Electromagnetic extraction of energy from merging black holes

Maxim Lyutikov

Department of Physics, Purdue University,

525 Northwestern Avenue, West Lafayette, IN 47907-2036

Abstract

We calculate the evolution of the prompt intrinsic Poynting power generated by merging black holes. Orbiting black holes induce rotation of the space-time. In a presence of magnetic field supported by an accretion disk outside of the orbit, this results in a generation of an electromagnetic outflow via the Blandford-Znajek-type process with luminosity $L_{EM} \sim G^3 M^3 B^2/(c^5 R_{orb})$ and reaching a fairly low maximum values of $L_{EM} = 10^{37} - 10^{39} m_6$ erg s^{-1} (m_6 is the masses of black holes in millions of Solar mass) at the time of the merger. Dissipation of the wind power may produce two types of observed signatures: a highly variable collimated emission coming from the internal dissipation within the jets and a broad-band near-isotropic emission generated at the termination shocks.

I. INTRODUCTION

Direct detection of gravitational waves of the associated merging events of compact objects is one of the prime goals of modern astrophysics. Observations of the corresponding electromagnetic signal is most desirable, as it will provide crucial information on the location and the physical properties of the event. In case of the merger of stellar-mass compact objects (neutron stars and/or black holes), the short Gamma Ray Bursts may be the corresponding electromagnetic event [1]. On the other hand, the detection of the merger of supermassive black holes in the centers of galaxies by the Laser Interferometer Space Antenna, is an exciting possibility [2, 3]. There are clear observational evidence for recent galaxy mergers [4] and direct observations evidence of binary black holes [5].

So far, the studies of a possible prompt electromagnetic signatures of the black hole mergers were attributed mostly to the perturbations that the black holes induce in the surrounding gas, [6–9]. The resulting electromagnetic signal is then subject to great uncertainty and naturally depends on the complicated non-linear fluid behavior of the system. One of the problems is that there should be little gas inside the orbit of the merging black holes since the timescale for shrinkage of the binary orbit by gravitational wave radiation becomes shorter than the timescale for mass inflow due to viscose stresses in the disk [8]. It is then hard to excite transient dissipative processes in the faraway accretion disk. In addition, the resulting long times scales of emission correspond more to the afterglow phase than to the prompt emission.

In this paper we take an alternative approach, which follows in spirit the seminal paper of Blandford & Znajek [10]. Blandford & Znajek demonstrated that the rotation of the space-time itself can generate a powerful electromagnetic outflow carried by a strongly magnetized wind. In case of merging black holes the rotation of the space-time is induced by the orbital motion. The properties of the resulting Poynting flux depend only on the black holes masses, their separation and the assumed magnetic field, which in turn can be estimated using a standard anzats of accretion physics.

Previously, in Ref. [11, 12] numerical simulations of the electromagnetic signal during the last orbit before the merger were performed, showing a production of collimated Poynting

flux jets. The jets are produced due to linear motion of the black holes though magnetic field via a mechanism that roughly resembles unipolar induction. The classical analogy of this mechanism is the motion of planet Io in Jupiter's magnetic field [13]. The mechanism we discuss in the current paper is different, relying on the principles of a Faraday disk, where the rotation of space-time plays a role of the conducing surface, drags the magnetic field lines and generates electromagnetic outflows [10].

II. FARADAY DISK IN ASTROPHYSICS

Many astrophysical sources are powered by the rotational energy of a central source using magnetic field as a "conveyor" belt to transport energy to large distances. Pulsars [14] are the prime examples of such process. Qualitatively, the neutron star surfaces permeated by the magnetic field works as a Faraday disk or unipolar inductor, generating a Poynting flux propagating away from the star. A qualitatively new application of this idea was proposed by Blandford and Znajek [10], who showed that, formally, there is no need for a hard conducting disk to launch the Poynting flux: rotation of the space itself can effectively act as a Faraday disk. In this paper we apply the ideas of Blandford and Znajek [10] to the space-time produced by orbiting black holes.

An important difference between the astrophysical applications mentioned above and the classical Faraday experiment is that in generating the Poynting flux large electric fields develop, which even in the absence of any pre-existing plasma will lead to vacuum breakdown due to various radiative effects. Thus, the system forms the plasma out of the vacuum, so that in general the problem should be describe by relativistic magnetohydrodynamics.

The magnetic energy density in the low density regions (from where the outflows are generated) is expected to greatly exceeds the plasma energy density, including rest mass. Charged particles carry current and generate charge density, that ensures ideal MHD condition $\mathbf{E} \cdot \mathbf{B} = 0$. In the limit of negligible inertial contribution, the system reaches a so-called force-free state, where electromagnetic forces balance itself. We stress that this is not vacuum: inertia-less charge particles ensure ideal MHD conditions. (Formally, in such plasma the stress tensor is diagonalizable; in addition, a condition $|B| > |E|$ must be satisfied.)

III. ELECTROMAGNETIC POWER OF ORBITING MASSES IN EXTERNAL MAGNETIC FIELD

Let us consider two black holes of equal mass M moving on a circular trajectory of radius R. Let us assume that there is a magnetized disk outside of the orbit that supports the electric currents, which produce a typical magnetic field B within the orbit. We are interested in the power produced specifically due to the orbital motion of black holes. If black holes are spinning, each will in addition act as a Faraday disk producing the Blandford-Znajek Poynting power [10]. In addition matter accreting onto black holes will produce some accretion luminosity both through viscous dissipation in the disk and by launching magnetized winds via the Blandford-Payne-type [15] mechanism (as seen in simulations [11]). Let us give qualitative estimates of the electromagnetic power specifically due to the orbital motion of black holes. (Below, for clarity, we omit numerical factors of the order of a few).

The power produced by the Faraday disk can be estimated as

$$L_{EM} \sim R^2 B^2 c (\Omega r/c)^2 \tag{1}$$

where Ω is the angular velocity of disk rotation. In the case of orbiting black holes, Ω is the typical angular velocity of the rotation of the space-time within the black hole orbit. In the weak field regime the angular velocity of the rotation of space-time within the orbit can be estimated as [$e.g.$ 16, Eq. 105.20]

$$\Omega \sim \frac{G|\mathbf{M}|}{c^2 R^3} = \frac{(GM)^{3/2}}{c^2 R^{5/2}} \tag{2}$$

where $|\mathbf{M}| \sim \sqrt{Gr} M^{3/2}$ is the total angular momentum of the orbiting black holes ($cf.$ Ref. [17], problem 13.18). Note, that since outside of the orbit $\Omega \propto R^{-3}$, the corresponding electromagnetic power is small.

Alternatively, we can calculate the rotation frequency of space-time due to orbiting black holes of equal masses M on a circular orbit if radius R well before the merger, when the motion can be treated in a linearized theory. As a time-averaged approximation, assume that two black holes may be represented as a ring of mass $2M$ rotating with velocity $v = \sqrt{GM/(2R)}$, producing a mass current $I_m = \sqrt{G} M^{3/2}/(\sqrt{2}\pi R^{3/2})$. In the linearized theory and assuming the time-independent field, the components of the metric tensor satisfy [18]

$$\Delta g_{0\phi} = -16\pi T_{0\phi} \tag{3}$$

The component $g_{0\phi}$ then can be found from the known expression for magnetic field produced by a current ring [19, p. 181]

$$g_{0\phi} = -\frac{2\sqrt{2}}{\pi c^2}(GM/r)^{3/2}\frac{1}{\sqrt{r^2 + R^2 + 2rR\sin\theta}}\left(\left(\frac{2}{k^2-1}\right)K(k^2) - \frac{2}{k^2}E(k^2)\right)$$

$$k^2 = \frac{4rR\sin\theta}{r^2 + R^2 + 2rR\sin\theta} \tag{4}$$

where K and E are elliptic integrals. The angular frequency of the rotation of space is then

$$\Omega = \frac{h_{0\phi}}{r\sin\theta} \tag{5}$$

(Note that there is no singularity at $r = 0$, see Fig. 1).

In the orbital plane, Ω is a slowly varying function of r with the value at $r = 0$ equal to $\Omega = (GM)^{3/2}/(\sqrt{2}c^2R^{5/2})$, in agreement with our estimate (2), see Fig. 1
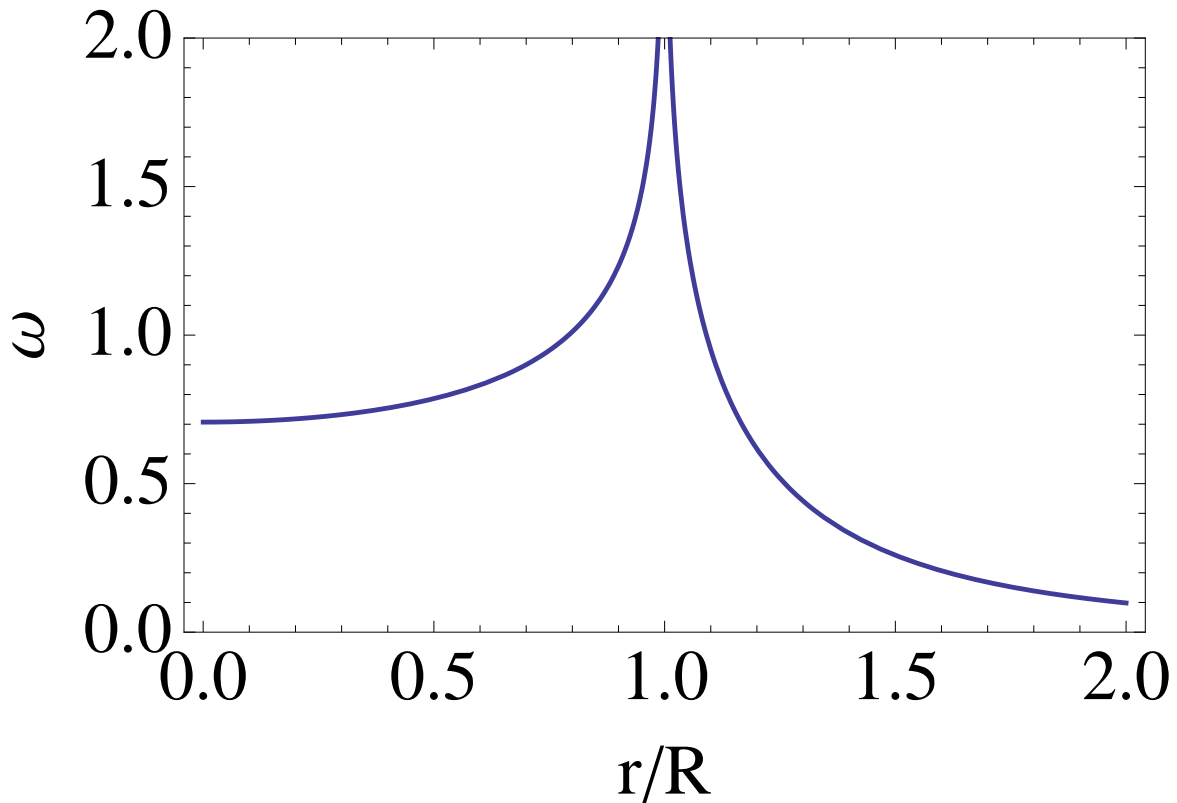

FIG. 1. Angular frequency of the rotation of space as a function of radius for a massive ring rotating with Keplerian velocity at radius R. Frequency is normalized to $(GM)^{3/2}/(c^2R^{5/2})$. Inside the ring Ω is nearly constant, falling off $\propto r^{-3}$ outside of the ring. Divergence at $r = R$ is an artifact of the weak field approximation used to calculate Ω.

Consider next a Faraday disk of radius R embedded in a highly magnetized plasma in the force-free approximation discussed in §II and permeated by a constant magnetic field aligned with the disk normal. The disk is rotating with angular velocity Ω. The following fields expressed in cylindrically collimated r, ϕ, z then satisfy Maxwell's equation and ideal condition $\mathbf{E} \cdot \mathbf{B} = 0$ [20, 21]:

$$B_r = 0, \ B_\phi = E_r = -\frac{\Omega r}{c} B_0, \ B_z = B_0 \tag{6}$$

for $r < R$ and $B_z = B_0$, $\mathbf{E} = 0$ for $r > R$. The Poynting flux is then

$$L_{EM} = \frac{1}{8} B_0^2 R^2 c (R\Omega/c)^2, \tag{7}$$

where Ω is the angular velocity of rotation of space (2), (and *not* the angular velocity of the Keplerian rotation Ω_K).

We stress that in the model problem considered above (of a rotating disk in a force-free plasma) *the rotation of field lines at $r < R$ does not break a force balance in the cylindrical radial direction r, resulting in a formation of cylindrically collimated outflow.* We expected that the resulting jets will have a low baryon contamination and thus will be moving at relativistic speeds before they start interacting with the surrounding medium.

Thus, the electromagnetic power due to orbiting black holes acting as a Faraday disk in the external magnetic field of strength B can be estimated as

$$L_{EM} \sim \frac{G^3 M^3}{c^5 R} B^2 = 6 \times 10^{38} \text{erg s}^{-1} \, b_3^2 m_6^2 \, (R/R_G)^{-1} \tag{8}$$

where $b_3 = B/10^3 \, \text{G}$, $m_6 = M/10^6 M_\odot$, $R_G = GM/c^2 = 1.5 \times 10^{11} \text{cm} \, m_6$. (In the numerical estimate in Eq. (8) and below the ratio (r/r_G) is treated as a parameter with no dependence on mass.)

(Note, that qualitatively, the power of the Blandford-Znajek process can be similarly estimated using $\Omega = ac/r_G$,

$$L_{BZ} \sim a^2 B^2 r_G^2 c \tag{9}$$

where a is the black hole spin parameters and $r_G = GM/c^2$ is the Schwarzschild radius. At the merger moment $r \sim GM/c^2$, so that Eq. (8) agrees with Eq. (9) for $a \sim 1$, a critically rotating black hole.)

Let us compare the electromagnetic power to the power emitted in gravitational waves (GWs). The GW power [16, §110]

$$L_{GW} \approx \frac{GM^2 \Omega_K^6 R^4}{c^5} = \frac{G^4 M^5}{c^5 R^5} \tag{10}$$

6

The ratio of EM to GW luminosities is

$$\frac{L_{EM}}{L_{GW}} = \frac{B^2 R^4}{GM^2} = \frac{E_B}{E_G} \tag{11}$$

$$E_B = B^2 R^3 \tag{12}$$

$$E_G = GM^2/R \tag{13}$$

where E_B is the energy of magnetic field within the orbit of BHs and E_G is the gravitational potential energy.

The two powers are equal when

$$r_{\rm eq} \approx \frac{G^{1/4}\sqrt{M}}{\sqrt{B}} = 2 \times 10^{16}\,{\rm cm}\, b_3^{-1/2} m_6^{1/2} \tag{14}$$

For smaller separations, the GW power dominates. The radius (14) is very large, with a merger time $\sim c^5 r_{\rm eq}^4/(G^3 M^3)$ larger than the age of the Universe.

IV. ESTIMATES OF MAGNETIC FIELD

In the above estimates we used the parametrization of the magnetic field somewhat arbitrarily. In this section we estimate the magnetic field within the orbit produced by the currents flowing in the accretion disk. As mentioned in the Introduction, two stages of disk dynamics may be identified. At early times the loss of energy via gravitational radiation is slow enough, so that due to viscous diffusion the inner edge of the disk will be located close to the orbital radius, $R_{\rm d} \approx 2R$. At later times, for $R_d \leq \xi_d GM/c^2$, $\xi_d \approx 40-100$ [8, 22], the binary will decouple from the disk, undergoing a merger, while the inner edge of the disk remains fixed at R_d. The decoupling occurs at time $t_d \sim \xi_d^4 GM/c^2 \approx 10\,{\rm yrs}\, m_6^{-3}$ before the merger.

Several different scalings can be used to estimate the magnetic field in the disk and within the orbit. Magnetic field is generated in the disk by the MRI dynamo [23], which in a thin accretion disk produces fields at equipartition with the turbulent motion, $B^2 \sim \rho v_{turb}^2$. In the standard α disk the turbulent velocity is a fraction of the local sound speed c_s, $v_{turb} \sim \sqrt{\alpha} c_s$, while the sound speed c_s is a function of the parameters of the central object, mass flow $\dot{M}$ and location in the disk [24] (*e.g.* in the radiation or matter-dominated parts of the disk). It is somewhat beyond the scope of this paper to discuss all the possible combinations of the parameters of the accretion disk. Instead, we estimate the upper limits on magnetic field

given various parametrizations of the accretion flow. As we will see, even the upper limits on the magnetic fields still produce fairly low luminosities, Eq. (19).

As a first approximation, we assume that the turbulent velocity is of the order of the sound speed, $v_{turb} \sim c_s$; in this case the magnetic field is in the equipartition with the particle energy-densities, $B^2 \approx \rho c_s^2$. Secondly, we assume thick accretion disk, so that the sound speed c_s at radius R is of the order of the free fall velocity, $c_s \approx \sqrt{GM/R}$. Two possible estimates then may be given for the plasma density ρ. The density can be found either from Bondi scaling, $\rho R^2 \sqrt{GM/R} \approx \dot{M} \rho_{ex} (GM)^2 / c_{s,ex}^3$, where ρ_{ex} and $c_{s,ex}$ are density and sound speed in the surrounding medium. This gives

$$B_B = \frac{(GM)^{5/4} \sqrt{\rho_{ex}}}{R^{5/4} c_{s,ex}^{3/2}} \approx 6 \times 10^4 \, \text{G} \, n_{-1}^{1/2} c_{s,ex,6}^{-3/2} \left(\frac{R}{R_G} \right)^{-5/4} \tag{15}$$

where the subscript indicates Bondi scaling and the external number density is scaled as $\rho_{ex} \approx m_p n = m_p (n/0.1 \text{cm}^{-3}) n_{-1}$ and the external sound speed as $c_{s,ex} \approx 10^7 \text{cm s}^{-1} c_{s,ex,7}$.

Alternatively, the density can be found from the requirement that the accreting matter powers an Eddington-type outflow $\eta_M \dot{M} c^2 = L_{\text{Edd}}$ with some efficiency $\eta_M \approx 0.1 (\eta_M / 0.1) \eta_{M,-1}$:

$$B_M = \frac{(GM)^{3/4} \sqrt{m_p}}{\sqrt{\eta_M c \sigma_T} R^{5/4}} \approx 3 \times 10^5 \, \text{G} \, m_6^{-1/2} \eta_{M,-1}^{-1/2} \left(\frac{R}{R_G} \right)^{-5/4} \tag{16}$$

Finally, magnetic field can be estimated assuming that a fraction of the Eddington luminosity is carried by magnetic field, $B^2 \approx \eta_E L_{\text{Edd}} / (c R^2)$,

$$B_E = \eta_E^{1/2} \frac{(GM)^{1/2} \sqrt{m_p}}{\sqrt{\sigma_T} R} \approx 3 \times 10^4 \, \text{G} \, m_6^{-1/2} \eta_{E,-1}^{1/2} \left(\frac{R}{R_G} \right)^{-1} \tag{17}$$

In all cases the magnetic field within the orbit should be evaluated at the inner edge of the accretion disk, located either at $R_d \sim R$ before decoupling or at $R_d \approx 100 R_G$ after decoupling. Accordingly, we find

$$L_B = \begin{cases} \frac{(GM)^{11/2} \rho_{ex}}{c^5 c_{s,ex}^3 R^{7/2}} & \text{for } \frac{R}{GM/c^2} > \xi_d \\ \frac{(GM)^3 \rho_{ex}}{c_{s,ex}^3 R \xi_d^{5/2}} & \text{for } \frac{R}{GM/c^2} < \xi_d \end{cases} \quad for \, B = B_B$$

$$L_M = \begin{cases} \frac{(GM)^{9/2} m_p}{c^6 \eta_M R^{7/2} \sigma_T} & \text{for } \frac{R}{GM/c^2} > \xi_d \\ \frac{(GM)^2 c m_p}{\eta_M \xi_d^{5/2} c R \sigma_T} & \text{for } \frac{R}{GM/c^2} < \xi_d \end{cases} \quad for \, B = B_M$$

$$L_E = \begin{cases} \eta_E \frac{(GM)^4 m_p}{c^5 R^3 \sigma_T} & \text{for } \frac{R}{GM/c^2} > \xi_d \\ \eta_E \frac{(GM)^2 m_p}{\xi_d^2 \sigma_T c R} & \text{for } \frac{R}{GM/c^2} < \xi_d \end{cases} \quad for \, B = B_E \tag{18}$$

The maximal powers, reached at the time of the merger are

$$L_B \sim 10^{37}\,\mathrm{erg\,s}^{-1} m_6^2 n_{-1} c_{s,ex,6}^{-3},\ L_M \sim 10^{39}\,\mathrm{erg\,s}^{-1} m_6 \eta_{M,-1}^{-1},\ L_E \sim 10^{38}\,\mathrm{erg\,s}^{-1} m_6 \eta_{E,-1} \quad (19)$$

Thus, various parametrizations of the magnetic field give a fairly consistent value of the electromagnetic luminocity.

The temporal evolution of the luminosity in each case can be found from the equation for the orbital separation at time $-t$ before the merger [16, §110],

$$R = ((GM)^3(-t)/c^5)^{1/4} \quad (20)$$

The total energies emitted after the decoupling are

$$E_B \approx \frac{(GM)^3 \rho_{\mathrm{ex}}}{cc_s^3} \approx 10^{43}\,\mathrm{erg}$$
$$E_M \approx \frac{(GM)^2 m_p \sqrt{\xi_d}}{\eta_M c^2 \sigma_T} \approx 5 \times 10^{44}\,\mathrm{erg}$$
$$E_E \approx \frac{(GM)^2 m_p \xi_d \eta_E}{c^2 \sigma_T} \approx 5 \times 10^{42}\,\mathrm{erg} \quad (21)$$

V. EXPECTED EMISSION

We foresee two types of electromagnetic signals coming from merging of black holes. First, internal dissipation within the jets can produce highly boosted emission similar to blazar jets and high energy emission from Gamma Ray Bursts (GRBs). (Blazars are types of Active Galactic Nuclei (AGNs) with relativistic jets directed nearly straight at the observer.) Blazars dominate the extragalactic sources in the *Fermi* catalogue [25, 26]. This type of emission will be highly variable due to relativistic boosting, similar to prompt GRB emission [27]. On the other hand, internal jet emission will be highly anisotropic, mostly aberrated along the direction of motion of the jets - perpendicular to the binary orbit. Thus, this type of emission will be detected only if the line of sight to the binary is nearly aligned with the binary normal.

Secondly, when the jet starts to interact with the circumbinary medium, its kinetic power will be dissipated in a termination shock, similar to the case of pulsar wind nebulae (magnetically-dominated jets, which lack strong termination shock, may produce similar observational signatures [28].) The post-shock flow will be only weakly relativistic, producing nearly isotropic emission.

9

By analogy with the emission from PWNe, one expects that a broad range of frequencies is produced by the Poynting-flux dominated wind. Unfortunately, our understanding of the dissipation and accelerations processes in such systems is not sufficient to make quantitative predictions. By comparison, *e.g.* with the Crab nebular, we expect that that the peak of the spectral energy density falls into the optical-UV-soft X-ray band, with the total dissipated power reaching tens of percent of the Poynting power.

VI. DISCUSSION

We have calculated the electromagnetic power expected from the merging black holes. The expected Poynting scales *not* as a square of the orbital frequency, but as a square of the rotational frequency of the space-time, Eq. (2). The maximum power is reached during the final merger orbit, when the orbital frequency and the rotational frequency of the space-time are nearly equal, since the resulting black hole has the spin parameter of the order of unity. Still, the distinction between the two frequencies is a principal point in our approach.

Overall, the corresponding peak Poynting powers (19) are fairly low, even though the total emitted energies are reasonably high (21). In addition, it is expected that only a fraction of the Poynting power is converted into radiation. Still, there is a chance of detection in the fortuitous case when the line of sight to the system is nearly aligned with the orbital normal and if the outflow reaches relativistic velocities. In this case most of the electromagnetic power will beamed along the direction of motion.

[1] R. Mochkovitch, M. Hernanz, J. Isern, and X. Martin, Nature (London) **361**, 236 (1993).

[2] M. C. Begelman, R. D. Blandford, and M. J. Rees, Nature (London) **287**, 307 (1980).

[3] R. N. Lang and S. A. Hughes, Astrophys. J. **677**, 1184 (2008), 0710.3795.

[4] M. Volonteri, F. Haardt, and P. Madau, Astrophys. J. **582**, 559 (2003), arXiv:astro-ph/0207276.

[5] M. Dotti, C. Montuori, R. Decarli, M. Volonteri, M. Colpi, and F. Haardt, MNRAS **398**, L73 (2009), 0809.3446.

[6] P. J. Armitage and P. Natarajan, ApJ Lett. **567**, L9 (2002), arXiv:astro-ph/0201318.

[7] J. H. Krolik, Astrophys. J. **709**, 774 (2010), 0911.5711.

[8] M. Milosavljević and E. S. Phinney, ApJ Lett. **622**, L93 (2005), arXiv:astro-ph/0410343.

[9] J. R. van Meter, J. H. Wise, M. C. Miller, C. S. Reynolds, J. Centrella, J. G. Baker, W. D. Boggs, B. J. Kelly, and S. T. McWilliams, ApJ Lett. **711**, L89 (2010), 0908.0023.

[10] R. D. Blandford and R. L. Znajek, MNRAS **179**, 433 (1977).

[11] C. Palenzuela, L. Lehner, and S. L. Liebling, Science **329**, 927 (2010), 1005.1067.

[12] C. Palenzuela, T. Garrett, L. Lehner, and S. L. Liebling, Phys. Rev. D **82**, 044045 (2010), 1007.1198.

[13] P. Goldreich and D. Lynden-Bell, Astrophys. J. **156**, 59 (1969).

[14] P. Goldreich and W. H. Julian, Astrophys. J. **157**, 869 (1969).

[15] Blandford and Payne, MNRAS **199**, 883 (1982).

[16] L. D. Landau and E. M. Lifshitz, *The classical theory of fields* (1971).

[17] A. P. Lightman, W. H. Press, R. H. Price, and S. A. Teukolsky, *Problem book in relativity and gravitation.* (1979).

[18] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (San Francisco: W.H. Freeman and Co., 1973, 1973).

[19] J. D. Jackson, *Classical electrodynamics* (1975).

[20] B. Punsly, *Black hole gravitohydromagnetics* (2001).

[21] S. S. Komissarov, ArXiv Astrophysics e-prints (2002), arXiv:astro-ph/0206076.

[22] S. M. O'Neill, M. C. Miller, T. Bogdanović, C. S. Reynolds, and J. D. Schnittman, Astrophys. J. **700**, 859 (2009), 0812.4874.

[23] S. A. Balbus and J. F. Hawley, Astrophys. J. **376**, 214 (1991).

[24] N. I. Shakura and R. A. Sunyaev, AAP **24**, 337 (1973).

[25] A. A. *et al.* . Abdo, Astrophys. J. **722**, 520 (2010), 1004.0348.

[26] T. Savolainen, D. C. Homan, T. Hovatta, M. Kadler, Y. Y. Kovalev, M. L. Lister, E. Ros, and J. A. Zensus, AAP **512**, A24+ (2010), 0911.4924.

[27] A. A. *et al.* . Abdo, Science **323**, 1688 (2009).

[28] M. Lyutikov, MNRAS **405**, 1809 (2010), 0911.0324.