A REMARK ON APPROXIMATION OF OPEN SETS WITH REGULAR BOUNDED ONES

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Abstract. We show that any open set in $\mathbb{R}^n$ is a union of an ascending sequence of bounded open sets with analytic boundary. This is just a technical result, which is probably known. We believe, however, that it can be useful for studing BVPs on irregular open sets.

A boundary of a *domain* (this word means any open set in $\mathbb{R}^n$) is called *analytic* if it is an analytic manifold and the domain is locally located on one side of it.

Theorem 1. *Any domain Ω is a union of an ascending sequence of bounded domains Ω_m with analytic boundary. Moreover, $\overline{\Omega}_m \subset \Omega$.*

Proof. a) If Ω is bounded and connected, then the statement of the theorem is a direct consequence of [1, Lemma 1] (see also [2, Section XI.14]).

b) Let Ω be any bounded domain. Then it is a union of at most countable number of open connected components ω_m. Each of them is a union of an ascending sequence $\omega_{m,k}$ of bounded domains with analytic boundary, and $\overline{\omega}_{m,k} \subset \omega_m$. Now, the sequence $\Omega_m = \{\bigcup_{l=1}^m \omega_{l,m}\}$ proves the claim. Observe that for any compact set $V \subset \Omega$ there exists $k = k(V, \Omega, \{\Omega_m\})$ such that $V \subset \Omega_k$.

c) Let Ω be an unbounded domain, and let $\Omega(m)$ be the intersections of Ω with the open balls of radii m centered at the origin. Let $\omega(m)$ denote the set of points x of $\Omega(m)$ such that the distance from x to $\partial\Omega(m)$ is larger than or equal to $1/m$. Every $\Omega(m)$ is a union of an ascending sequence $\Omega_{m,k}$ of bounded domains with analytic boundary. Then the required sequence $\Omega_m = \Omega_{m,k_m}$ is determined by the

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$$k_1 = k(\omega(1), \Omega(1), \{\Omega_{1,k}\}),$$
$$k_m = k(\omega(m) \cup \overline{\Omega}_{m-1}, \Omega(m), \{\Omega_{m,k}\}).$$

□

References

[1] C. J. Amick. Some remarks on Rellich's theorem and the Poincaré inequality. *J. London Math. Soc. (2)*, 18(1):81–93, 1978.

[2] O. D. Kellogg. *Foundations of potential theory*. Reprint from the first edition of 1929. Die Grundlehren der Mathematischen Wissenschaften, Band 31. Springer-Verlag, Berlin, 1967.

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