Exploring universality in nuclear clusters with Halo EFT *

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Abstract I present results and highlight aspects of halo EFT to loosely bound systems composed of nucleons and alpha particles, with emphasis on Coulomb interactions.

Keywords Halo nuclei · Cluster systems · Effective Field Theory

1 Introduction

The physics of exotic nuclei still keeps motivating several research initiatives worldwide. Dedicated ongoing and future experiments promise to deliver more intense beams of rare isotopes along with new sophisticated detection techniques, paving the way to explore the limits of existence of several unknown nuclear systems and their unusual properties.

Halo nuclei and nucleon-alpha clusters are particular examples of exotic nuclei and constitute the focus of this talk. They are normally characterized by a large structure relative to the typical size of each of its components, nucleons and/or stable nuclei. The large-distance physics of those systems is a response to the shallowness of their separation binding energies, $B_{lo} \sim 0.1$ MeV. To them is associated a low-momentum scale $M_{lo} \approx \sqrt{2\mu B_{lo}}$ which contrasts with a high-momentum M_{hi} set by the energy required to excite a core, usually of the order of a few MeV. This separation of scales matches quite well with the ideas of effective field theories (EFTs), where the ratio of scales sets an expansion parameter that provides systematic and model-independent predictions, as well as more rigorous control over theoretical uncertainties.

Halo/cluster EFT has been developed to account for certain aspects of loosely bound nuclear clusters, namely, low-energy resonances and Coulomb interactions. In the following I explain how these features are handled, with $\alpha\alpha$ and $p\alpha$ systems as examples.

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2 $\alpha\alpha$ and $p\alpha$ systems

The power counting for low-energy narrow resonances was developed in [1]. Unlike shallow bound states, a higher amount of fine-tuning is required to produce the expected energy dependence of the amplitude. That means a non-static two-body propagator or, equivalently, the sum of effective range contributions to all orders. As an example let us start with the $\alpha\alpha$ interaction, whose strong part is described by the following Lagrangian

$$\mathscr{L} = \phi^{\dagger} \left[i\partial_0 + \frac{\nabla^2}{2m_{\alpha}} \right] \phi - d^{\dagger} \left[i\partial_0 + \frac{\nabla^2}{4m_{\alpha}} - \Delta \right] d + g \left[d^{\dagger} \phi \phi + (\phi \phi)^{\dagger} d \right] + \cdots$$
 (1)

We introduce an auxiliary (dimer) field *d* with "residual mass" Δ , carrying the quantum numbers of two alphas in *S*-wave and coupling with their fields via the coupling constant *g*. For a non-static *d* propagator Δ has to scale as the kinetic energy, $\sim M_{lo}^2/4m_{\alpha}$, already two orders away from the natural scaling $\sim M_{hi}^2/4m_{\alpha}$. The dots stand for higher order terms in a derivative expansion. Electromagnetic interactions are introduced via minimal substitution, with Coulomb forces being the dominant ones [2]. The Sommerfeld parameter $\eta = Z_{\alpha}^2 \alpha_{em} m_{\alpha}/2k = k_C/k$ sets the magnitude of Coulomb interactions, where k_C is the inverse of the $\alpha \alpha$ Bohr radius. The fact that k_C is numerically of $O(M_{hi})$ requires the sum of Coulomb photons to all orders. The technique to handle this problem was given by Kong and Ravndal, using established ideas of Coulomb Green's function and two-potential scattering [3]. The Coulomb-distorted strong amplitude acquires the form of a Coulomb-modified effective range expansion, with the shape parameter term treated as a perturbation [2].

The $\alpha\alpha$ system is remarkable for having a scattering length of the order of 2000 fm, while the effective range and shape parameter obey natural dimensional analysis, $r_0 \approx$ 1 fm ~ M_{hi}^{-1} and $\mathcal{P}_0 \approx 1.5 \text{ fm}^3 \sim M_{hi}^{1-3}$. The $\alpha \alpha$ resonance energy of $E_R \simeq 92 \text{ keV}$ sets the low-momentum scale to $M_{lo} \sim \sqrt{m_{\alpha} E_R} \approx 20$ MeV, well below the high-momentum scale set by the pion mass or the excitation energy of the alpha particle, $M_{hi} \sim m_{\pi} \sim$ $\sqrt{m_{\alpha}E_{\alpha}^{*}} \approx 140$ MeV. With an expansion parameter around 1/7, the strong scattering length $a_{0,s} \sim m_{\alpha}g^2/\Delta \sim M_{hi}/M_{lo}^2$ can at most be of the order of few hundreds of MeV. We found in our study [2] that most of the remaining scaling factor comes from a detailed cancellation between strong and electromagnetic interactions, an incredible amount of fine tuning. This is the outcome of an exponentially suppressed resonance width due to a large Coulomb barrier, entangled with the location of the resonance very close to the $\alpha\alpha$ threshold. This fine tuning was not expected and is not well understood, but leads to some interesting scenarios. First, an increase on the strong force by a few percent is enough to produce a bound ⁸Be, which could have drastic astrophysical consequences. Second, the already large fine tuning in the strong parameters lead to an unitary amplitude at leading order (LO) when Coulomb is turned off. The ⁸Be would then be a bound state at zero energy, and the system with three α 's would exhibit an exact Efimov spectrum [2,4]. Although Coulomb forces are highly non-perturbative, the fact that both ⁸Be and the Hoyle state (a 12 C excited state ~ 400 keV above the 3α threshold) remains very close to threshold supports such picture close to the unitary limit. The Hoyle state is essential to describe the correct abundance of 12 C in the universe. Its existence is usually given as an example of large fine tuning in the parameters of the underlying theory [5], but how that relates to the fine tuning in the $\alpha\alpha$ system is almost an unexplored subject. Our study provides a humble step to address this issue.

Despite this puzzling fine tuning, the associated power counting seems to be the correct one to describe the scattering data. At LO we have no free parameters, since we use the latest measurements of the resonance position and width [6] as input. At NLO, the extra

Table 1 $\alpha \alpha$ effective range parameters.



Fig. 1 S-wave $\alpha \alpha$ scattering phase shifts (left panel) and $p\alpha$ differential cross-section at 140° CM angle (right panel).

parameter is fitted to the $\alpha\alpha$ phase shifts. The large cancellation between strong and electromagnetic forces allows us to extract the effective range parameters with an accuracy better than previous determinations [7].

The EFT approach to $p\alpha$ scattering follows steps similar to the $\alpha\alpha$ case [8]. However, in this situation the envelope of the resonance is mainly given by the angular momentum barrier — Coulomb forces provide just a correction to the strong part. At LO the amplitude receives contributions of both the $S_{1/2}$ and $P_{3/2}$ except around the $P_{3/2}$ resonance, which is enhanced. At NLO the $S_{1/2}$ effective range and $P_{3/2}$ shape parameter enter as perturbations. The $P_{1/2}$ partial wave contributes only at higher orders [1,8]. Preliminary result is shown in the right panel of Fig. 1 using the effective range parameters from Arndt *et al.* [9], compared to the measurements performed by Nurmela *et al.* [10]. The shape of the resonance peak reflects the smaller values obtained by Ref. [10, 11] relative to previous measurements used in Ref. [9].

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