Statistical and dynamical fluctuations in the ratios of higher cumulants in relativistic heavy ion collisions

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The statistical and dynamical fluctuations in the ratios of higher net-proton cumulants are firstly estimated and discussed. It is shown that the statistical fluctuations dominate the behavior of the ratios measured at RHIC. Both the correlation between proton and antiproton and the dynamical ratios of higher net-proton cumulants are suggested. Using the transport and statistical models, it is demonstrated that the correlation show directly if proton and antiproton are emitted independently or not. The dynamical ratios measure the fluctuations in additional to the statistical ones. So, they are more relevant to the critical related phenomena.

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I. INTRODUCTION

One of the main goals of relativistic heavy ion collisions is to locate the critical point of QCD phase diagram. The critical point is characterized by the long-wavelength fluctuations. The fluctuations of final state particles are supposed to be largely enhanced if the freeze-out of the formed system is close to the region of phase transition. Therefore, the correlation length related observables are great interested in heavy ion collisions [1].

Recently, it is shown that the net-baryon cumulants are directly related to the susceptibilities [2–4], i.e.,

$$\langle \delta N^i \rangle = VT\chi_i. \tag{1}$$

Where $\langle \delta N^i \rangle = \langle (N-\overline{N})^i \rangle$ is the ith net-baryon cumulants. The third and forth cumulants,

$$K_3 = \langle \delta N^3 \rangle, \quad K_4 = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2, \tag{2}$$

are argued to be proportional to the higher power of correlation length, i.e., $\xi^{4.5}$ and ξ^7, respectively [3]. So the higher cumulants are more sensitive to the correlation length, and highly recommended [3–6].

On experiments, the proton is a good approximation of the baryon [3]. Moreover, the properly normalized ratios, the net-proton Skewness and Kurtosis,

$$\begin{aligned} S &= K_3/K_2^{3/2} = \frac{\langle \delta N^3 \rangle}{\langle \delta N^2 \rangle^{3/2}}, \\ K &= K_4/K_2^2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle^2} - 3, \end{aligned} \tag{3}$$

are actually presented [7]. They measure the symmetry and sharpness of net-proton distribution, respectively.

The data from RHIC/STAR [7] show that both Skewness and Kurtosis decrease with increasing number of participants (centrality). This trend is understood by invoking the central limit theorem (CLT) [7]. It means that the net-proton approaches to a Gaussian distribution at central collisions.

On the other hand, the results from various models [8] are all surprisedly converged to the data. It seems that the Skewness and Kurtosis are insensitive to the mechanisms of particle production implemented in different models.

More recently, F. Karsch and K. Redlich [9] have derived simple relations between the ratios and thermal parameters,

$$\begin{aligned} R_{2,1} &= \frac{\chi_2}{\chi_1} = 1/\tanh(\mu_B/T), \\ R_{3,2} &= S\sigma = \frac{\chi_3}{\chi_2} = \tanh(\mu_B/T), \\ R_{4,2} &= K\sigma^2 = \frac{\chi_4}{\chi_2} = 1. \end{aligned} \tag{4}$$

These relations are obtained in hadron resonance gas (HRG) model under Boltzmann approximation. Where the system is well thermalized and no internal interactions for both Fermions and Bosons. The temperature, T, and chemical potential of baryon, μ_B, at chemical freeze-out are determined by incident energy [10]. The σ^2 is the second cumulant.

They found that the data at different RHIC energies are well described by these relations. Meanwhile, they stress that these relations should be observed as long as the freeze-out of the formed system is close to the region of the phase transition. It is not necessary that the system has reached the equilibrium and has completely lost its memory in the expansion.

However, before we understand the physics of the measured Skewness and Kurtosis, the contributions from statistical fluctuations [5, 11] and all non-thermal sources (minijets, resonance decay, etc.) [12] should be taken into account and properly eliminated.

In the paper, we first estimate the statistical fluctuations in the ratios and show how it dominates the behavior of net-proton Kurtosis at RHIC energies. Then we suggest the correlations between proton and antiproton in Section III, and the dynamical ratios of higher net-proton cumulants in section IV. The centrality dependencies of the correlations and the dynamical ratios from two versions of AMPT [13], UrQMD [14], and Therminator [15] are presented and discussed. Finally, the summary and conclusions are given in section V.

II. STATISTICAL FLUCTUATIONS IN THE RATIOS OF HIGHER CUMULANTS

As we know that statistical fluctuation is caused by insufficient number of particles. It is usually a Poisson liked distribution [5, 11, 17]. If the proton and antiproton are two independent Poisson liked distributions, the net-proton is a Skellam (SK) distribution [8, 16],

$$f(N;\langle N_p\rangle,\langle N_{\bar{p}}\rangle) = e^{-(\langle N_p\rangle+\langle N_{\bar{p}}\rangle)}(\langle N_p\rangle/\langle N_{\bar{p}}\rangle)^{\frac{N}{2}} I_k\left(2\sqrt{\langle N_p\rangle\langle N_{\bar{p}}\rangle}\right). \quad (5)$$

Where $I_k(2\sqrt{\langle N_p\rangle\langle N_{\bar{p}}\rangle})$ is the first kind modified Bessel function. The $\langle N_p\rangle$ and $\langle N_{\bar{p}}\rangle$ are the mean values of proton and antiproton, respectively.

The statistical fluctuations of the ratios of higher net-proton cumulants can be directly deduced from Skellam distribution,

$$\begin{aligned} S_{\rm stat} &= \frac{\langle N_p\rangle-\langle N_{\bar{p}}\rangle}{[\langle N_p\rangle+\langle N_{\bar{p}}\rangle]^{3/2}}, \\ K_{\rm stat} &= \frac{1}{\langle N_p\rangle+\langle N_{\bar{p}}\rangle}, \\ R_{3,2,\rm stat} &= \frac{\langle N_p\rangle-\langle N_{\bar{p}}\rangle}{\langle N_p\rangle+\langle N_{\bar{p}}\rangle}, \\ R_{4,2,\rm stat} &= 1, \\ R_{2,1,\rm stat} &= \frac{\langle N_p\rangle+\langle N_{\bar{p}}\rangle}{\langle N_p\rangle-\langle N_{\bar{p}}\rangle}. \end{aligned} \quad (6)$$

They are completely determined by the mean values of proton and antiproton. As the mean values usually increase with incident energy and centrality, the statistical fluctuations of the Skewness and Kurtosis decrease with incident energy and centrality.

For comparison, the statistical fluctuations of net-proton Kurtosis (SK) in APMT default, AMPT with string melting [13] and Therminator [15] are presented together with corresponding Kurtosis in Fig. 1(a). We can see that in both transport and statistical models, the statistical fluctuations (all kinds of open points) are almost coincided with corresponding net-proton Kurtosis (all kinds of solid points) and data (red stars). So the statistical fluctuations dominate the behavior of net-proton Kurtosis at RHIC. This is why the results from various of models converge to the data.

In Fig. 1(b), the energy dependence of three ratios of net-proton, $R_{2,1}$, $R_{4,2}$, and $R_{3,2}$, obtained from RHIC/STAR data, HRG, AMPT default, AMPT with string melting, and Therminator, are presented together with corresponding statistical fluctuations (SK) determined by Eq. (6). We can see that at 200 GeV, the results from Therminator (magenta squares) are also close

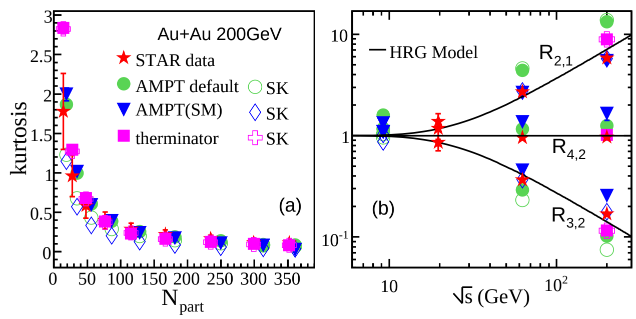

FIG. 1: (Color online) (a) Centrality dependence of net-proton Kurtosis, and (b) Energy dependence of $R_{2,1}$, $R_{4,2}$, and $R_{3,2}$ for Au + Au collisions at 200 GeV. The results are obtained from RHIC/STAR data [7], HRG, two versions of AMPT and Therminator models, and are compared with corresponding statistical fluctuations estimated by Skellam distribution (SK) in Eq. (6), respectively.

to that of HRG (the lines). The statistical fluctuations (open magenta crosses) of three ratios in Therminator are almost coincident with the corresponding ratios.

This is because the Therminator is a statistical model with only constrains on kinetics. It is similar to the HRG model, where the system reaches equilibrium and is no internal interactions. They are both indistinguishable from a completely random system, where the proton and antiproton are emitted independently by Poisson liked distributions. So the data from RHIC/STAR, which are dominated by the statistical fluctuations, are very close to the results from HRG and Therminator models.

We can also see from Fig. 1(b) that the results from two versions of AMPT models (green points and blue down triangles) are close to the lines given by HRG model. The statistical fluctuations (open green circles and open purple rhombus) are slightly deviated from the corresponding ratios.

So the data described by HRG model do not necessarily imply that the formed system is close to the transition region, or approach to the thermal equilibrium. It shows that the influences of statistical fluctuations are not negligible in the ratios of higher net-proton cumulants at RHIC energies.

III. CORRELATIONS BETWEEN PROTON AND ANTIPROTON

In order to see if the proton and antiproton are emitteded independently or not, we should directly measure the correlations between them. It is usually defined as,

$$C(N_p, N_{\bar{p}}) = \frac{\langle N_p N_{\bar{p}}\rangle}{\langle N_p\rangle\langle N_{\bar{p}}\rangle} - 1. \quad (7)$$

If proton and antiproton are independent, the correlation is zero.

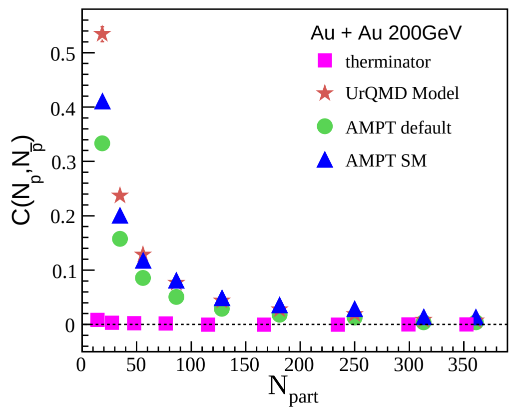

FIG. 2: (Color online) Centrality dependence of proton antiproton correlations for Au + Au collisions at 200 GeV, given by transport models, AMPT and UrQMD, and statistical model, Therminator.

The centrality dependence of the correlations from two versions of AMPT, UrQMD and Therminator models for Au + Au collisions at 200 GeV are presented in Fig. 2. It shows clearly that the correlations are zero at all centralities in Therminator model. So the proton and antiproton are indeed produced independently as assumed in the model.

While, in transport models, the correlation decrease with centrality. The stronger correlations are at peripheral collisions, and very weak correlations are at central collisions. This general trend are observed in different transport models. Therefore, the proton and antiproton are not produced independently in transport models.

So it is interesting to measure the correlations directly in relativistic heavy ion collisions. Then we can see if the proton and antiproton are emitted completely independently, like the case of Therminator, or correlatively as assumed in transport models.

In order to see the dynamical fluctuations related to the correlations in the ratios of higher cumulants, the corresponding statistical fluctuations should be subtracted.

IV. DYNAMICAL RATIOS OF HIGHER CUMULANTS

It is a long effort in eliminating statistical fluctuations in elementary collisions [11, 18]. For raw moments, it is known that the factorial moments can get rid of the statistical fluctuations and obtain the dynamical ones [2, 11]. But this method can not be directly generalized to the difference of two variables with Poisson distributions.

From what we have shown above, it is easily to estimate the statistical fluctuations in the ratios of higher cumulants. The dynamical ratios of net-proton cumulants can be simply defined as a deviation from the statistical

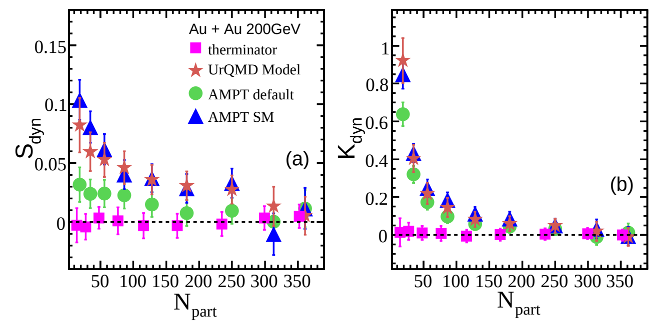

FIG. 3: (Color online)Centrality dependence of dynamical Skewness (left row) and Kurtosis (right row) for Au + Au collisions at 200 GeV, given by transport models, AMPT and UrQMD, and statistical model, Therminator.

ones [17], i.e.,

$$\begin{aligned} S_{\rm dyn} &= S - S_{\rm stat}, \\ K_{\rm dyn} &= K - K_{\rm stat}, \\ R_{4,2,\rm dyn} &= R_{4,2} - R_{4,2,\rm stat}, \\ R_{3,2,\rm dyn} &= R_{3,2} - R_{3,2,\rm stat}, \\ R_{2,1} &= R_{2,1} - R_{2,1,\rm stat}. \end{aligned} \tag{8}$$

Where the statistical parts are given in Eq. (6).

The centrality dependence of dynamical net-proton Skewness and Kurtosis from AMPT default, AMPT with string, UrQMD, and Therminator models for Au + Au collisions at 200 GeV are shown in Fig. 3(a) and (b), respectively. We can see that both Skewness and Kurtosis from Therminator are zero at all centralities. These show that both the symmetry and sharpness of net-proton distribution are identical with corresponding Skellam distribution in the model.

While, in transport models, both Skewness and Kurtosis are large than zero in peripheral collisions, and only approach to zero in central collisions. The positive dynamical Kurtosis implies that the peak of the net-proton distribution is sharper than that of the statistical one, and positive dynamical Skewness indicates that the net-proton distribution has a longer tail at large net-proton side in comparison with corresponding Skellam distribution.

So from the dynamical ratios of higher order net-proton cumulants, we can clearly see how the net-proton distribution deviates from the statistical one. It is interesting to see how the experimental data behave at RHIC energy scan.

If the deviation is zero, the proton and antiproton are emitted independently as statistical model assumed. If the deviation is not zero, but keep changeless with incident energy, like the case of transport model shown, then no critical phenomena is observed. However, if the deviation change significantly, like the sign change at the third, or fourth cumulant, which means that the symmetry, or sharpness, of net-proton distribution deviates from corresponding statistical fluctuations in two contrasted directions with the variation of incident energy [6, 19, 20], this may imply the critical related phenomena nearby [20].

V. SUMMARY

It is argued that before we understand the underlying physics in measured ratios of higher net-proton cumulants at RHIC, the statistical fluctuations should be taken into account. It is demonstrated that statistical fluctuations dominates the behavior of the ratios measured at RHIC. This is why the results of all kinds of models are converged to the data.

In order to see the internal interactions, the correlations between proton and antiproton, and dynamical ratios of high net-proton cumulants are suggested and discussed. It is shown that the correlations and dynamical ratios are zero at all centralities in statistical model. While, they are all larger than zero in transport models. These indicate that proton and anti-proton are not emitted independently in transport models. The deviations from statistical distribution are well demonstrated in suggested dynamical ratios. They are more relevant to the critical related phenomena.

Therefore, the behavior of the suggested correlations between proton and anti-proton, and dynamical ratios of higher net-proton cumulants are more relevant to the location of critical point, and are looking forward at RHIC beam energy scan plan.

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