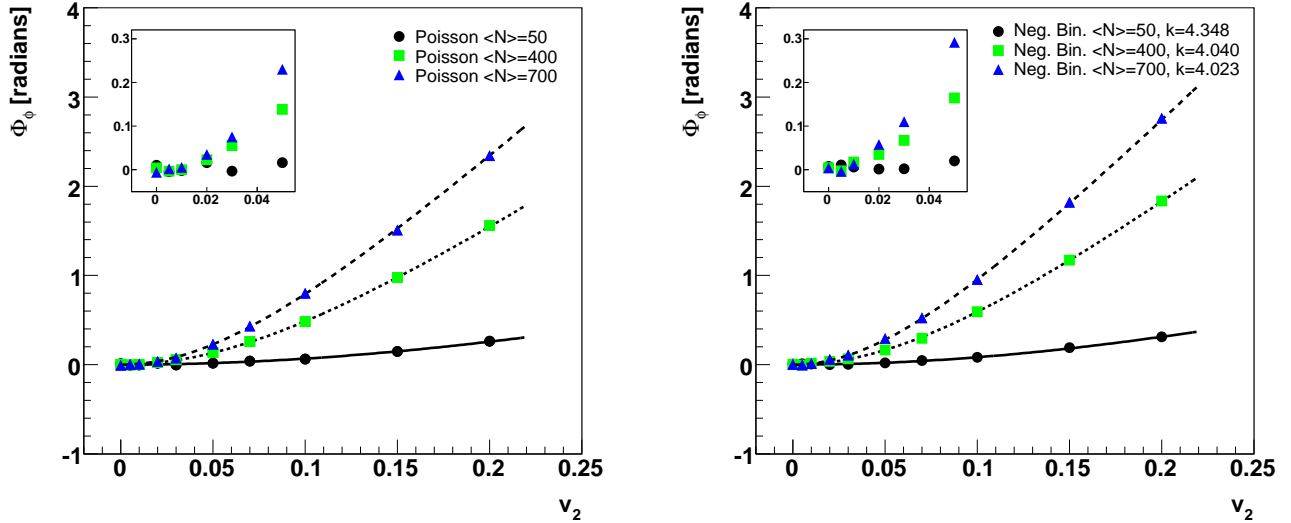
FIG. 1: (Color online) Φ_ϕ as a function of the second Fourier coefficient v_2 for the Poisson (left panel) and NB (right panel) multiplicity distributions. The lines represent the analytical formula (2). The insets show Φ_ϕ for small values of v_2.

III. COLLECTIVE FLOW

Particles produced in relativistic heavy-ion collisions reveal a collective behavior which is naturally described in terms of hydrodynamics where the collective flow is caused by pressure gradients. The flow, which is transverse to the collision beam axis, is usually decomposed into Fourier harmonics with the amplitudes denoted as v_1, v_2, v_3, ... The collective flow as quantified by the measure Φ_ϕ was studied in [8]. The azimuthal correlations caused by the flow were found to give

$$\Phi_\phi = \sqrt{\frac{\pi^2}{3} + \left(\frac{\langle N^2\rangle - \langle N\rangle}{\langle N\rangle}\right)S} - \frac{\pi}{\sqrt{3}} \; . \qquad (2)$$

where $\langle N^m\rangle$ is the $m-$th moment of multiplicity distribution and

$$S \equiv 2\left\langle \sum_{n=1}^{\infty}\left(\frac{v_n}{n}\right)^2 \right\rangle. \qquad (3)$$

We have first verified the effect of second Fourier coefficient v_2 on Φ_ϕ by Monte Carlo simulations. For this purpose we have generated events of particle multiplicity given by either Poisson or Negative Binomial (NB) distribution. The latter is defined as

$$P_N = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)}\frac{\langle N\rangle^N k^k}{\left(\langle N\rangle + k\right)^{N+k}} \; , \qquad (4)$$

where $\Gamma(k)$ is the Gamma function, which for positive integer arguments equals $\Gamma(k) = (k-1)!$; the parameter k can be expressed through the variance of the distribution $\mathrm{Var}(N) \equiv \langle N^2\rangle - \langle N\rangle^2$ and the average value $\langle N\rangle$ as

$$k = \frac{\langle N\rangle^2}{\mathrm{Var}(N) - \langle N\rangle} \; . \qquad (5)$$

The parameter k is chosen in such a way in our all simulations that $\sqrt{\mathrm{Var}(N)} = \langle N\rangle/2$. Then, the multiplicity distribution approximately obeys the Wróblewski's formula [10] which is known to hold for proton-proton interactions in a wide collision energy range. The simulations are performed for both the Negative Binomial and Poisson distributions as the former distribution is much broader than the latter one for $\langle N\rangle \gg 1$. We note here that the width of multiplicity distributions in relativistic heavy-ion collisions strongly depends on centrality selection criteria. Thus, it is important to see how the correlation signal changes with the width of multiplicity distribution.

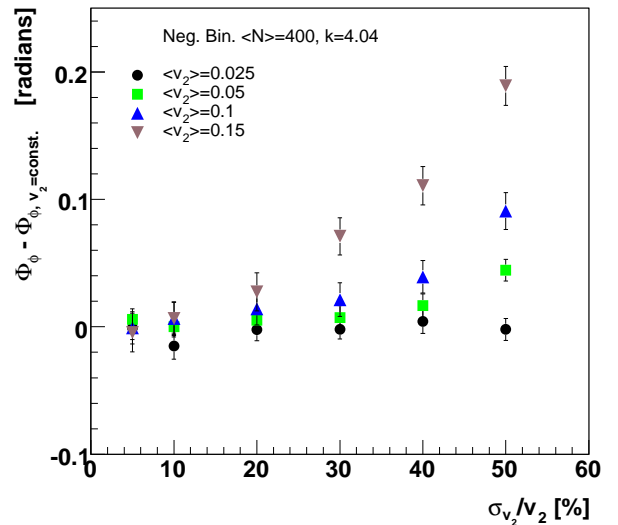The azimuthal angle of each particle has been gener-

FIG. 2: (Color online) The difference of Φ_ϕ computed for the fluctuating and fixed v_2.
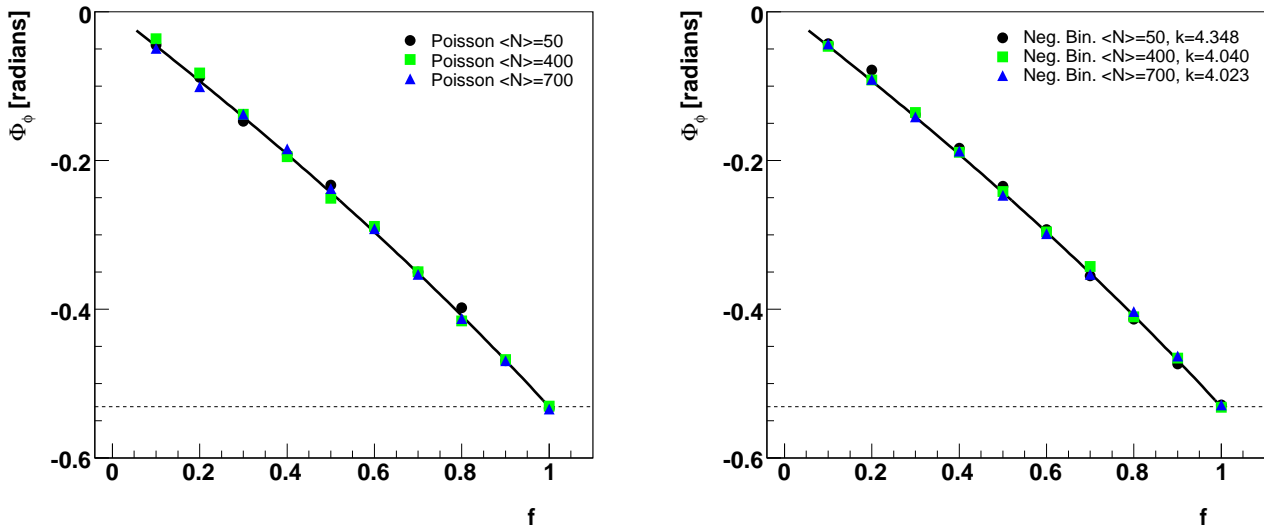
FIG. 3: (Color online) Φ_ϕ as a function of fraction of particles coming from the back-to-back resonance decays. The particle multiplicity is distributed according to the Poisson (left panel) or NB (right panel) distribution. The dashed and solid lines represent the analytical formulas (7) and (8), respectively.

ated from the distribution

$$P(\phi) = \frac{1}{2\pi}\Big(1 + 2v_2\cos\big(2(\phi - \phi_R)\big)\Big),\qquad (6)$$

where $0 \le \phi \le 2\pi$; the reaction plane angle ϕ_R of a given event has been generated from the flat distribution. The results of our simulations are shown in Fig. 1 for both the Poisson (left panel) and NB (right panel) multiplicity distributions. As seen, the analytical formula (2) works perfectly well.

There are large ($\sim 40\%$) event-by-event fluctuations of v_2 observed [2] at BNL RHIC. The v_2 fluctuations are dominated by the fluctuations of eccentricity of the overlap region of colliding nuclei, see e.g. [11]. We have introduced the v_2 fluctuations in our simulations in the following way. For each event the value of v_2 has been generated from the Gaussian distribution of the dispersion σ_{v_2}. The fluctuations have been restricted to vary within $2\sigma_{v_2}$ around the mean $\langle v_2 \rangle$ that is $\langle v_2 \rangle - 2\sigma_{v_2} \le v_2 \le \langle v_2 \rangle + 2\sigma_{v_2}$. Then, v_2 remains positive unless $\sigma_{v_2}/\langle v_2 \rangle$ exceeds 0.5.

In Fig. 2 we demonstrate the effect of flow fluctuations relative the effect of flow. Specifically, we show the difference of the correlation measures computed for the fluctuating v_2 and fixed $v_2 = \langle v_2 \rangle$. The particle multiplicity has been generated from the NB distribution with $\langle N \rangle = 400$. As seen, the flow fluctuations of relative magnitude of $\sim 40\%$ noticeably increase the value of Φ_ϕ if $\langle v_2 \rangle$ is not too small.

IV. RESONANCE DECAYS

Let us start the discussion of effects of resonance decays with the toy model where *all* produced particles come from heavy resonances which have vanishing transverse velocity and decay back to back into pairs of particles.

The particle multiplicity is arbitrary but fixed even number. Then, as shown in Appendix, we have

$$\Phi_\phi = \frac{1 - \sqrt{2}}{\sqrt{6}}\,\pi \approx -0.531.\qquad (7)$$

When only a fraction f of all produced particles comes from the back-to-back decays of resonances while the remaining particles are produced independently from each other, the calculations presented in Appendix lead to

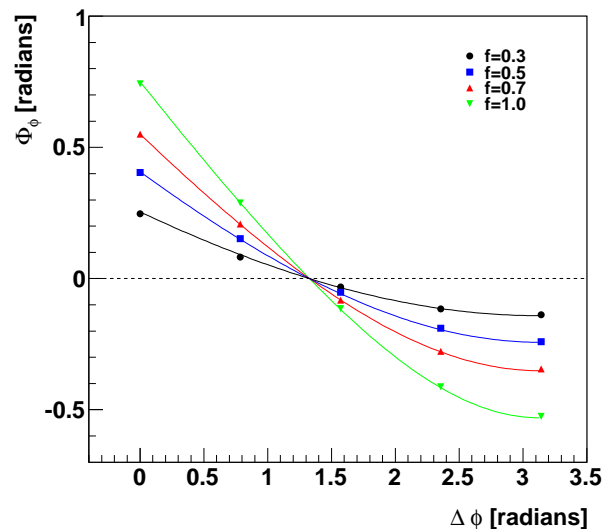$$\Phi_\phi = \frac{\sqrt{2 - f} - \sqrt{2}}{\sqrt{6}}\,\pi\;.\qquad (8)$$

FIG. 4: (Color online) Φ_ϕ as a function of the correlation angle $\Delta\phi$ for varying fraction f of particles emitted in pairs. The particle multiplicity is given by the NB distribution.
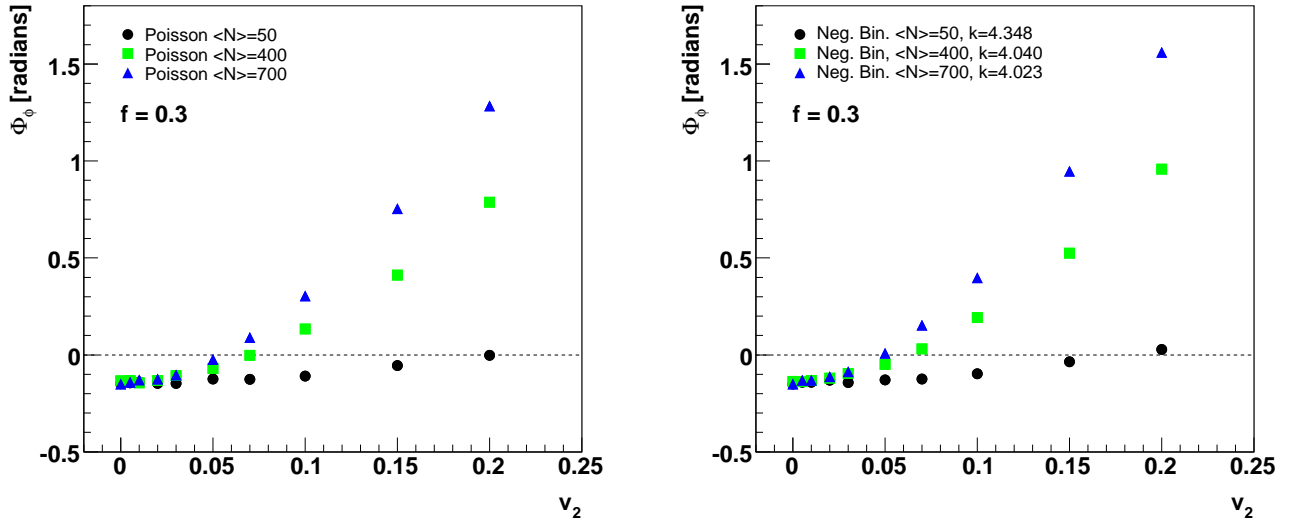
FIG. 5: (Color online) Φ_ϕ resulting from the combined effect of elliptic flow and resonance decays. The particle multiplicity is distributed according to the Poisson (left panel) or NB (right panel) distribution.

As seen, for $f = 0$ the formula (8) gives, as expected, $\Phi = 0$ and for $f = 1$ we get the value (7).

We have checked the formula (8) by Monte Carlo simulations and then we have considered the model where the particle multiplicity is not fixed but it is given by either Poisson or NB distribution with the average multiplicity equal 50, 400, or 700 particles. For a given fraction f of particles coming from the back-to-back decays of resonances, the number of particles coming from the decays in the event of multiplicity N has been the even number which is the nearest to fN. The measure Φ_ϕ as a function of f is shown in Fig. 3. As seen, the formula (8) still works very well.

When a resonance, which is at rest, decays back to back, the difference of azimuthal angles of the decay products is $\Delta\phi = \pi$. When the resonance has a finite velocity, the difference of azimuthal angles of the decay products is smaller than π. When the resonance's kinetic energy is much larger than the energy released in its decay, $\Delta\phi$ is zero. Therefore, we consider a model where a fraction of particles comes from the resonance decays and the particles are emitted in pairs with the difference of their azimuthal angles $\Delta\phi$ varying from 0 to π. We first assume that *all* particles are emitted in pairs and the relative azimuthal angle of two correlated particles equals $\Delta\phi$. As shown in Appendix, we then have

$$\Phi_\phi = \sqrt{\frac{2}{3}\pi^2 - \Delta\phi\,\pi + \frac{1}{2}(\Delta\phi)^2} - \frac{\pi}{\sqrt{3}}. \qquad (9)$$

As seen that Φ_ϕ changes its sign from positive to negative with growing $\Delta\phi$; Φ_ϕ vanishes when

$$\Delta\phi = \pi\left(1 - \frac{1}{\sqrt{3}}\right) \approx 1.328 \qquad (10)$$

and for $\Delta\phi = \pi$ we deal with the model described by the formula (7). Further on, we have considered a model

where only a fraction f of particles is emitted in correlated pairs. Then, as explained in Appendix, Eq. (9) gets the form

$$\Phi_\phi = \sqrt{\frac{\pi^2}{3} + f\left(\frac{\pi^2}{3} - \Delta\phi\,\pi + \frac{1}{2}(\Delta\phi)^2\right)} - \frac{\pi}{\sqrt{3}}. \qquad (11)$$

As previously, Φ_ϕ changes its sign and $\Phi_\phi = 0$ for $\Delta\phi$ given by Eq. (10).

In Fig. 4 we compare the formula (11) with the results of Monte Carlo simulation of Φ_ϕ as a function of $\Delta\phi$. The fraction of particles emitted in pairs equals 0.3, 0.5, 0.7 or 1.0. The remaining particles, which are not emitted in pairs, carry no correlations. The particle multiplicity is generated from the NB distribution with $\sqrt{\mathrm{Var}(N)} = \langle N\rangle/2 = 50$. As seen, the formula (11) works perfectly well.
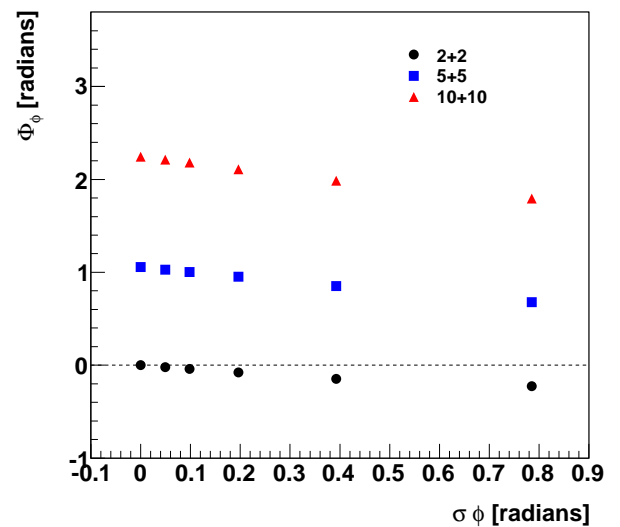

FIG. 6: (Color online) Φ_ϕ as a function of the jet opening angle $\sigma\phi$ for several numbers of particles in a jet.
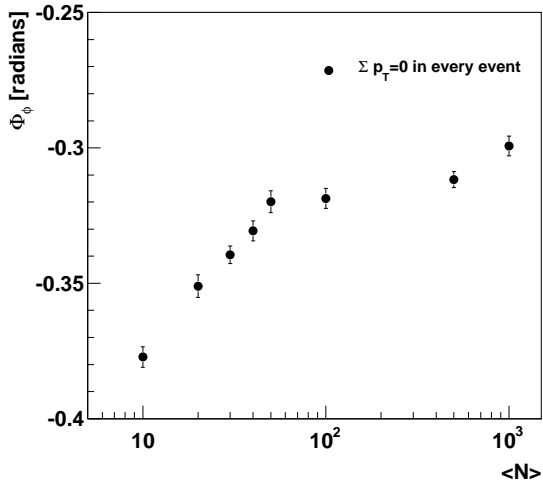
FIG. 7: Φ_ϕ as a function of average multiplicity in events where total transverse momentum exactly vanishes.
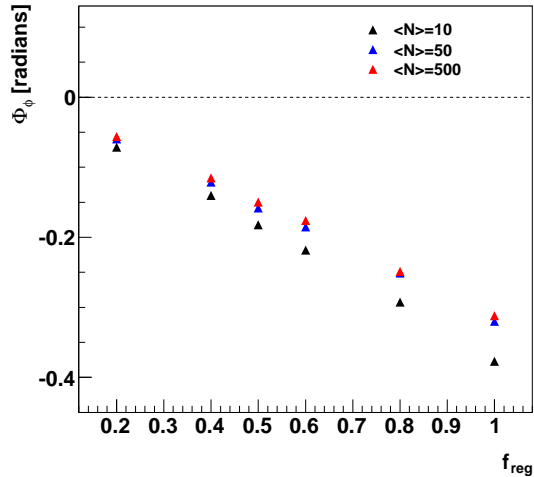

FIG. 8: (Color online) Φ_ϕ as a function of fraction of registered particles f_{reg}. The correlation results from the transverse momentum conservation.

We next discuss the combined effect of resonance decays and elliptic flow. The multiplicity of events is generated from the Poisson or NB binomial distribution. For each event 30% of particles is assumed to originate from heavy resonances which decay back to back into pairs of particles. Neither resonances nor their decay products experience any flow, but the remaining 70% of particles manifest the collective elliptic flow according to Eq. (6). The results of the simulation are shown in Fig. 5. When the particle multiplicity or v_2 are sufficiently small, the effects of resonance decays dominates and Φ_ϕ is negative. It becomes positive when the effect elliptic flow takes over.

V. 'DIJETS'

We call a dijet the two groups (jets) of particles flying in exactly opposite directions. Particles from each jet are distributed within a cone of the azimutal angle $\sigma\phi$. We have considered the dijets of 2+2, 5+5 and 10+10 particles. The total particle multiplicity is correspondingly 4, 10, 20 as there is exactly one dijet per event and there are no other particles. The results of dijet simulation are shown in Fig. 6.

We have here two sources of azimuthal correlations which counteract each other. As we already know, the back-to-back emission of particles generates negative correlations while the collinear emission leads to positive ones. When the particle's multiplicity of dijets is sufficiently high and $\sigma\phi$ is sufficiently small, the effect of collinear emission wins and Φ_ϕ is positive.

VI. MOMENTUM CONSERVATION

The momentum conservation obviously leads to interparticle correlations. We have studied the effect on Φ_ϕ,

generating the sets of particles of multiplicity N. The azimuthal angle distribution of a single particle is assumed to be flat while the transverse momentum distribution is chosen in the form

$$P(p_T) = \beta^2 p_T e^{-\beta p_T} \qquad (12)$$

with the slope parameter $\beta^{-1} = 200$ MeV. For each particle the x and y components of its momentum have been computed as $p_x = p_T \cos\phi$ and $p_y = p_T \sin\phi$. To make the total transverse momentum of N particles vanish, the x and y component of momentum of each particle has been shifted as

$$p_x \to p_x - \frac{1}{N}\sum_{i=1}^N p_x^i, \qquad p_y \to p_y - \frac{1}{N}\sum_{i=1}^N p_y^i. \qquad (13)$$

The simulation showing the effect of transverse momentum conservation is illustrated in Fig. 7. The particle multiplicity has been generated according to NB distribution with $\sqrt{\text{Var}(N)} = \langle N \rangle/2$. As seen, the effect of momentum conservation is sizable and it survives to large multiplicities.

In real experiments only a fraction of all produced particles is observed due to a finite detector efficiency and acceptance. We model the effect of detector efficiency by randomly loosing particles independently of their azimuthal angle. In Fig. 8 we show how the effect of detector efficiency modifies the correlations caused by the transverse momentum conservation. As seen, the random losses of particles lead to the dilution of the correlation that is Φ_ϕ monotonically goes to zero as the fraction of registered particles $f_{\text{reg}} \to 0$.

VII. PROTON-PROTON COLLISIONS IN PYTHIA

After the discussion of various mechanisms responsible for azimuthal correlations, let us now consider
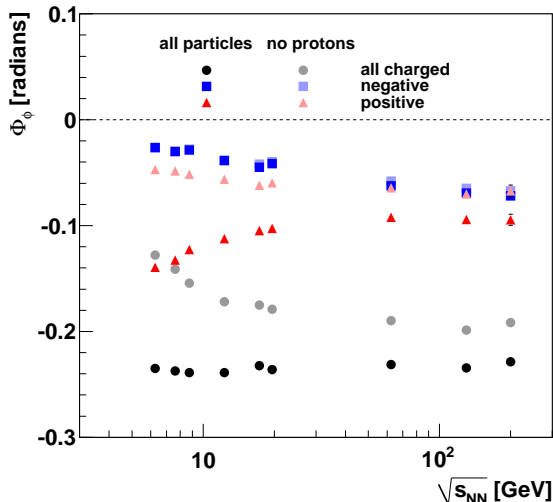
FIG. 9: (Color online) The energy dependence of Φ_ϕ for positive, negative and all charged particles in the PYTHIA simulations of p-p collisions. The pale symbols correspond to the results with protons excluded from the positive and all charged particles.
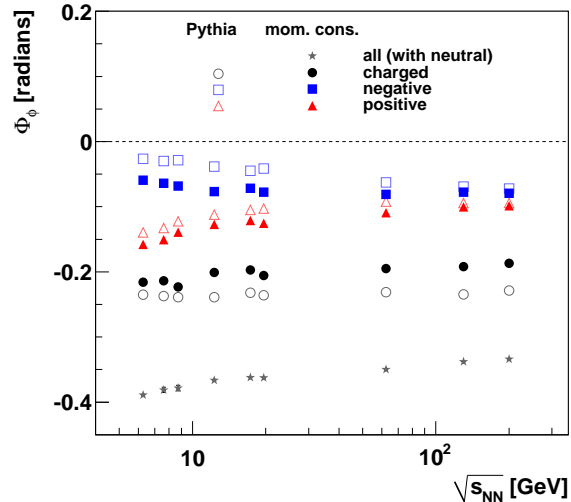

FIG. 10: (Color online) Φ_ϕ for the PYTHIA events (open symbols) compared to the results of toy-model simulations (full symbols) which take into account only the effect of transverse momentum conservation. The asterisks show the toy-model results for all (neutral and charged) particles.

more realistic situation where several mechanisms of azimuthal correlations are present at the same time. We used the PYTHIA generator [12] to simulate p-p collisions at several collision energies accessible at SPS ($\sqrt{s_{NN}}$ = 6.27, 7.62, 8.73, 12.3, 17.3 GeV) and RHIC ($\sqrt{s_{NN}}$ = 19.6, 62.4, 130, 200 GeV). For every energy a set of minimum bias events was collected. We treated as stable the following particles: μ^-, π^0, π^+, K^0, K^+, K^0_L, K^0_S, Λ, Σ^+, Σ^-, Ξ^0, Ξ^-, Ω^- and their antiparticles. No acceptance cuts were applied.

For every energy we computed Φ_ϕ separately for positive, negative and all charged particles. The results are shown in Fig. 9. As seen, Φ_ϕ is negative and weakly depends on collision energy. To understand why Φ_ϕ is so different for negative and for positive particles, we excluded protons from all charged and from positive particles. The corresponding values of Φ_ϕ are also shown in Fig. 9. After excluding protons, the correlations among negative particles and among positive are very similar to each other.

What is the mechanism responsible for negative values of Φ_ϕ? We first checked that high p_T particles play no role here, as Φ_ϕ does not significantly change when particles with $p_T > 1.5$ GeV are excluded. The correlations among charged particles can be caused by the effect resonance decays but the effect is certainly very minor for same-sign particles, as there are very a few resonances decaying into two positive or two negative particles.

The transverse momentum conservation, which is discussed in Sec. VI, is another possible source of negative values of Φ_ϕ. We checked that the PHYTHIA events indeed obey the transverse momentum conservation. Specifically, we proved vanishing of the total momentum in x and in y directions of all particles (charged and neutral) form every event. To quantitatively study

the effect of transverse momentum conservation we proceeded as follows. For every collision energy we determined the average multiplicity of positive, negative and neutral particles. Then, we performed the simple simulations described in Sec. VI, generating events of a given total multiplicity which satisfy the transverse momentum conservation. Then, a fraction of particles was randomly eliminated to get the multiplicity of charged, positive or negative particles. The values of Φ_ϕ computed for such events are shown in Fig. 10. As seen, the values of Φ_ϕ for the PYTHIA events agree quite well with the results of our toy-model simulations which take into account only the effect of transverse momentum conservation. It is somewhat surprising that the agreement for same-sign particles is not much better that that for all charged particles. It means that the resonance decays do not generate strong correlations in the PYTHIA events. We note, however, that the effect of transverse momentum conservation overshoots the correlations of the like-sign particles and it undershoots the correlations of all charged particles. The latter results presumably signals presence of resonances decaying into pairs of one positive and one negative particle.

VIII. NUCLEUS-NUCLEUS COLLISIONS IN HIJING

We have also performed simulations of nucleus-nucleus collisions using the HIJING [13] event generator. We have simulated the collisions of p-p, C-C, Si-Si and Pb-Pb at $\sqrt{s_{NN}} = 17.3$ GeV and Φ_ϕ has been computed separately for positive, negative and all charged particles coming from minimum bias events. The results are shown in Fig. 11. As seen, Φ_ϕ is almost independent of the mass
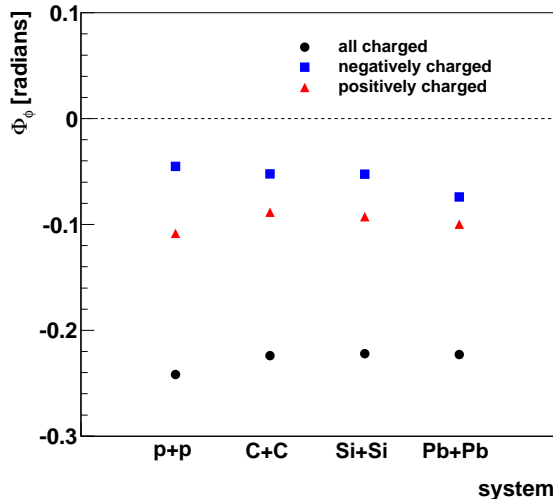
FIG. 11: The Hijing simulation of the system size dependence of Φ_ϕ for nucleus-nucleus collisions at $\sqrt{s_{NN}} = 17.3$ GeV with no acceptance cuts.

number of colliding nuclei and the values of Φ_ϕ are very close to those found using PYTHIA. It is by no means accidental. When a nucleus-nucleus collision is a simple superposition of nucleon-nucleon interactions, the value of Φ is exactly the same for p-p interactions and nucleus-nucleus collisions at any centrality. In the HIJING model a nucleus-nucleus collision is not exactly a superposition of nucleon-nucleon collisions but it is almost so. And the treatment of proton-proton interactions is essentially the same in PYTHIA and HIJING. For these reasons our analysis of PYTHIA events presented in the previous section fully applies here.

IX. SUMMARY AND OUTLOOK

Azimuthal correlations of final state particles from high-energy collisions carry valuable information on the collision dynamics. It motivates the analysis of experimental data collected by the NA49 and NA61 Collaborations which is in progress with some preliminary results already published [9]. The integral measure Φ_ϕ, which proved to be very sensitive to various dynamical correlations, is used in the analysis. To interpret the experimental results it should be understood how different sources of correlations manifest themselves when measured by means of Φ_ϕ. This was the aim of our study. We performed several simulations to analyze separately the azimuthal correlations caused by the elliptic flow, resonance decays, jets and transverse momentum conservation. We also discussed how the correlations are diluted due to randomly lost particles. Finally we used the PYTHIA and HIJING event generators to produce a big sample of events which mimic experimental data from p-p and nucleus-nucleus collisions at the SPS and RHIC collision energies. Φ_ϕ appeared to be surprisingly independent of the collision energy and of the size of colliding systems. Applying some kinematic cuts and selection criteria of particles, we showed that the azimuthal cor-

relations present are dominated by rather trivial effect of transverse momentum conservation which appeared to be almost independent of particle's multiplicity which changes dramatically for collision energies and system's sizes under consideration.

The experience gathered in the course of this theoretical study will be used to better understand experimental data. Quantitative analysis of several simple mechanisms of azimuthal correlations we discussed will facilitate an observation of possible new phenomena like critical fluctuations at phase boundaries of strongly interacting matter or plasma instabilities from the early stage of relativistic heavy-ion collisions.

Appendix: Toy model of resonance decays

The inclusive distribution of azimuthal angle is assumed to be flat that is

$$P_{\text{inc}}(\phi) = \frac{1}{2\pi}\,\Theta(\phi)\,\Theta(2\pi - \phi)\,, \qquad \text{(A.1)}$$

which gives $\bar{\phi} = \pi$ and $\bar{\phi}^2 = 4\pi^2/3$. Consequently, $\bar{z}^2 = \pi^2/3$.

Let us first assume that *all* produced particles come from heavy resonances which are at rest and decay back to back into two particles. When one particle is emitted at the azimuthal angle ϕ_1 and $0 \leq \phi_1 < \pi$, the second particle is emitted at $\phi_2 = \phi_1 + \pi$. When $\pi \leq \phi_1 < 2\pi$, then $\phi_2 = \phi_1 - \pi$. Therefore, the two-particle distribution of azimuthal angles reads

$$\begin{aligned} P_2(\phi_1, \phi_2) &= \frac{1}{2\pi}\,\Theta(\pi - \phi_1)\,\delta(\phi_1 - \phi_2 + \pi) \quad \text{(A.2)} \\ &+ \frac{1}{2\pi}\,\Theta(\phi_1 - \pi)\,\delta(\phi_1 - \phi_2 - \pi)\,. \end{aligned}$$

One observes that

$$\int d\phi_1 P_2(\phi_1, \phi) = \int d\phi_2 P_2(\phi, \phi_2) = P_{\text{inc}}(\phi)\,, \quad \text{(A.3)}$$

and computes

$$\int d\phi_1 d\phi_2 \phi_1 \phi_2 P_2(\phi_1, \phi_2) = \frac{5}{6}\pi^2\,. \qquad \text{(A.4)}$$

We further assume that the particle multiplicity is arbitrary but fixed even number N. Then, the $N-$particle distribution of azimuthal angles is

$$P_N(\phi_1, \phi_2, \ldots, \phi_N) = P_2(\phi_1, \phi_2)\, P_2(\phi_3, \phi_4)\, \cdots\, P_2(\phi_{N-1}, \phi_N)\, . \tag{A.5}$$

The variable Z is defined as $Z = \phi_1 + \phi_2 + \ldots + \phi_N - N\pi$ and one computes $\langle Z^2 \rangle$ in the following way

$$\langle Z^2 \rangle = \int d\phi_1 d\phi_2 \ldots d\phi_N (\phi_1 + \phi_2 + \ldots + \phi_N - N\pi)^2 P_N(\phi_1, \phi_2, \ldots, \phi_N) \tag{A.6}$$

$$= N\bar{\phi^2} + N \int d\phi_1 d\phi_2 \phi_1 \phi_2 P_2(\phi_1, \phi_2) + N(N-2)\bar{\phi}^2 - 2N^2\bar{\phi}\pi + N^2\pi^2 = \frac{N}{6}\pi^2\, .$$

Using the result (A.6) and keeping in mind that $\bar{z}^2 = \pi^2/3$, we find the formula (7) from the definition (1).

Let us now assume that N_1 particles come from resonances and additional N_2 particles are produced independently from each other and from resonances. Then, we still have $\bar{z}^2 = \pi^2/3$ and $\langle Z^2 \rangle$ is computed as

$$\langle Z^2 \rangle = N\bar{\phi^2} + N_1 \int d\phi_1 d\phi_2 \phi_1 \phi_2 P_2(\phi_1, \phi_2) + N_1(N_1 - 2)\bar{\phi}^2 \tag{A.7}$$

$$+\ 2N_1 N_2 \bar{\phi}^2 + N_2(N_2 - 1)\bar{\phi}^2 - 2N^2\bar{\phi}\pi + N^2\pi^2 = \frac{\pi^2}{6}\left(2N - N_1\right)\, ,$$

where $N \equiv N_1 + N_2$. The result (A.7) with $f \equiv N_1/N$ gives the formula (8).

The model can be easily generalized to the situation when the particles from a correlated pair are not emitted back to back but their relative azimuthal angle is fixed and equal $\Delta\phi$. Then, the two-particle distribution of azimuthal angles is

$$P_2(\phi_1, \phi_2) = \frac{1}{2\pi}\, \Theta(2\pi - \Delta\phi - \phi_1)\, \delta(\phi_1 - \phi_2 + \Delta\phi) + \frac{1}{2\pi}\, \Theta(\phi_1 - 2\pi + \Delta\phi)\, \delta(\phi_1 - \phi_2 + \Delta\phi - 2\pi)\, , \tag{A.8}$$

and instead of Eq. (A.4) we have

$$\int d\phi_1 d\phi_2 \phi_1 \phi_2 P_2(\phi_1, \phi_2) = \frac{4}{3}\pi^2 - \Delta\phi\,\pi + \frac{1}{2}(\Delta\phi)^2\, . \tag{A.9}$$

Substituting the result (A.9) to Eq. (A.6) or Eq. (A.7), one finds $\langle Z^2 \rangle$ which finally gives the formula (9) or (11), respectively.

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