Novel Term in Fermion-Boson Mapping for Nuclear Rotation

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We introduce a novel term in fermion-boson mapping method for the Interacting Boson Model (IBM) for rotational nuclei. The rotation of a given deformed nucleon system is similar to the corresponding one of the boson system, but there is a certain difference in quantitative details. This requires the additional rotational correction to the boson system. We implement the rotational $L \cdot L$ term into the IBM Hamiltonian, and determine its coupling constant based on the mean-field calculation with Skyrme Energy Density Functional. The validity of the method is examined for rare-earth (Sm) and actinoid (U) nuclei. Experimental rotational spectra for these nuclei are shown to be reproduced remarkably well without any phenomenological correction.

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Atomic nucleus is a strongly interacting many-body quantal system which has a distinct shape. The shape, i.e., the surface of the nucleus, deforms with low excitation energies, depending on the number of nucleons. Such a deformed nucleus can rotate, and, as a consequence, remarkable regularity shows up as characteristic rotational band, like in many other physical systems. In spite of simple structure of the rotational band, the underlying multi-nucleon dynamics is so complex that the fully microscopic understanding of the rotational motion has been one of the most challenging problems in nuclear physics [1, 2]. In this Letter, we propose a novel way of describing the nuclear rotational spectra in the context of the fermion-boson mapping idea.

The Interacting Boson Model (IBM) [3] has made tremendous success in reproducing low-lying quadrupole collective states of nuclei. The beauty of the IBM lies in its simplicity, describing highly complex nuclear collectivity in numerically economical way, and indeed has a wide applicability in and outside of nuclear physics [3, 4]. The IBM is comprised of bosons which reflect, in a microscopic sense, the collective pairs of valence nucleons [5]. The derivation of an IBM Hamiltonian has been extensively studied [5–7], and it is now of interest to shed light upon this problem from a modern perspective.

Mean-field calculation with Skyrme Energy Density Functional (EDF) gives reasonable and universal description of nuclear properties [2, 8–10], and has been often useful to start with for nuclear spectroscopy [2, 10]. For instance, a distinct way of deriving the IBM Hamiltonian, relevant to the quadrupole collectivity, from the Skyrme EDFs has been proposed recently [11]. Multi-fermion dynamics of surface deformation is simulated by boson degrees of freedom. The simplicity of IBM and the universality of Skyrme EDF can be combined, which allows spectroscopic calculations.

Along this line, we here implement a kinetic/mass term into the IBM Hamiltonian necessary for the description of rotational spectra. Although we mainly discuss deformed rotor, the method presented here is a further development of the mapping scheme of [11] and is one-step closer to a complete picture of the quadrupole collectivity. We take as representative samples Sm and U nuclei, which have typical examples of rotational SU(3) limit of IBM [12]. These nuclei are adequate for showing the generality of the method, because they are in completely different mass regions on the nuclear chart with different values of moment of inertia (MOI).

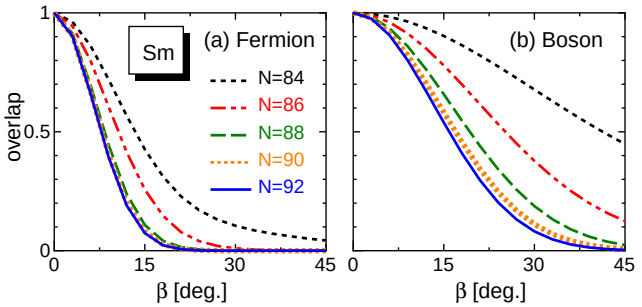

FIG. 1: (Color online) Overlap between the intrinsic state and its rotation at angle β for Sm nuclei for (a) fermion and (b) boson systems. Skyrme SkM* interaction is used.

We start with a basic picture in terms of the rotation of the intrinsic state, i.e., the wave function in the body-fixed frame. The axial symmetry is assumed, i.e., the intrinsic state is invariant with respect to the rotation around the symmetry (z) axis. As in the usual convention, we consider the rotation around the y-axis. Figure 1 shows the overlap between the intrinsic state $|\phi_X\rangle$ and its rotation $|\phi'_X\rangle = e^{-iL_y\beta}|\phi_X\rangle$ for $^{146-154}$Sm nuclei for (a) fermion ($X = F$) and (b) boson ($X = B$) systems. Here L_y and β denote the y-component of the angular momentum operator and the angle of rotation around the y-axis, respectively. The nucleon wave function $|\phi_F\rangle$ is calculated by the Hartree-Fock plus BCS (HF+BCS) approach. Skyrme SkM* interaction [14] is used throughout, while different Skyrme forces do not alter our conclu-

sion. For boson system, we discuss the IBM-2, comprised of proton $L = 0^+$ (s_π) and 2^+ (d_π) bosons, and of neutron $L = 0^+$ (s_ν) and 2^+ (d_ν) bosons [5]. Here the number of proton (neutron) bosons, denoted by n_π (n_ν), is equal to half of the valence proton (neutron) numbers. The boson image $|\phi_B\rangle$ is given as the so-called coherent state [15], $|\phi_B\rangle \propto \prod_{\rho=\pi,\nu}(s^\dagger_\rho + \beta_\rho d^\dagger_{\rho 0})^{n_\rho}|0\rangle$, where $|0\rangle$ and β_ρ represent the boson vacuum (inert core) and amplitude, respectively. We assume $\beta_\pi = \beta_\nu \equiv \beta_B$, for simplicity. β_B is the boson image of geometrical deformation variable [1] (denoted by β_F), and is assumed to be $\beta_B \propto \beta_F$. In Fig. 1 the overlap becomes narrower in β with neutron number N. This is a consequence of growth of deformation. There is a striking difference between fermion and boson wave functions. For each nucleus, the overlap of the fermion system is sharply peaked at the origin $\beta = 0^\circ$, whereas boson one looks wider in β. The fermionic rotated intrinsic state changes more rapidly, as the angle β increases, than the boson one, because the degrees of freedom is more limited for bosons.

We here consider the standard IBM-2 Hamiltonian [11, 13]

$$H_B = \epsilon(n_{d\pi} + n_{d\nu}) + \kappa Q_\pi \cdot Q_\nu, \tag{1}$$

where $n_{d\rho} = d^\dagger_\rho \cdot \tilde{d}_\rho$ and $Q_\rho = [s^\dagger_\rho \times \tilde{d}_\rho + d^\dagger_\rho \times \tilde{s}_\rho]^{(2)} + \chi_\rho [d^\dagger_\rho \times \tilde{d}_\rho]^{(2)}$ stand for the d-boson number operator and the quadrupole operator with the coupling constants ϵ and κ, respectively. χ_ρ determines either prolate or oblate deformation. The ϵ, κ, and $\chi_{\pi,\nu}$ values are derived by mapping the potential energy surface (PES) of the constrained HF+BCS method, characterized by axial and triaxial (i.e., non-axially symmetric) deformations, onto the corresponding PES of IBM system (expectation value of H_B in the intrinsic state) [11, 13]. Here, some studies deriving a collective Hamiltonian from modern EDFs treat the mean-field PES, supplemented with zero point rotational and vibrational corrections, as an effective potential [16–18]. A generalized kinetic energy term also emerges in such approximations. On the other hand, we compare the total HF+BCS energy with the corresponding total energy of the IBM. All terms are included in both the HF+BCS and the IBM results. By reproducing the Skyrme-EDF PES by the IBM one as much as possible, the nucleonic kinetic energies should be included in the boson Hamiltonian. If the nucleus is well deformed, however, the difference in the rotational response, i.e., cranking, between fermion and boson systems becomes sizable due to the difference illustrated by Fig. 1, leading to some systematic deviation of the IBM rotational spectra from the fermionic ones. Note that the response to cranking may not be fully reproduced by the boson Hamiltonian fixed by the PES, as the present PES has nothing to do with the rotational cranking. In order to remedy this difference, we introduce a novel term, which changes rotational response of a boson system so that it can be similar to the rotational response of the corresponding fermionic system without changing intrinsic wave function. The rotational response of fermions is evaluated in a manner discussed later. The term can take the form of $L \cdot L$, where $L(= L_\pi + L_\nu)$ is the angular momentum operator with L_ρ being

$$L_\rho = \sqrt{10}[d^\dagger_\rho \times \tilde{d}_\rho]^{(1)}. \tag{2}$$

We then adopt, hereafter, a Hamiltonian which includes the $L \cdot L$ term (with coupling constant α) and denote the Hamiltonian as H'_B

$$H'_B = H_B + \alpha L \cdot L, \tag{3}$$

where the first term H_B appeared in Eq. (1). The $L \cdot L$ term contributes straightforwardly to the rotational spectra with its eigenvalue being $L(L+1)$, and is thus reasonable for our present purpose. Note that the $L \cdot L$ term has no contribution to the PES, if $d_{\pm 1}$ is not included in the boson intrinsic state $|\phi_B\rangle$ as usual, including the present work. This means that the interaction strengths of the Hamiltonian H_B, fixed already by the mapping from the Skyrme PES, are kept unchanged. This is a practical advantage of the $L \cdot L$ term which allows one to keep the merit of the existing scheme of Refs. [11, 13]. In fact, this form coincides with the so-called $L \cdot L$ term in phenomenology, which has been introduced without knowing its origin [12].

Later on, we use the ϵ, κ, and $\chi_{\pi,\nu}$ values of Ref. [13] for Sm nuclei, and $\epsilon \approx 0.100$ MeV, $\kappa \approx -0.180$ MeV, and $\chi_\pi \approx \chi_\nu \approx -1.000$ for U nuclei. These are determined by the application of the Wavelet analysis [19]. The fermion PES and the corresponding boson PES for U isotopes exhibit pronounced prolate deformation with $\beta_2 \sim 0.25$, similarly to those of Sm isotopes in Ref. [13].

Based on these results, we now turn to the derivation of the $L \cdot L$ coupling constant α. We consider the Hamiltonian matrix element between the intrinsic state $|\phi_X\rangle$ and its rotation $|\phi'_X\rangle$ with an infinitesimal rotation angle $\beta \ll 1$. The cranking model is a simpler form for this quantity and would be feasible to deal with. Then, we first perform cranking model calculation for fermion system in a usual way [2] and obtain the MOI using the Inglis-Belyaev (IB) formula [20, 21], denoted by $\mathcal{I}_F$. The relevant quasi-particle state and energy, as well as the occupation probabilities, are produced by the HF+BCS method of Ref. [22]. The corresponding boson image, denoted by $\mathcal{I}_B$, is given by the cranking formula of [23]. Now that the ϵ, κ and $\chi_{\pi,\nu}$ values are known, $\mathcal{I}_B$ has one free parameter α. The α value is determined for individual nucleus so that $\mathcal{I}_B$ becomes equal to $\mathcal{I}_F$.

The upper panel of Fig. 2 shows the MOIs for Sm and U isotopes. The MOIs calculated by IBM, where the $L \cdot L$ term is (*w/ LL*) and is not (*w/o LL*) included, and by the IB formula, are compared with the experimental value taken from the 2^+_1 energy [25]. Note that the IB formula no longer works well for vibrational nuclei. Therefore, we divide the discussion on Sm nuclei into two categories: one is for vibrational and transitional nuclei, $^{146-152}$Sm, in Figs. 2(a) and 2(d), which are shown just for reference,

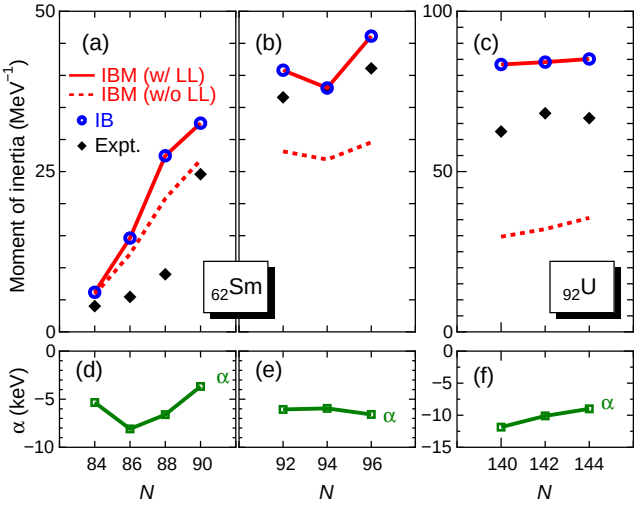

FIG. 2: (Color online) (Upper panels) MOIs in the intrinsic state for (a) $^{146-152}$Sm, (b) $^{154-158}$Sm and (c) $^{232-236}$U, calculated by IBM with (w/) and without (w/o) including the $L \cdot L$ term and by IB formula. Experimental data taken from 2_1^+ energy [25]. (Lower panel) The derived α value for (d) $^{146-152}$Sm, (e) $^{154-158}$Sm and (f) $^{232-236}$U. Skyrme SkM* force is used.

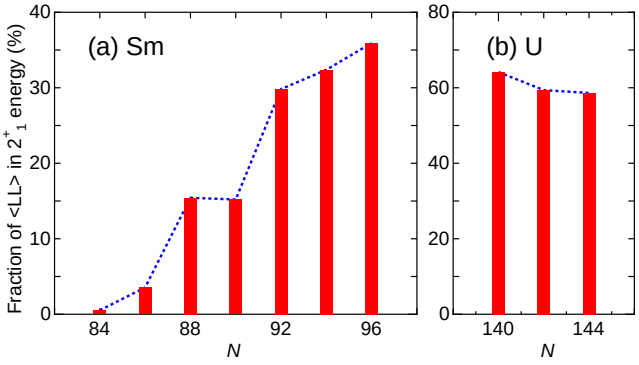

FIG. 3: (Color online) Fraction of the $L \cdot L$ matrix element $\langle LL \rangle$ in the 2_1^+ energy for (a) Sm and (b) U isotopes.

and the other is for the deformed rotational $^{154-158}$Sm nuclei close to SU(3) limit in Figs. 2(b) and 2(e). The criterion for this division is quite simple. It is whether the $\mathcal{I}_B$ (*w/o LL*) is larger (former) or smaller (latter) than the experimental data. In addition, we denote hereafter the difference between $\mathcal{I}_F$ and $\mathcal{I}_B$ (*w/o LL*) as $\Delta\mathcal{I} \equiv \mathcal{I}_F - \mathcal{I}_B$ (*w/o LL*).

A nucleus close to the mid shell is supposed to show rotational character. This is reflected in the increase of $\mathcal{I}_F$ and $\mathcal{I}_B$ with N in Fig. 2(a). The experimental MOI exhibits rapid change from N =88 to 90. $\mathcal{I}_B$ and $\mathcal{I}_F$ do not follow this trend, but show gradual evolution. For $N \geqslant 92$ Sm nuclei in Fig. 2(b), on the other hand, the experimental MOI becomes larger and looks nearly flat, being approximately $35 \sim 40$ MeV^{-1}. The IB formula reproduces this trend well. $\Delta\mathcal{I}$s for $N = 92 \sim 96$ Sm nuclei are much larger than those for $N = 84 \sim 90$ ones. Conversely, the $L \cdot L$ term appears to make crucial contribution to reproducing the $\mathcal{I}_F$ value for deformed rotational nuclei. One can also observe, in Fig. 2(c), the flat systematics of the MOIs for $^{232-236}$U nuclei. The experimental MOI for U isotopes is approximately 70 MeV^{-1}, which is about twice as large as those of $^{154-158}$Sm nuclei in Fig. 2(b). Thus, U nuclei exhibit more pronounced rotational feature, compared to Sm.

The lower panels of Fig. 2 shows the derived α value for (d) $^{146-152}$Sm, (e) $^{154-158}$Sm and (c) $^{232-236}$U nuclei. In Figs. 2(e) and 2(f), the magnitude of α is nearly proportional to $\Delta\mathcal{I}$. The rule does not apply for vibrational $N = 84 \sim 88$ Sm nuclei. While $\Delta\mathcal{I}$ is relatively small ($\lesssim 5$ MeV^{-1}) in these nuclei, the derived α value is almost comparable in magnitude to the heavier ($N = 92 \sim 96$) ones. In other words, $\mathcal{I}_B$ does not depend much on α.

This becomes more apparent by studying to what extent the 2_1^+ energy is lowered by the $L \cdot L$ term. Figure 3 shows fraction of the $L \cdot L$ matrix element $\langle LL \rangle$ in the 2_1^+ energy (*w/o LL*) for (a) Sm and (b) U nuclei. Levels are calculated by NPBOS code [24]. More than 30% (60%) of the 2_1^+ energy is lowered by the $L \cdot L$ term for $N = 92 \sim 96$ ($N = 140 \sim 144$) Sm (U) nuclei. However, its effect does almost nothing for $N = 84$ and 86 Sm nuclei, and is also much less for $N = 88$ and 90 compared with $N = 92 \sim 96$ ones. Thus, as expected, the $L \cdot L$ term can be neglected for the $N \leqslant 90$ nuclei, because the $\mathcal{I}_B$ (*w/o LL*) value is larger than the experimental one in Fig. 2(a).

Figure 4(a) shows the low-lying yrast spectra for Sm isotopes. The calculated spectra decrease with N and exhibit rotational feature for $N \geqslant 92$ similarly to the experimental trend [25]. One sees remarkable agreements for all calculated levels of $N \geqslant 92$ nuclei with the experimental data. $N = 88$ and critical-point $N = 90$ [26, 27] nuclei exhibit rather pronounced rotational feature as compared with the experimental data. The deviation arise because the peculiar topology of Skyrme-EDF PES may not always show phase transitional change in the considered Sm isotopic sequence [13]. This is inherited in the corresponding spectra.

Figures 4(b) and 4(c) show level schemes of selected nuclei, ^{154}Sm and ^{232}U, respectively. The $L \cdot L$ term remarkably improves the agreement between the theoretical and the experimental spectra [25]. Particularly for ^{154}Sm, the present work gives almost identical result to the experimental spectra. While there is quantitative deviation as in ^{232}U, the results are promising. One also sees remarkable agreements for ^{156,158}Sm and ^{234,236}U nuclei. It should be emphasized that all these agreements are not accidental, but are observed in many other deformed nuclei from rare-earth to actinoid regions.

Lastly, we comment that the $L \cdot L$ term slightly improves agreement of side-band energies. The calculated β- and γ-bandhead energies are lowered by tens of keV.

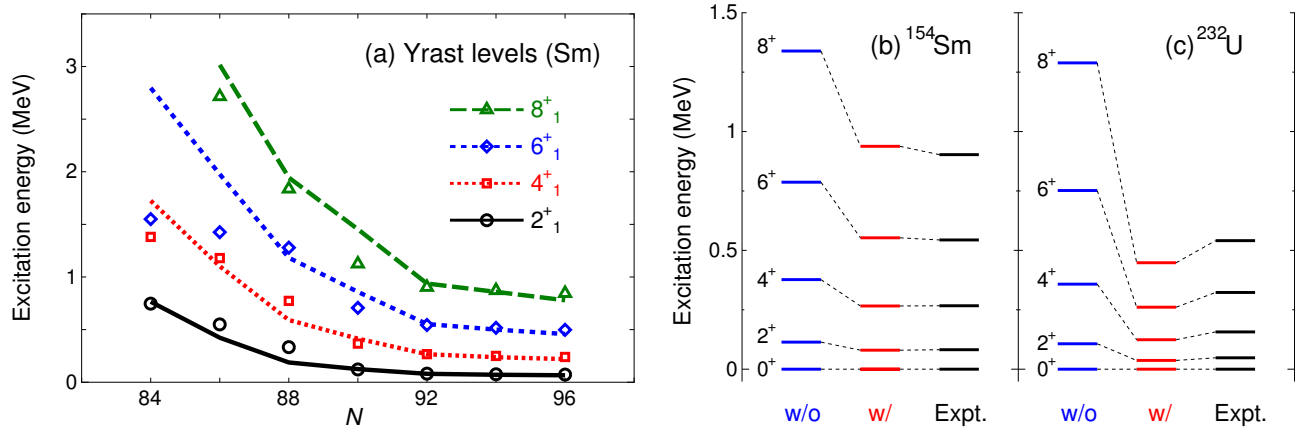

FIG. 4: (color online) (a) Experimental [25] (symbols) and calculated (curves) yrast spectra for Sm isotopes as functions of N. Level schemes for (b) ^{154}Sm and (c) ^{232}U nuclei. Calculated spectra with (w/) and without (w/o) the $L \cdot L$ term, and experimental spectra [25]. Skyrme SkM* force is used.

The relative spacing inside the bands is reduced by a hundred of keV. However, these energies are still much larger than the experimental ones. More studies are then needed for side-band energies.

In summary, we have introduced a novel term in fermion-boson mapping for the IBM for nuclear rotation. The rotation of deformed nucleon system differs in its response to the cranking from its boson image by the present mapping method [11]. This difference requires the $L \cdot L$ term in the IBM Hamiltonian. We emphasize once more that the determination of IBM Hamiltonian by the PES is valid basically for static properties and their excitations, while the rotational response is a specific time-dependent mode and thereby may not be fully taken into account by the analysis of the PES. The $L \cdot L$ term has been introduced microscopically and shown to make essential contribution to rotational spectra, solving the long-standing problem of too small moment of inertia. Experimental data are reproduced remarkably well, without any phenomenological correction. The feature discussed in this Letter can be closely related to another open question as to whether the IBM can be applied to deformed nuclei or not [28]. Studies on such intriguing issues considering the present result are about to begin.

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