How many of the scaling trends in pp collisions will be violated at $\sqrt{s_{\text{NN}}} = 14 \text{ TeV}$? -Predictions from Monte Carlo quark-gluon string model.

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(Dated: November 12, 2010)

Multiplicity, rapidity and transverse momentum distributions of hadrons produced both in inelastic and non-diffractive pp collisions at energies from $\sqrt{s} = 200 \text{ GeV}$ to 14 TeV are studied within the Monte Carlo quark-gluon string model. Good agreement with the available experimental data is obtained and predictions are made for the collisions at top LHC energy $\sqrt{s} = 14 \text{ TeV}$. The model indicates that Feynman scaling and extended longitudinal scaling remain valid in the fragmentation regions, whereas strong violation of Feynman scaling is observed at midrapidity. The KNO scaling in multiplicity distributions is violated at LHC also. The origin of both maintenance and violation of the scaling trends is traced to short range correlations of particles in the strings and interplay between the multi-string processes at ultra-relativistic energies.

PACS numbers: 24.10.Lx, 13.85.-t, 12.40.Nn

I. INTRODUCTION

The recent interest to general features of elementary hadronic interactions, especially to characteristics of ppcollisions, at ultrarelativistic energies is manifold. First of all, these collisions are conventionally used as reference ones to reveal the nuclear matter effects, such as strangeness enhancement, nuclear shadowing, collective flow etc., attributed to formation of a pattern of hot and dense nuclear matter and the quark-gluon plasma (QGP) in the course of ultrarelativistic heavy-ion collisions (see [1] and references therein). Although the formation of the QGP and/or collective behavior was not found yet in pp collisions at energies up to the Tevatron energy $\sqrt{s} = 1.8 \,\text{TeV}$, physicists are discussing the possibility to observe, e.g., elliptic flow in pp interactions at $\sqrt{s} = 7 \text{ TeV}$ accessible for the Large Hadron Collider (LHC) at CERN at present. This limit may be raised to $\sqrt{s} = 14 \text{ TeV}$ in the nearest future. Because of the huge amount of energy deposited in the overlapping region the pp systems might be similar to A + A collisions at nonzero impact parameter at lower energies [2, 3]and, therefore, demonstrate collectivity. An alternative approach developed in [4] considers the flow effects in hadronic interactions as initial state effects linked to correlation between the transverse momentum and position in the transverse plane of a parton in a hadron. This problem should definitely be clarified in future. Then, the problem of multiparticle production in elementary

hadronic collisions is not fully solved yet. Here for hard processes with large momentum transfer the running coupling constant α_S is small, that allows for application of the perturbative quantum chromodynamics (QCD). For soft processes with small momentum transfer, which give dominant contribution to high energy hadronic interactions, the α_S is close to unity and, therefore, nonperturbative methods should be applied. Many microscopic models [5–13] based on the string picture of particle production [14] have been successfully employed to describe gross features of hadronic collisions at relativistic and ultrarelativistic energies, whereas the statistical approach pioneered more than 50 years ago by Fermi and Landau [15, 16] is not ruled out. To make predictions for the LHC in the latter case one has to extrapolate the data obtained at lower energies to the high energy region. It was found quite long ago that, despite the complexity of a reaction with tens or more particles in a final state, multiparticle production in pp collisions exhibits several universal trends, such as $(\ln \sqrt{s})^2$ dependence of total charged particle multiplicity [17], Feynman scaling [18] and related to it extended longitudinal scaling [19], KNO scaling [20] and so on. Similar trends were found later on in proton-nucleus and nucleus-nucleus collisions as well (for review see, e.g., [21]). On the other hand, the description of ultrarelativistic hadronic interactions in the framework of Color Glass Condensate (CGC) theory [22] leads to a universal power-law behavior of, e.g., density of produced charged particles per unit of rapidity and their transverse momentum [23].

The aim of the present article is to study the main characteristics of pp interactions at energies from $\sqrt{s} =$ 200 GeV to top LHC energy $\sqrt{s} = 14$ TeV. We employ the Monte Carlo (MC) realization [7] of the quark-gluon string model (QGSM) [24] based on Gribov's Reggeon field theory (RFT) [25] that obeys both analyticity and

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unitarity requirements. The features of the model are described in Sec. II in details. Comparisons with available experimental data for $\bar{p}p$ and pp collisions at energies $\sqrt{s} \geq 200 \text{ GeV}$, including the recently measured pp interactions at $\sqrt{s} = 2.36 \text{ TeV}$ and $\sqrt{s} = 7 \text{ TeV}$, and predictions for the top LHC energy $\sqrt{s} = 14 \text{ TeV}$ are presented in Sec. III. Here exclusive contributions of soft and hard processes to particle rapidity and transverse momentum spectra are studied. Special attention is given to the origin of violation of the KNO-scaling, violation of the Feynman scaling at midrapidity and its maintenance in the fragmentation region. Obtained QGSM results are also confronted to the predictions of other microscopic and macroscopic models. Conclusions are drawn in Sec. IV.

II. QUARK-GLUON STRING MODEL AND ITS MONTE CARLO REALIZATION

As was mentioned in the Introduction, the description of soft hadronic processes cannot be done within the perturbative QCD. Therefore, the quark-gluon string model [24] employs the so-called 1/N series expansion [26, 27] of the amplitude for processes in QCD, where N is either number of colors N_c [26] or number of flavors N_f [27]. It this approach the amplitude of a hadronic process is represented as a sum over diagrams of various topologies, so the method is often called *topological expansion*. It appears that at high energies and small momentum transfer the arising diagrams are similar [28, 29] to processes describing the exchange of Regge singularities in the tchannel. For instance, planar diagrams correspond to the exchange of Reggeons, and cylinder diagrams correspond to reactions without quantum number exchange in the t-channel, i.e., taking place via the Pomeron exchange, where Pomeron is a composite state of the reggeized gluons. Processes with many-Pomeron or many-Reggeon exchanges are also possible. To find the amplitude of multiparticle production one has to cut the diagrams in the s-channel, and the physical picture of quark-gluon strings arises. Namely, new particles are produced through the formation and break-up of quark-gluon strings or exited objects consisting of quarks, diquarks and their antistates connected by a gluon string.

Figure 1 shows the subprocesses with particle creation taken into account in the current Monte Carlo version of the QGSM [7] for pp collisions at ultrarelativistic energies. The inelastic cross section consists of three terms

$$\sigma_{inel}^{pp}(s) = \sigma_P(s) + \sigma_{SD}(s) + \sigma_{DD}(s) , \qquad (1)$$

where $\sigma_P(s)$ is the cross section for the multi-chain processes described by the cylinder diagram and diagrams with multi-Pomeron scattering [Fig. 1(a)], $\sigma_{SD}(s)$ by the single diffractive processes with small [Fig. 1(b)] and large [Fig. 1(c)] mass excitation, corresponding to the triple-Reggeon and triple-Pomeron limit, respectively, and $\sigma_{DD}(s)$ by the double diffractive diagram [Fig. 1(d)].



FIG. 1: Diagrams of particle production processes included in the modeling of pp interactions at ultrarelativistic energies. See text for details.

Other diagrams that are relevant at low and intermediate energies, such as undeveloped cylinder diagram or diagram with quark rearrangement [7], play a minor role here because their cross sections rapidly drop with rising s. The statistical weight of each subprocess is expressed in terms of the interaction cross section for the given subprocess $\sigma_i(s)$

$$\omega_i = \sigma_i(s) / \sigma_{inel}(s) . \tag{2}$$

Then, the hadron inelastic interaction cross section $\sigma_{inel}(s) = \sigma_{tot}(s) - \sigma_{el}(s)$ is split into the cross section for single diffractive interactions $\sigma_{SD}(s)$ and the cross section for non-diffractive reactions $\sigma_{ND}(s)$, as it is usually done in analysis of experimental data. By means of the Abramovskii-Gribov-Kancheli (AGK) cutting rules [30] the inelastic non-diffractive interaction cross section $\sigma_{ND}(s)$ can be expressed via the sum of the cross sections for the production of $n = 1, 2, \ldots$ pairs of quark-gluon strings, or cut Pomerons, and the cross section of double diffractive process

$$\sigma_{ND}(s) = \sum_{n=1}^{\infty} \sigma_n(s) + \sigma_{DD}(s) .$$
 (3)

To find $\sigma_n(s)$ one can rely on the quasi-eikonal model [29, 33] which states that

$$\sigma_{tot}(s) = \sum_{n=0}^{\infty} \sigma_n(s) = \sigma_P f\left(\frac{z}{2}\right) , \qquad (4)$$

$$\sigma_n(s) = \frac{\sigma_P}{nz} \left(1 - \exp\left(-z\right) \sum_{k=0}^{n-1} \frac{z^k}{k!} \right), \ k \ge 1 \quad (5)$$

$$\sigma_0 = \sigma_P \left(f\left(\frac{z}{2}\right) - f(z) \right) , \qquad (6)$$

$$f(z) = \sum_{\nu=1}^{\infty} \frac{(-z)^{\nu-1}}{\nu\nu!} , \qquad (7)$$

Here

$$\sigma_P = 8\pi\gamma_P \exp\left(\Delta\xi\right), \qquad (8)$$

$$z = \frac{2C\gamma_P}{(R_P^2 + \alpha'_P \xi)} \exp\left(\Delta\xi\right) . \tag{9}$$

The cross section σ_0 corresponds to diffraction contribution. The parameters γ_P and R_P are Pomeron-nucleon vertex parameters, quantity $\Delta \equiv \alpha_P(0) - 1$, and $\alpha_P(0)$ and α'_P is the intercept and the slope of the Pomeron trajectory, respectively. The quantity C takes into account the deviation from the pure eikonal approximation (C = 1) due to intermediate inelastic diffractive states, $\xi = \ln (s/s_0)$ and s_0 is a scale parameter.

For the diffractive processes displayed in Fig. 1(b) and Fig. 1(c) the fractions of momenta of initial hadrons carried by the sea quark pairs $x_{q\bar{q}}$ are determined according to distribution

$$u_{q\bar{q}}^{h}(x_{q\bar{q}}) \propto \frac{(1 - x_{q\bar{q}})^5}{x_{q\bar{q}}}$$
 (10)

with the cut for the minimum values

$$x_{q\bar{q}} \le \frac{0.14}{\sqrt{s}} \ . \tag{11}$$

The transverse momentum distribution of (anti)quarks in a proton in the low-mass excitation process shown in Fig. 1(b) is given by

$$f_q(\vec{p_T}) \, dp_T = \frac{b_1}{\sqrt{\pi}} \, \exp\left(-b_1 p_T^2\right) d\vec{p_T} \,\,, \qquad (12)$$

where the slope parameter $b_1 = 20 \,(\text{GeV}/c)^{-2}$. Then, it is assumed that the valence (anti)diquark in the (anti)proton carries a transverse momentum equal in magnitude and opposite in sign to the sum of transverse momenta of the other (anti)quarks. The number of quark-gluon strings increases with collision energy, thus the average transverse momentum of the (anti)diquark rises also.

The quantitative description of single diffractive and double diffractive processes at high energies was done in QGSM recently [31, 32] in terms of dressed tripple-Reggeon and loop diagrams. The results obtained in [31] for the cross sections of the diffractive processes are utilized in our MC model via the parametrizations

$$\sigma_{SD}(s) = 0.68 \left(1 + \frac{36}{s}\right) \ln\left(0.6 + 0.2\,s\right) \,, \quad (13)$$

$$\sigma_{DD}(s) = 1.65 + 0.27 \ln s . \tag{14}$$

Soft processes dominate the particle production in hadronic interactions at intermediate energies. With

the rise of the collision energy hard processes resulting to formation of hadronic jets with large transverse momenta p_T become important also. To take into account the jet formation and, on the other hand, to describe simultaneously the increase of the total and inelastic hadronic interaction cross section with rising \sqrt{s} the eikonal model was properly modified in [34] by introducing the new term that represents the hard Pomeron exchange. The cut of the hard Pomeron leads to formation of two hadronic jets, see Fig. 2. Therefore, the



FIG. 2: String formation in hard gluon-gluon scattering and soft Pomeron exchange in proton-proton collision.

eikonal u(s, b), that depends on the center-of-mass energy \sqrt{s} and the impact parameter b, can be decomposed onto the terms corresponding to soft and hard Pomeron exchange:

$$u(s,b) = u_{soft}(s,b) + u_{hard}(s,b)$$
. (15)

The inelastic hadronic cross section $\sigma_{inel}(s)$ is connected to the real part of the eikonal $u^R(s, b)$ as

$$\tau_{inel}(s) = 2\pi \int_{0}^{\infty} \left\{ 1 - \exp\left[-2u^{R}(s,b)\right] \right\} \, bdb \; . \tag{16}$$

Recall that the concept of (semi)hard Pomeron is nowadays a common feature of all RFT-based MC models [8, 12, 13, 35, 36] designed for the description of hadronic and nuclear interactions at ultrarelativistic energies. Other microscopic MC models also rely on the picture of soft+hard eikonal approach [10].

Following [34, 35], both soft and hard eikonal can be expressed as

$$u_{soft/hard}^{R}(s,b) = z_{soft/hard}(s) \exp\left[-\frac{\beta^2}{4\lambda_{soft/hard}(s)}\right],$$
(17)

where [cf. Eqs. (4)-(9)]

$$z_{soft/hard}(s) = \frac{\gamma_P}{\lambda_{soft/hard}(s)} \left(\frac{s}{s_0}\right)^{\alpha_P(0)-1}$$
(18)

$$\lambda_{soft/hard}(s) = R_P^2 + \alpha'_P \ln\left(\frac{s}{s_0}\right). \tag{19}$$

TABLE I: Parameters of the soft and hard Pomeron used in the current version of the QGSM.

Parameter	Soft Pomeron	Hard Pomeron
$\alpha_P(0)$	1.1855	1.3217
α'_P	0.175	0
γ_P	0.832	0.021
R_P	2.6	2.4

Numerical values of the slopes and intercepts of the Pomeron trajectories and parameters of the hadron coupling to the Pomeron used in the model fit to experimental data are listed in Table I. Note that these values deviate from the parameters of the soft and the hard Pomeron obtained in [34, 37] from the cross section of minijets measured by the UA1 Collaboration. To describe the LHC data at energies above $\sqrt{s} = 900 \text{ GeV}$ it was necessary to increase the soft Pomeron intercept to $\alpha_P(0) - 1 \approx 0.185$ and simultaneously to reduce the value of its slope parameter α'_P to 0.175.

Then, the AGK cutting rules enable one to express the inelastic cross section as

$$\sigma_{inel}(s) = \sum_{i,j=0; i+j \ge 1} \sigma_{ij}(s) , \qquad (20)$$

where

$$\sigma_{ij}(s) = 2\pi \int_{0}^{\infty} bdb \exp\left[-2u^{R}(s,b)\right] \qquad (21)$$
$$\times \frac{\left[2u^{R}_{soft}(s,b)\right]^{i}}{i!} \frac{\left[2u^{R}_{hard}(s,b)\right]^{j}}{j!} .$$

The last expression can be used to determine the number of quark-gluon strings and hard jets via the number of cut soft and hard Pomerons, respectively. At very high energies one has to take into account the effects of shadowing of partonic distributions both in nucleons and in nuclei. In the Reggeon calculus such processes correspond to the so-called enhanced diagrams [38] describing the interactions between Pomerons. These diagrams are not implemented yet in the current MC version of the QGSM.

It is worth noting here that the multi-Pomeron exchanges become very important with increasing c.m. energy of hadronic collision. For instance, the contribution of a single-cylinder diagram to the scattering amplitude is proportional to $(s/s_0)^{\alpha_P(0)-1}$, $\alpha_P(0) > 0$. In contrast, the contributions coming from the *n*-Pomeron exchanges grow as $(s/s_0)^{n\Delta}$. Although in the framework of the 1/N-expansion the *n*-Pomeron exchange amplitudes are suppressed by factor $1/N^{2n}$, the quickly rising term $s^{n\Delta}$ dominates over the suppression factor at ultrarelativistic energies.

There is no unique theoretical prescription for modeling the fragmentation of a string with a given mass,

TABLE II: Total, elastic, multi-chain, single diffraction and double diffraction cross sections of pp collisions calculated by the QGSM.

\sqrt{s} (GeV)	$\sigma_{tot} (mb)$	$\sigma_{el} \ (mb)$	$\sigma_P \ ({\rm mb})$	σ_{SD} (mb)	σ_{DD} (mb)
200	51.62	9.67	31.12	6.12	4.51
546	60.83	12.51	35.72	7.48	5.05
630	62.25	12.97	36.42	7.67	5.13
900	65.85	14.15	38.19	8.16	5.32
1800	72.97	16.55	41.61	9.10	5.70
2360	75.74	17.50	42.92	9.47	5.84
7000	86.60	21.31	47.91	10.95	6.43
14000	93.07	23.61	50.76	11.89	6.80

momentum and quark content into hadrons. In the presented version of the QGSM the Field-Feynman algorithm [39] is employed. It enables one to consider emission of hadrons from both ends of the string with equal probabilities. The break-up procedure invokes the energy-momentum conservation and the preservation of the quark numbers. The transverse momentum of the (di)quarks in the vacuum pair is determined by the power-law probability

$$f(p_T^2) dp_T^2 = \frac{3 D b_2(s)}{\pi (1 + D p_T^2)^4} dp_T^2 , \qquad (22)$$

$$b_2(s) = 0.325 + 0.016 \ln s$$
, (23)

with $D = 0.34 \, (\text{GeV}/c)^{-2}$.

Further details of the MC version of QGSM and its extension to h + A and A + A collisions can be found in [7, 40, 41].

III. COMPARISON WITH DATA AND PREDICTIONS FOR LHC

For the comparison with model results concerning the pseudorapidity and transverse momentum distributions we used experimental data obtained by the UA5 Collaboration for proton - antiproton collisions at c.m. energies $\sqrt{s} = 200 \text{ GeV}$, 546 GeV and 900 GeV [42], by the UA1 Collaboration for $\bar{p}p$ collisions at $\sqrt{s} = 546 \text{ GeV}$ [43], by the CDF and the E735 Collaborations for $\bar{p}p$ collisions at $\sqrt{s} = 1800 \text{ GeV}$ [44, 45], and recent CERN LHC data obtained in pp interactions at $\sqrt{s} = 900 \text{ GeV}$, 2360 GeV, and 7 TeV by the ALICE Collaboration [46–48] and by the CMS Collaboration [49, 50]. At such high energies the annihilation cross section is almost zero and the main characteristics of particle production in pp and $\bar{p}p$ interactions are essentially similar.

Total and elastic cross sections are listed in Table II together with the cross sections of multi-chain, singleand double-diffraction processes for energies ranging from $\sqrt{s} = 200 \text{ GeV}$ to $\sqrt{s} = 14 \text{ TeV}$. Compared to those at $\sqrt{s} = 900 \text{ GeV}$, σ_{tot} , σ_{el} and σ_{SD} increase at $\sqrt{s} = 14 \text{ TeV}$ by nearly 50%, whereas σ_{DD} increases by less than 30%.



FIG. 3: (Color online) Transverse momentum distributions of the invariant cross section of charged particles in NSD pp collisions obtained in QGSM at $|y| \leq 2.5$ for all energies in question. Experimental data are taken from [42–44, 50].

The transverse momentum distributions of the invariant cross section $E \frac{d^3\sigma}{dp^3}$ divided to σ_{tot} for charged particles in non-single diffractive (NSD) pp collisions at all energies in question are presented in Fig. 3. We see that the QGSM reproduces the experimental data in a broad energy range pretty well. The spectra become harder with increasing \sqrt{s} , thus the average transverse momentum of produced hadrons should grow also. Fig. 4 displays the $\langle p_T \rangle$ of charged particles in NSD pp events calculated in QGSM and compared to experimental data. We assume here 5% systematic errors for the extraction of mean p_T because we do not apply any extrapolation procedure to the generated spectra, as it is usually done in the experiments. Results of the fit of model simulations to quadratic logarithmic dependence and to power-law dependence are as follows

$$\langle p_T \rangle = 0.42 - 0.006 \ln s + 0.0022 \ln^2 s ,$$

 $\langle p_T \rangle = 0.27 + 0.212 E^{0.115} .$

In the last expression $E = \sqrt{s}/2$ and the exponent 0.115 is not a free parameter. According to [23], this exponent is just half of the exponent of power-law fit to $dN/d\eta$ distribution (see below). As one can see in Fig. 4, the difference between two parametrizations of mean p_T is negligible even for top LHC energy $\sqrt{s} = 14$ TeV.

To study the interplay between the soft and hard processes we show separately in Fig. 5 their fractional contributions and combined results for pp collisions at $\sqrt{s} = 900 \text{ GeV}$, 2.36 TeV, 7 TeV and at top LHC energy



FIG. 4: (Color online) Average transverse momentum as function of \sqrt{s} . Squares present the QGSM results, other symbols denote experimental data from [42–45, 48, 50]. Solid and dashed lines are fit to logarithmic and power-law dependences, respectively. See text for details.

 $\sqrt{s} = 14 \,\mathrm{TeV}$. Moreover, the p_T -dependence of the underlying soft processes from the collisions with at least one hard Pomeron is displayed in these plots as well together with the experimental data of the CMS Collaboration. As expected, the soft processes dominate at low and intermediate transverse momenta, whereas at higher transverse momenta the major contribution to the cross section comes from the minijets. The crossover between the hard and soft branches takes place at $p_T \approx 2.8 \,\mathrm{GeV}/c$ for the reactions at $\sqrt{s} = 900 \,\text{GeV}$. It is shifted to $p_T \approx 2.2 \,\mathrm{GeV}/c$ at $\sqrt{s} = 14 \,\mathrm{TeV}$. The slopes of the p_T spectra for both soft and underlying soft processes are similar. At $\sqrt{s} = 7 \text{ TeV}$ and 14 TeV both lines coincide, i.e., the contributions to the invariant cross sections from barely soft Pomeron processes are equal to those from the soft Pomerons exchanges, accompanied by one or more hard Pomeron ones.

Let us briefly recall the main assumptions and predictions of the hypothesis of Feynman scaling [18]. It requires scaling behavior of particle spectra within the whole kinematically allowed region of the Feynman scaling variable $x_{\rm F} \equiv p_{||}/p_{||}^{max}$ or, alternatively, c.m. rapidity y^* at ultrarelativistic energies $s \to \infty$. In addition, the existence of non-vanishing central area $|x_{\rm F}| \leq x_0$, $x_0 \sim$ 0.1 is postulated. In terms of rapidity this central region increases with rising \sqrt{s} as

$$(\Delta y^*)_{centr} \approx 2 \ln \left[x_0 \sqrt{s}/m_T \right]$$
 (24)

provided the transverse mass $m_T = \sqrt{m_0^2 + p_T^2}$ is finite. In contrast, the fragmentation region remains constant

$$(\Delta y^*)_{frag} \approx \ln\left(1/x_0\right) \,. \tag{25}$$

From here it follows that (i) in the central area the particle density $\rho_{cent}(y^*, p_T, s)$ depends on neither y^* nor



FIG. 5: (Color online) Transverse momentum distributions of the invariant cross section of charged particles in NSD ppcollisions at (a) $\sqrt{s} = 900 \text{ GeV}$, (b) $\sqrt{s} = 2360 \text{ GeV}$, (c) $\sqrt{s} =$ 7 TeV and (d) $\sqrt{s} = 14$ TeV calculated in QGSM. Combined contribution of all processes and, separately, of only soft, hard and underlying soft sub-processes are shown by solid, dashdotted, dashed and dotted lines, respectively (see text for details). Experimental data plotted in (a), (b) and (c) panels are taken from [49, 50].

 \sqrt{s} , i.e. $\rho_{cent} \equiv \rho_{cent}(p_T)$, and rapidity spectra of particles have, therefore, a broad plateau; (ii) this area gives a main contribution to average multiplicity of produced hadrons; (iii) contribution to the average multiplicity from the fragmentation regions is energy independent.

The charged particle pseudorapidity spectra 1 $d\sigma_{inel}$ $d\sigma_{NSD}$ 1 for inelastic and and $d\eta$ σ_{NSD} σ_{inel} $d\eta$ non-single-diffractive events, respectively, are displayed in Fig. 6(a) and Fig. 6(b) together with the $pp(\bar{p}p)$ data at $\sqrt{s} = 200 \,\text{GeV}, 546 \,\text{GeV}, 900 \,\text{GeV}, 2.36 \,\text{TeV}$ and 7 TeV. QGSM predictions for $\sqrt{s} = 14$ TeV are plotted here also. The model gives good description of these distributions within the indicated energy range except, maybe, not very distinct dip at midrapidity for the lowest energy in question $\sqrt{s} = 200 \,\text{GeV}$. For ppcollisions at top LHC energy QGSM predicts further



FIG. 6: (Color online) The charged particle pseudorapidity spectra for (a) inelastic and (b) non-single-diffractive events calculated in QGSM for pp collisions at $\sqrt{s} = 200 \text{ GeV}$ (6), 546 GeV (5), 900 GeV (4), 2.36 TeV (3), 7 TeV (2) and 14 TeV (1). Data are taken from [42, 47, 50].

increase of the central particle densities to

$$\frac{dN_{inel}}{d\eta}|_{\eta=0} = 6.2 \quad , \quad \frac{dN_{NSD}}{d\eta}|_{\eta=0} = 7.1$$

Compared to the $\sqrt{s} = 7 \text{ TeV}$, the rise of the central particle density at $\sqrt{s} = 14 \text{ TeV}$ is expected to be about 25%.

In Fig. 7 the charged particle density at $\eta = 0$ is presented as a function of the c.m. energy \sqrt{s} for inelastic (upper plot) and non-single diffractive (bottom plot) events. The experimental data for inelastic collisions below $\sqrt{s} = 546 \text{ GeV}$ are well described by a linear dependence on $\ln s$ [42]. The striking evidence of first LHC results for pp interactions at $\sqrt{s} = 900 \text{ GeV}$, 2.36 GeV and 7 TeV is the quadratic dependence of the increase of midrapidity density of charged particles with rising $\ln s$ [50]. The theory of CGC suggests power-law rise [23]. In the QGSM these trends hold also, and the fitting parametrizations for c.m. energies from 200 GeV to 14 TeV are

$$\frac{dN_{inel}}{d\eta} |_{\eta=0}(s) = 5.87 - 0.74 \ln s + 0.39 \ln^2 s ,$$

$$\frac{dN_{NSD}}{d\eta} |_{\eta=0}(s) = 6.58 - 0.84 \ln s + 0.45 \ln^2 s ,$$

$$\frac{dN_{NSD}}{d\eta} |_{\eta=0}(s) = 0.77 E^{0.23} .$$



FIG. 7: (Color online) The charged particle density at midrapidity as a function of \sqrt{s} for (a) inelastic and (b) non-single diffractive collisions. Dashed lines show the results of the fit to expression $a + b \ln s + c \ln^2 s$, dotted line (b) - to the power-law dependence $d\sqrt{s}^{\lambda}$.

As in the mean- p_T case, there is a hairwidth difference between the two curves representing the logarithmic and the power-law fit, respectively. Indicating further increase of particle density at $\eta = 0$ with rising energy, the model favors violation of the Feynman scaling at midrapidity, otherwise the particle density there should not depend on \sqrt{s} .

It is interesting to compare the QGSM predictions for the charged particle multiplicity in pp collisions at LHC with that obtained by the extrapolation of pseudorapidity distributions measured at lower energies. This method [21] employs the energy independence of the slopes of pseudorapidity spectra combined with logarithmic proportionality to \sqrt{s} of both the width and the height of the distributions. Therefore, any data set from Fig. 6(a) and Fig. 6(b) can be used for the extrapolation, and the results are [21]

$$\frac{dN_{NSD}}{d\eta}|_{\eta=0} = 4.6 \pm 0.4 \quad , \quad \frac{dN_{NSD}}{d\eta}|_{\eta=\pm 2} = 5.25 \pm 0.7$$

These predictions are significantly lower than the recent experimental data from LHC and the QGSM calculations.

Another feature that is closely related to Feynman scaling is the so-called extended longitudinal scaling [19] exhibited by the slopes of (pseudo)rapidity spectra. In the QGSM these slopes are identical in the fragmentation region $y_{beam} \geq -2.5$ as shown in Fig. 8, where the distributions $\frac{1}{\sigma_{NSD}} \frac{d\sigma_{NSD}}{dy}$ are expressed as functions $y - y_{max}$.



FIG. 8: (Color online) The distributions $\frac{1}{\sigma_{NSD}} \frac{d\sigma_{NSD}}{dy}$ as functions of rapidity difference $y-y_{max}$ obtained in QGSM for energies $\sqrt{s} = 200 \text{ GeV}$, 546 GeV, 900 GeV, 1.8 TeV, 2.36 TeV, 7 TeV and 14 TeV.

QGSM indicates that the extended longitudinal scaling remains certainly valid at LHC. This result contradicts to the prediction based on the statistical thermal model [51]. The latter fits the measured rapidity distributions to the Gaussian, extracts the widths of the Gaussians and implements the energy dependence of the obtained widths to simulate the rapidity spectra at LHC. The extrapolated distribution was found to be much narrower [51] compared to that presented in Fig. 8. We are eagerly awaiting the LHC measurements of pp collisions in the fragmentation regions to resolve the obvious discrepancy. Note, that experimentally the extended longitudinal scaling was found to hold to 10% in a broad energy range from $\sqrt{s} = 30.8$ GeV to 900 GeV [42].

The emergence of the extended longitudinal scaling as well as Feynman scaling in the QGSM is not accidental. It arises due to short range correlations in rapidity space. The correlation function of particle i and particle j, produced in the string fragmentation, drops exponentially with rising rapidity difference

$$C(y_i, y_j) = \frac{d^2 \sigma}{\sigma_{inel} \, dy_i dy_j} - \frac{d\sigma}{\sigma_{inel} \, dy_i} \frac{d\sigma}{\sigma_{inel} \, dy_j}$$

$$\propto \exp\left[-\lambda \left(y_i - y_j\right)\right], \qquad (26)$$

and, therefore, the particles with large rapidity difference are uncorrelated. Consider now the inclusive process $1 + 2 \rightarrow i + X$. Its single particle inclusive cross section

$$f_i \equiv E \, \frac{d^3 \sigma_i}{d^3 p} = \frac{d^2 \sigma(y_1 - y_i, y_i - y_2, p_{iT}^2)}{dy_i d^2 p_{iT}} \tag{27}$$

becomes independent of $y_i - y_2$ at sufficiently high collision energy in the fragmentation region of particle 1, provided $y_1 - y_i \approx 1$ and $y_i - y_2 \approx y_1 - y_2 \gg 1$. Thus, the inclusive densities $n_i \equiv f_i / \sigma_{inel}$ are determined by only two variables

$$n_i = \phi(y_1 - y_i, p_{iT}^2) . (28)$$

Recalling that the Feynman variable

$$x_{iF} \equiv \frac{p_i}{p_{\parallel}^{max}} \approx \exp\left[-(y_1 - y_i)\right], \qquad (29)$$

one arrives from Eq.(23) to the condition of Feynman scaling

$$n_i = \psi(x_F^{(i)}, \, p_{i\,T}^2) \ . \tag{30}$$

The invariant distribution $F(x_F) = \frac{2}{\pi\sqrt{s}} \int E_{cm} \frac{d^2\sigma}{dx_F dp_T^2} dp_T^2$ is displayed in Fig. 9 for all charged particles from the pp collisions at energies from $\sqrt{s} = 200 \,\text{GeV}$, to 14 TeV. The scaling seems to hold within 20% of accuracy in the fragmentation region at $0.1 < x_F < 0.2$ only.



FIG. 9: (Color online) The invariant distribution $F(x_{\rm F})$ in non single diffractive *pp* collisions obtained in QGSM at $\sqrt{s} =$ 200 GeV, 546 GeV, 900 GeV, 1.8 TeV, 2.36 TeV and 14 TeV.

Another scaling dependence is known as Koba-Nielsen-Olesen or KNO-scaling [20]. Initially it was also derived from the hypothesis of Feynman scaling, but later on it appeared that both hypotheses are of independent origin. The KNO-scaling claims that at $\sqrt{s} \to \infty$ the normalized multiplicity distribution just scales up as $\ln s$ or, equivalently, that

$$\frac{\langle n \rangle \,\sigma_n}{\Sigma \,\sigma_n} = \Psi \left(\frac{n}{\langle n \rangle} \right) \,\,, \tag{31}$$

with σ_n being the partial cross section for *n*-particle production, $\langle n \rangle$ - the average multiplicity and $\Psi(n/\langle n \rangle)$ - energy independent function. KNO-scaling was found to hold up to ISR energies, $\sqrt{s} \leq 62 \,\text{GeV}$, despite the apparent failure of the Feynman scaling hypothesis in the central region $|x_{\rm F}| \leq x_0$. Violation of KNO-scaling was predicted within the QGSM in [24]. Later on the violation was observed experimentally by the UA5 and UA1 collaborations in $\bar{p}p$ collisions at $\sqrt{s} = 546 \,\text{GeV}$ [42]. The origin of this phenomenon in the model is the following. At ultrarelativistic energies the main contribution to particle multiplicity comes from the cut-Pomerons, and each cut results to formation of two strings. Short range correlations inside a single string lead to a Poissonlike multiplicity distribution of produced secondaries. At energies below 100 GeV the multi-string (or chain) processes are not very abundant and invariant masses of the strings are not very large. Therefore, different contributions to particle multiplicity overlap strongly, and KNOscaling is nearly fulfilled. With rising \sqrt{s} the number of strings increases as $(s/s_0)^{\Delta}$ and their invariant masses increase as well. This leads to enhancement of high multiplicities, deviation of the multiplicity distribution from the Poisson-like behavior and violation of KNO-scaling [7, 24].



FIG. 10: (Color online) Charged-particle multiplicity distributions in $|\eta| < 0.5$, $|\eta| < 1.0$ and $|\eta| < 1.3$ intervals, obtained in QGSM for pp collisions at $\sqrt{s} = 900 \text{ GeV}$ (upper plot) and at $\sqrt{s} = 2360 \text{ GeV}$ (bottom plot). Open symbols show the corresponding ALICE data [47].

Before studying the violation of KNO-scaling at LHC, we compare in Fig. 10 the QGSM calculations with the ALICE data. In this figure the multiplicity distributions of charged particles calculated in NSD pp events at $\sqrt{s} = 900 \text{ GeV}$ and $\sqrt{s} = 2.36$ TeV in three central pseudorapidity intervals are plotted onto the experimental data. The agreement between the model results and the data is good. Moreover, the QGSM demonstrates a kind of a wavy structure mentioned in [47]. As we will see below, such a wavy behavior in the model can be linked to processes going via the many-Pomeron exchanges.



FIG. 11: (Color online) Charged-particle multiplicity distributions in the KNO-variables in QGSM nondiffractive ppcollisions at $\sqrt{s} = 546$ GeV, 900 GeV, 1.8 TeV and 14 TeV.

The multiplicity distributions of charged particles obtained in QGSM for NSD pp collisions at all energies in question is presented in Fig. 11 for the interval $|\eta| < 2.4$. Although the differences between the neighbor energies seem not to be very dramatic, the tendency in the modification of the distributions is quite clear. - The high-multiplicity tail is pushed up, maximum of the distribution is shifted towards small values of $n_{ch}/\langle n_{ch}\rangle$ and the characteristic "shoulder" in the spectrum becomes quite distinct, as presented by the distribution for top LHC energy. Another interesting observation is the unique intersection point for all distributions. All curves cross each other at $z \approx 2.3$ as can be clearly seen in the bottom plot of Fig. 11, where the ratio $\langle n_{ch} \rangle P(z)|_{546 \, GeV} / \langle n_{ch} \rangle P(z)|_{7 \, TeV}$ is displayed. Note that the aforementioned pseudorapidity range $|\eta| < 2.4$ is not sufficient to observe the multi-hump structure in the KNO-plot predicted in [29] (see also [32]) for the full phase-space. To clarify the role of multi-Pomeron processes in violation of KNO-scaling explicitly, Fig. 12 shows the contribution to particle multiplicity diagram coming from the processes with different number of soft Pomerons in pp collisions at $\sqrt{s} = 14$ TeV. The maxima of distributions for multi-Pomeron processes are moved in the direction of high multiplicities thus lifting the high-multiplicity tail. The pronounced peak in the low-



FIG. 12: (Color online) Charged-particle multiplicity distribution (solid line) for processes going via the exchange of $n \ge 1$ soft Pomerons in pp collisions at $\sqrt{s} = 14$ TeV. Contributions of first four terms are shown by dash-dotted (n = 1), double-dash-dotted (n = 2), dashed (n = 3) and dotted (n = 4) lines, respectively.

multiplicity interval arises solely due to single Pomeron exchange.

IV. CONCLUSIONS

Multiplicity, momentum transverse and (pseudo)rapidity distributions of hadrons produced in pp interactions at energies from $\sqrt{s} = 200 \,\text{GeV}$ to 14 TeV are studied within the Monte Carlo guark-gluon string model. Parameters of soft and hard Pomerons are determined from the fit to recent LHC pp data. Compared to the fit to lower energies, it was found necessary both to increase the intercept of soft Pomeron and to reduce its slope parameter. Other parameters, such as total cross sections, cross sections of single diffractive and double diffractive processes etc, are taken from theoretical considerations. The model simulations of pseudorapidity, transverse momentum and multiplicity spectra of secondaries are in a good agreement with the corresponding experimental data obtained in $\bar{p}p$ and pp collisions at Tevatron and at CERN energies. Predictions are made for pp interactions at top LHC energy $\sqrt{s} = 14 \,\text{TeV}$. It is shown that within the examined energy range one cannot distinguish between

the standard" logarithmic dependence ($\propto \ln^2 s$) and novel power-law approximation ($\propto E^{\lambda}$), employed for particle densities and for their mean p_T in Regge theory and in theory of Color Glass Condensate, respectively.

Several scaling properties observed in particle production at relativistic energies have been examined. QGSM favors violation of Feynman scaling in the central rapidity region and its preservation in the fragmentation areas. Extended longitudinal scaling is shown to hold at LHC. This scaling is also attributed to heavy-ion collisions at energies up to $\sqrt{s} = 200 \text{ GeV}$. Extrapolations based on statistical thermal model predict its violation at LHC, thus implying vanishing of Feynman scaling for nuclear collisions in the fragmentation regions as well. This important problem should be resolved experimentally in the nearest future. Finally, further violation of

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the KNO-scaling in multiplicity distributions is demonstrated in QGSM. The origin of both conservation and violation of the scaling trends is traced to short range correlations of particles in the strings and interplay between the multi-Pomeron processes at ultra-relativistic energies.

Acknowledgments

Fruitful discussions with K. Boreskov, L. Csernai, O. Kancheli and L. McLerran are gratefully acknowledged. This work was supported by the Norwegian Research Council (NFR) under contract No. 185664/V30.

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