ELLIPTICAL MOTIONS OF STARS IN CLOSE BINARY SYSTEMS

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Abstract.

Motions of stars in close binary systems with a conservative mass exchange are examined. It is shown that Paczynski-Huang model widely used now for obtaining the semi-major axis variation of a relative stars orbit is incorrect, because it brings about large mistakes. A new model suitable for elliptical orbits of stars is proposed. Both of reactive and attractive forces between stars and a substance of the flowing jet are taken into account. A possibility of a mass exchange at presence of accretion disk is considered.

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INTRODUCTION

The research of stars motions in close binary systems began in the 60-s' of the last century in a cycle of works by Kruszevski [1], Hadjidemetriou [2], Piotrovski [3], Huang [4], Paczynski [5] and others. As a result for a case of conservative mass exchange the simplified dependence was received for semi-major axis a of a circular relative stars orbit from constant mass increase of an accepting star $\dot{M}_2$ as

$$\dot{a} = 2a\dot{M}_2\left(\frac{1}{M_1} - \frac{1}{M_2}\right). \tag{1}$$

This dependence, which we shall call for by Paczynski-Huang name and up to this day it is used in all researches dedicated to close binary systems with a conservative mass exchange when $M_1 + M_2 = M = \text{const}$.

The derivation of formula (1) is based on the assumption that equations of motion for stars with variable masses admit the angular momentum integral

$$\mathbf{J} = M_1\mathbf{R}_1 \times \mathbf{V}_1 + M_2\mathbf{R}_2 \times \mathbf{V}_2 = \mathbf{const}, \tag{2}$$

where $M_i, \mathbf{R}_i, \mathbf{V}_i,\ (i = 1, 2)$ — the mass, radius-vector and velocity of stars motion. By the most an assumption is made that the close binary stars form a closed mechanical system which admits integrals of momentum and angular momentum. Differential equations of motion thus are usually not written out.

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However the assumption on existence of integral of momentum is erroneous. We shall be convinced of it, proceeding from most general view of the Mestschersky equations for a two-body problem with variable masses [6]:

$$M_1 \frac{d\mathbf{V}_1}{d\,t} = G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{Q}_1, \qquad M_2 \frac{d\mathbf{V}_2}{d\,t} = -G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{Q}_2, \tag{3}$$

where $\mathbf{R} = \mathbf{R_2} - \mathbf{R_1}$, and $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are jet forces acting on stars S_1 and S_2.

Let's consider three opportunities of reception of integral of momentum (2) from equations (3).

1. At first we shall take into account of closed mechanical system, described by the problem of Mestschersky-Levy-Civita [7], differential equations of which turn out from (3), if jet forces put equal to:

$$\mathbf{Q}_1 = -\dot{M}_1 \mathbf{V}_1, \qquad \mathbf{Q}_2 = -\dot{M}_2 \mathbf{V}_2, \tag{4}$$

where $\dot{M}_1 = -\dot{M}_2 = \text{const}$, $\dot{M}_2 > 0$.

This model is usually used in astronomy at study of motions of mutually attracted pair stars, taking place in dust cloud with account of jet forces, arising owing to sticking substances dust of cloud on stars.

The Mestschersky-Levy-Civita problem represents a closed mechanical system, therefore there are integrals of momentum and of angular momentum (2). However to use model Mestschersky-Levy-Civita for study on motions of stars in close binary system with a conservative exchange of mass is not admitted, so as the jet forces (4) in close binary system do not exist. The true jet forces have other directions and others absolute values: on a donor-star the jet force acts, directed in the opposite side to radius $\mathbf{R}$ and is equal to product of velocity of sound on $\dot{M}_1$, and on accreted-star acts the jet force, directed on tangent to a trajectory of relative motion of flowing particles at the moment of their hit on a surface of a star. These jet forces sharply differ from values (4) in the Mestschersky-Levy-Civita problem.

For this reason the model of Mestschersky-Levy-Civita is not suitable for study of stars motion in close binary system.

2. Any other problems of two bodies with variable masses, admitting integral of angular momentum, does not exist. But it is possible to consider an opportunity of the approximate reception of integral (2) from equations (3). For this purpose we shall consider, that the jet forces acting on stars, are small values in comparison with forces of mutual attraction between stars, and consequently they can be neglected

$$\mathbf{Q}_1 = 0, \qquad \mathbf{Q}_2 = 0. \tag{5}$$

Then we shall receive a well known Gylden-Mestschersky model differential equations of which are

$$M_1 \frac{d\mathbf{V}_1}{d\,t} = G\frac{M_1\,M_2}{R^3}\mathbf{R}, \qquad M_2 \frac{d\mathbf{V}_2}{d\,t} = -G\frac{M_1\,M_2}{R^3}\mathbf{R}, \tag{6}$$

in which integral of the angular momentum (2) in absolute motion does not exist, but a quasi-integral does exist

$$\mathbf{J}' = M_1 \mathbf{R}_1 \times \mathbf{V}_1 + M_2 \mathbf{R}_2 \times \mathbf{V}_2 - \int (\dot{M}_1 \mathbf{R}_1 \times \mathbf{V}_1 + \dot{M}_2 \mathbf{R}_2 \times \mathbf{V}_2)\, dt = \mathbf{const}. \qquad (7)$$

If in this quasi-integral one neglect small members, containing as multiplier the velocity of mass change $\dot{M}_1$ and $\dot{M}_2$, we shall receive integral (2), but thus a problem of Gylden-Mestschersky will be transformed to a two-body problem with constant masses, with the help of which it is impossible to study a transfer of substance between stars.

Nevertheless the conservative ($M = \text{const}$) problem of Gylden-Mestschersky represents the certain interest for study conservative exchange in close binary system, so as it is possible with the help of it to characterize approximately motions of stars at presence of accretion disk. The equations of relative motion in the Gylden-Mestschersky problem are

$$\frac{d\mathbf{V}}{dt} = -\frac{GM}{R^3}\mathbf{R} \qquad (8)$$

and at $M = \text{const}$ exactly coincide with the equations of relative motions of the classical two-body problem with constant masses [8], therefore for relative motion of bodies in the problem of Gylden-Mestschersky there is a strict angular momentum integral

$$\sqrt{GMa(1-e^2)} = \text{const}, \qquad (9)$$

and all Kepler's elements of an orbit in this problem are constants, including $a = \text{const}$.

Thus, the exclusion from consideration all of jet forces, i.e. the transition to a conservative problem of Gylden-Mestschersky, results to invariance of the semi-major axis of a relative orbit of stars, but does not result in the formula of Paczynski-Huang.

3. We shall consider now the third opportunity of a "formal conclusion" the rules (1), which is usually used in the literature. Proceeding from the lack of external forces acting on a system, and invariance of its complete mass, one asserts, that the system is closed and, hence, admits the existence of integral (2).

Such statement is valid for stars with constant masses, but is not valid for stars with variable masses. The point is that for systems of bodies with variable masses to be closed except of absence of external forces and the invariance of complete mass is needed it is necessary a presence some additional "internal" jet forces, which make system to be closed.

Really, according to the second law of Newtons mechanics the velocity speeds of change of momentum of stars are determined by equalities:

$$\frac{d(M_1\mathbf{V}_1)}{dt} = G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{F}_1, \quad \frac{d(M_2\mathbf{V}_2)}{dt} = -G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{F}_2, \qquad (10)$$

where $\mathbf{F}_1$ and $\mathbf{F}_2$ are external forces, acting on stars.

Let's rewrite these equations as

$$M_1 \frac{d\mathbf{V}_1}{dt} = G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{Q}_1 + \mathbf{F}_1, \quad M_2 \frac{d\mathbf{V}_2}{dt} = -G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{Q}_2 + \mathbf{F}_2, \qquad (11)$$

where $\mathbf{Q}_1 = -\dot{M}_1\mathbf{V}_1$ and $\mathbf{Q}_2 = -\dot{M}_2\mathbf{V}_2$ are "internal" jet forces, active on stars S_1 and S_2.

From there it is visible, that integral of momentum $M_1\mathbf{V}_1 + M_2\mathbf{V}_2 = \mathbf{const}$ and together with it those of angular moment (2), will exist at absence of external forces, $\mathbf{F}_1 = \mathbf{F}_2 = 0$, but it is obviously necessary a preservation of "internal" jet forces $\mathbf{Q}_1 = -\dot{M}_1\mathbf{V}_1$ and $\mathbf{Q}_2 = -\dot{M}_2\mathbf{V}_2$.

Therefore for existence of integral (2) it is necessary not only the absence of external forces and invariance of complete mass of the system, but also the presence of "internal" jet forces of a kind (4) too is necessary. These requirements are carried out only for the problem of Mestschersky-Levy-Civita, considered in item 1. For a problem on motion of stars in close binary system with a conservative exchange of mass these requirements are not carried out, so as the jet forces have another vector values which are distinct from "internal" jet forces, namely, $\mathbf{Q}_1 = -\dot{M}_1\mathbf{W}_1$ and $\mathbf{Q}_2 = -\dot{M}_2\mathbf{W}_2$, where $\mathbf{W}_1 \neq \mathbf{V}_1$ and $\mathbf{W}_2 \neq \mathbf{V}_2$ are relative velocities of the outflow (inflow) of mass on stars. Therefore the integral of angular momentum does not here exist.

Thus, the assumption on existence of integral of angular momentum (2) is infaithfull, i.e. those about a problem on stars motion in close binary system with a conservative exchange of mass being closed, is erroneous. In a consequence of that the model of Paczynski-Huang is incorrect, and it cannot be used for study of motions in close binary stars. As show results of numerical integration of equations of motion with taking into account of true jet forces, acting on a star, the use of Paczynski-Huang model results in significant mistakes in definition of the semi-major axis, down to an opposite sign of derivative $\dot{a}$.

For this reason any conventional correct model, determining motions of close binary stars, now does not exist. By the aim of purpose our work is the such model creation. For circular motions of stars such a model was offered in the work [9], in which except for forces of mutual attraction of stars both the true jet forces, and forces of attraction on stars from jet stream of substance are taken into account also. The definition of stars motions is carried out with the help of numerical integration. In the present work a similar research, but under assumption, that the orbits of stars are elliptic will be carried out.

THE RESTRICTED ELLIPTICAL THREE-BODY PROBLEM

For definition of motion of flowing particles of masses we shall use the restricted elliptical three-body problem, the equations of which in rotating and pulsating, barycentric system of coordinates of Schepner (Scheibner, Petr, Nechvil, Rein) x, y look like [10]

$$\begin{aligned} \frac{d^2x}{dv^2} - 2\frac{dy}{dv} = \rho\frac{\partial U}{\partial x}, \\ \frac{d^2y}{dv^2} + 2\frac{dx}{dv} = \rho\frac{\partial U}{\partial y}, \end{aligned} \tag{12}$$

where the x-axis is always directed on a accreted-star S_2, v is true anomaly of stars, $\rho = 1/(1 + e\cos v)$ is the dimensionless distance of a star S_2 comparatively S_1. Through U the Jacobi function in the restricted elliptic three-body problem is designated

$$U = \frac{x^2 + y^2}{2} + p^3\left(\frac{1-m}{r_1} + \frac{m}{r_2}\right) + \frac{p^2}{2}(3 + m^2 - m), \tag{13}$$

and r_1 and r_2 are distances from a flowing particle of a jet accordingly up to the centers of masses of the first and second stars

$$r_1 = \sqrt{(x + pm)^2 + y^2}, \quad r_2 = \sqrt{(x + pm - p)^2 + y^2}, \tag{14}$$

where $1 - m = M_1/M$ and $m = M_2/M$ are relative masses of stars, $M = M_1 + M_2$, $p = a(1-e^2)$ is the focal parameter of an orbit, a and e are semi-major axis and eccentricity accordingly.

The stream of substance from a donor-star occurs through a vicinity of internal Euler libration point L_1, x_L with which is conclude in limits $-pm < x_L < p - pm$ and is determined numerically as a root of the nonlinear equation $\partial U/\partial x = 0$ at $y = 0$, i.e. of equation

$$x - p^3\left(\frac{1-m}{\sqrt{x + pm}} + \frac{m}{\sqrt{x + pm - p}}\right) = 0. \tag{15}$$

Roche lobes in the planar elliptic restricted three-body problem are determined with the help of equations of curves of minimal energy [11]

$$x^2 + y^2 + 2p^3\left(\frac{1-m}{r_1} + \frac{m}{r_2}\right) - p^2(3 + m^2 - m) = C(1 + e\cos v), \tag{16}$$

where C is Jacobi's constant.

Let's consider, that the accepting star has the form of sphere

$$\rho^2[(x + pm - p)^2 + y^2] = P^2, \tag{17}$$

the radius P of which in stream process of substance changes according to the dependence

$$P = P_0\sqrt[3]{\frac{m}{m_0}}. \tag{18}$$

The stream of substance begins with a donor-star after achievement by flowing particles a level of energy more than that of in the lipration point L_1. As it has been shown in

[11], the stream of substance through the vicinity of a point L_1 has a pulsating character and occurs in the vicinity of apoastr of orbits. The velocity V_0 of the stream of particles masses from a star S_1 is always directed along the x-axis to the star S_2 and has the value

$$V_0(v) = V_{00} V_1(v), \quad V_1(v) = m\sqrt{\frac{GM}{p}}\sqrt{1 + 2e\cos v + e^2}, \tag{19}$$

where $V_{00} \simeq 0.03$ is the coefficient, established from observations. The apoastr vicinity, in which occurs the stream of mass, is determined on a level of energy of particles (Jacobi's constant C, see [11]) by range of true anomaly $\pi - v_a \leq v \leq \pi + v_a$, where $0 < v_a < \pi$. Therefore the mean velocity V_c of stream of mass from the star S_1 during one orbital period of stars is possible to determine with the help of formula

$$V_c = \frac{1}{2v_a}\int\limits_{\pi - v_a}^{\pi + v_a} V_0\, dv = V_{00} m\sqrt{\frac{GM}{p}}\frac{1}{2v_a}\int\limits_{\pi - v_a}^{\pi + v_a}\sqrt{1 + 2e\cos v + e^2}\, dv. \tag{20}$$

All stream lines of a jet, having scatter both on coordinates and on time during one "emission" of substance for an orbital period, we shall approximate by one trajectory, outgoing from singular point L_1. The numerical integration of equations (12) is carried out with such initial conditions:

$$\begin{gathered} v_0 = \pi, \; v_a = \frac{\pi}{2}, \; x(v_0) = x_L, \; x'(v_0) = \frac{V_c}{\dot{v}}, \; y(v_0) = 0, \; y'(v_0) = 0, \\ m = m_0, \; \dot{m} = \text{const}, \; P = P_0, \end{gathered} \tag{21}$$

where "prime" signifies differentiation on true anomaly, $(\ldots)' = d(\ldots)/dv$.

The numerical integration is carried out on an interval $v_0 \leq v \leq v_0 + \tau$, where the value of true anomaly $v_0 + \tau$ is determined by the moment of hit of a particle on the second star surface in the point $x_2 = x(v_0 + \tau), y_2 = y(v_0 + \tau)$. If such a moment does not exist, the process of stream of substance occurs with forming of an accretion disk.

As a result of numerical integration there are components of outflow velocity of $\mathbf{W}_1$ from the star S_1 and the inflow mass velocity $\mathbf{W}_2$ on the star S_2:

$$W_{1x} = V_c, \; W_{1y} = 0, \; W_{2x} = x'(v_0 + \tau)\dot{v}, \; W_{2y} = y'(v_0 + \tau)\dot{v}. \tag{22}$$

Jet forces acting on stars S_1 and S_2 are considered to be applicable to their centers of masses and are determined with these formulas

$$\mathbf{Q}_1 = \dot{m}M\{-V_c, 0\}, \quad \mathbf{Q}_2 = \dot{m}M\{x'(v_0 + \tau)\dot{v}, y'(v_0 + \tau)\dot{v}\}. \tag{23}$$

With the help of numerical integration also the mass of established stream S_3 and coordinates x_3, y_3 of its center of masses are determined as:

$$M_3 = \frac{\tau}{\dot{v}_c}\dot{m}M, \quad X_3 = \frac{1}{\tau}\int\limits_{v_0}^{v_0 + \tau} x(v)dv, \quad Y_3 = \frac{1}{\tau}\int\limits_{v_0}^{v_0 + \tau} y(v)dv, \tag{24}$$

where

$$\dot{v}_c = \frac{1}{2\pi}\int\limits_0^{2\pi} \dot{v}\, dv = \frac{1}{2\pi}\int\limits_0^{2\pi} \sqrt{\frac{GM}{p^3}}\;(1 + e\cos v\,)^2\; dv = \sqrt{\frac{GM}{p^3}}\;\left(1 + \frac{e^2}{2}\right)$$

is the mean angular velocity of orbital motion.

With the help of formulas (23) and (24) the jet forces, the mass of jet and coordinates of its center of masses are determined. These values enter in the right-hand sides of the differential equations of relative motion of stars.

DIFFERENTIAL EQUATIONS OF MOTION OF STARS

As initial differential equations of motion of stars in an inertial coordinate system we shall consider system of equations:

$$\begin{aligned} M_1\frac{d\mathbf{V}_1}{d\,t} &= G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{Q}_1 + G\frac{M_1 M_3}{r_{13}^3}\mathbf{r}_{13}, \\ M_2\frac{d\mathbf{V}_2}{d\,t} &= -G\frac{M_1\,M_2}{R^3}\mathbf{R} + \mathbf{Q}_2 + G\frac{M_2 M_3}{r_{23}^3}\mathbf{r}_{23}, \end{aligned} \tag{25}$$

where $r_{13} = \sqrt{(x_3 + pm)^2 + y_3^2}$, $r_{23} = \sqrt{(x_3 + pm - p)^2 + y_3^2}$ distances between centers of masses of stars and that of the jet. The equation of a star S_2 relative S_1 motion from there turns out to be as

$$\frac{d\mathbf{V}}{d\,t} = -\frac{GM}{R^3}\mathbf{R} + \frac{\mathbf{Q}_2}{M_2} - \frac{\mathbf{Q}_1}{M_1} + GM_3\left(\frac{\mathbf{r}_{23}}{r_{23}^3} - \frac{\mathbf{r}_{13}}{r_{13}^3}\right), \tag{26}$$

where $\mathbf{V} = \mathbf{V}_2 - \mathbf{V}_1$ is the velocity of a star S_2 relative to S_1 motion.

Differential equations in osculating elements for semi-major axis of a relative orbit a and its eccentricity e are represented as [10]:

$$\begin{aligned} \frac{da}{dt} &= \frac{2a^2}{\sqrt{GMa(1-e^2)}}[e\sin vS + (1 + e\cos v)T], \\ \frac{de}{dt} &= \sqrt{\frac{a(1-e^2)}{GM}}\left[\sin vS + \left(\cos v + \frac{\cos v + e}{1 + e\cos v}\right)T\right]. \end{aligned} \tag{27}$$

Perturbing accelerations S and T are happened to be known after integration of equations (12):

$$\begin{aligned} S &= \dot{m}\left(\frac{W_{2x}}{m} + \frac{V_c}{1-m}\right) + GM\tau\frac{\dot{m}}{\dot{v}_c}\left(\frac{x_3 + pm - p}{r_{23}^3} - \frac{x_3 + pm}{r_{13}^3}\right), \\ T &= \dot{m}\frac{W_{2y}}{m} + GM\tau\frac{\dot{m}}{\dot{v}_c}\,Y_3 r_3, \quad r_3 = \frac{1}{r_{23}^3} - \frac{1}{r_{31}^3}. \end{aligned} \tag{28}$$

Choosing as the independent variable the true anomaly v of stars, the equations (27) can be rewritten as follows

$$\begin{aligned} \frac{da}{dv} &= \frac{2a^2}{\dot{v}\sqrt{GMa(1-e^2)}}[e\sin vS + (1 + e\cos v)T], \\ \frac{de}{dv} &= \frac{1}{\dot{v}}\sqrt{\frac{a(1-e^2)}{GM}}\left[\sin vS + \left(\cos v + \frac{\cos v + e}{1 + e\cos v}\right)T\right], \end{aligned} \tag{29}$$

where

$$\dot{v} = \sqrt{\frac{GM}{p^3}}(1 + e\cos v)^2 + \sqrt{\frac{p}{GM}}\left[\frac{\cos v}{e}S - \frac{\sin V}{e}T\left(1 + \frac{1}{1 + e\cos v}\right)\right].$$

After averaging on the true anomaly v, equations of stars motion result in the form

$$\begin{aligned}
\frac{d\tilde{a}}{d\tilde{v}} &= \frac{1}{2\pi}\int\limits_0^{2\pi} \frac{2a^2}{\dot{v}\sqrt{GMa(1-e^2)}}[e\sin v\, S + (1 + e\cos v)T]dv, \\
\frac{d\tilde{e}}{d\tilde{v}} &= \frac{1}{2\pi}\int\limits_0^{2\pi} \frac{1}{\dot{v}}\sqrt{\frac{a(1-e^2)}{GM}}\left[\sin v\, S + \left(\cos v + \frac{\cos v + e}{1 + e\cos v}\right)T\right]dv.
\end{aligned} \tag{30}$$

Besides if instead of v we choose the new independent variable m with the help of the formula $\dfrac{d}{d\tilde{v}} = \dfrac{\dot{m}}{\dot{v}_c}\dfrac{d}{dm}$, then equations of motion for averaged elements $\tilde{a}$ and $\tilde{e}$ will happened to be as

$$\begin{aligned}
\frac{d\tilde{a}}{dm} &= \frac{\dot{v}_c}{2\pi\dot{m}}\int\limits_0^{2\pi} \frac{2a^2}{\dot{v}\sqrt{GMa(1-e^2)}}[e\sin vS + (1 + e\cos v)T]dv, \\
\frac{d\tilde{e}}{dm} &= \frac{\dot{v}_c}{2\pi\dot{m}}\int\limits_0^{2\pi} \frac{1}{\dot{v}}\sqrt{\frac{a(1-e^2)}{GM}}\left[\sin vS + \left(\cos v + \frac{\cos v + e}{1 + e\cos v}\right)T\right]dv.
\end{aligned} \tag{31}$$

After calculation of definite integrals in right-hand sites of equations (31) we shall receive the final form of differential equations for determination of secular perturbations of semi-major axis and eccentricity of the stars relative orbit:

$$\begin{aligned}
\frac{d\tilde{a}}{dm} &= \frac{2 + \tilde{e}^2}{(1-\tilde{e}^2)^2}\frac{\widetilde{W}_{2y}}{m} + 2\tilde{a}^3\sqrt{1-\tilde{e}^2}\,\tau y_3 r_3, \\
\frac{d\tilde{e}}{dm} &= \frac{2 + \tilde{e}^2}{2\tilde{a}(1-\tilde{e}^2)}\frac{\tilde{e}}{1 + \sqrt{1-\tilde{e}^2}}\frac{\widetilde{W}_{2y}}{m} - \frac{3}{2}\tilde{e}\,\tilde{a}^2\sqrt{1-\tilde{e}^2}\,\tau y_3 r_3,
\end{aligned} \tag{32}$$

where the designation $\widetilde{W}_{2y} = y'(v_0 + \tau)$ is accepted.

If to put $\widetilde{W}_{2y} = 0$, i.e. not to take into account jet forces, the equations (32) are simplified and accept the form

$$\begin{aligned}
\frac{d\tilde{a}}{dm} &= 2\tilde{a}^3\sqrt{1-\tilde{e}^2}\,\tau y_3 r_3, \\
\frac{d\tilde{e}}{dm} &= \frac{3}{2}\tilde{e}\,\tilde{a}^2\sqrt{1-\tilde{e}^2}\,\tau y_3 r_3,
\end{aligned} \tag{33}$$

which admin the existence of such first integral

$$\tilde{a}^3\,\tilde{e}^4 = \text{const} = \tilde{a}_0^3\,\tilde{e}_0^4 \tag{34}$$

with initial conditions $\tilde{a}_0 = \tilde{a}(m_0)$ and $\tilde{e}_0 = \tilde{e}(m_0)$.

Further for simplicity the overlined index "tilde" above values a and e will be omitted.

NUMERICAL RESULTS

The relative orbit of stars was determined by means of numerical integration of differential equations (32) and (33). The integration was carried out on an interval by independent variable m from 0.1 up to 0.8.

In fig. 1 are given dependencies of the semi-major axis a and eccentricity e of relative orbit of a accreted-star from value $q = m/(1-m)$ for various values of initial radius stars P_0 at initial value of eccentricity $e_0 = 0.1$. From this figure it is visible, that the change of an orbit strongly depends on P_0. At small values P_0 the flowing jet of substance gets on the surface of a star S_2 with the large velocity and also creates enough the large jet force accelerating motion of a star, while for large P_0, on the contrary, the jet force is directed in the opposite side and consequently drags the motion of the star.

The influence of initial value of eccentricity e_0 on elements of star S_2 is represented in fig. 2 for fixed value $P/a = 0.075$. As it is seen stars motions strongly depend on the initial value of eccentricity e_0. Under small e_0 and P_0 secular changes of the semi-major axes can reach significant values.

In fig. 3 on the plane $(q, P/a)$ the curves are represented describing a close binary system. The curve 1 determines border $P_{max}/a = 0.49\sqrt[3]{m}$, after which the binary forms contact system, when the sizes of accreted-star reach the singular Euler point. The solid curves 2-5 for various e_0 determine border for P_{min}/a below of which at close binary system an accreted disk is formed. Between curves P_{max}/a and P_{min}/a the half-divided phase of close binary system settles down. On dot and dash curves 6-9 for various e_0 the first derivative of the semi-major axis turns into zero. Above these curves a close binary system extends, and downwords it is compressed. The model of Paczynski-Huang on this figure is represented by a vertical straight line $q = 1$. According to the formula (1) at $q < 1$ a binary is compressed, and at $q > 1$ it extends, that sharply differs from results which has been received by means of numerical integration.

In fig. 4 on the same plane $(q, P/a)$ curves of change are represented for value P/a depending on q for various values P_0. The curve 1 crosses the border of contact binary system formation. The further account for this curve is not carried out. The curves 6 and 7 cross a border of accreted disk formation. If one considers, that the accreted disk is formed instantly, it is possible to continue these curves further, using the differential equations (33), i.e. believing, that any jet stream does not create jet force on a star S_2, giving back energy on rotary motion for particles of accreted disk. Continuations of curves 6-7 on the figure is shown by dashed lines.

In fig. 5 the comparison of change of the semi-major axis and eccentricity of an orbit of stars is carried out at presence of accreted disk and at its absence for various initial values of eccentricity. As it is visible from figure changes a and e at presence of accreted disk are not rather great, that is explained by absence of a jet force and small forces of attraction on stars by the stream. Therefore for, approximate estimations of evolution of close stars orbit with presence of accreted disk, taking into account small sizes of a jet and, hence, small right hand sides of equations (33), it is possible to consider, that

$$a = \text{const}, \quad e = \text{const}, \tag{35}$$

i.e. instead of Paczynski-Huang model one can use that of Gylden-Mestschersky.

Dependencies between the semi-major axis a end eccentricity e along trajectories of stars motion are represented in fig. 6. For the accreted disk these dependencies turn out from the first integral (34).

It is difficult to carry out comparison of the present work results with those of other authors, so as jet forces, created by a stream, actually nobody takes into account (see, for example, [11]), but in the present work the jet forces play the basic role.

It is doubtless, that all received results require a further revision and their conformity to observations.

CONCLUSION

During the last half-century for definition a relative orbit of close binary stars the incorrect Paczynski-Huang model was used, that was marked in the paper [8]. However up to this day in works, dedicated to close binary systems this model continues to be used. Therefore in the present work once again, but by means of other methods, the inaccuracy of this model has been shown.

For determination of relative motion of stars in close binary system in the present work the numerical integration of equations of motion is used with taking into account of jet forces and forces of attraction of stars by the flowing jet. The above calculations of elliptic orbits of close binary stars show that the eccentricity of orbits can change strongly enough. The influence of jet force on orbital evolution of stars can be various. If the accepting star has smaller mass and little sizes, and the accreted disk is absent, then the eccentricity is increased strongly, reaching values 0.5-0.8. Probably just the such influence of jet force explains rather large number of close double stars with great orbital eccentricity. If the accepting star has the large mass and the great size, the jet force creates the braking effect, and the eccentricity decreases down to zero.

It is shown that for approximate determination of orbital evolution of close binary system with a generated accreted disk instead of the Paczynski-Huang model it is necessary to use the model of Gylden-Mestschersky, according to which all Keplerien elements remain constant, in particular, $a = \text{const}$ and $e = \text{const}$.

It is doubtless, that obtained results can be specified, taking into account other perturbation factors and making new assumptions based on observations.

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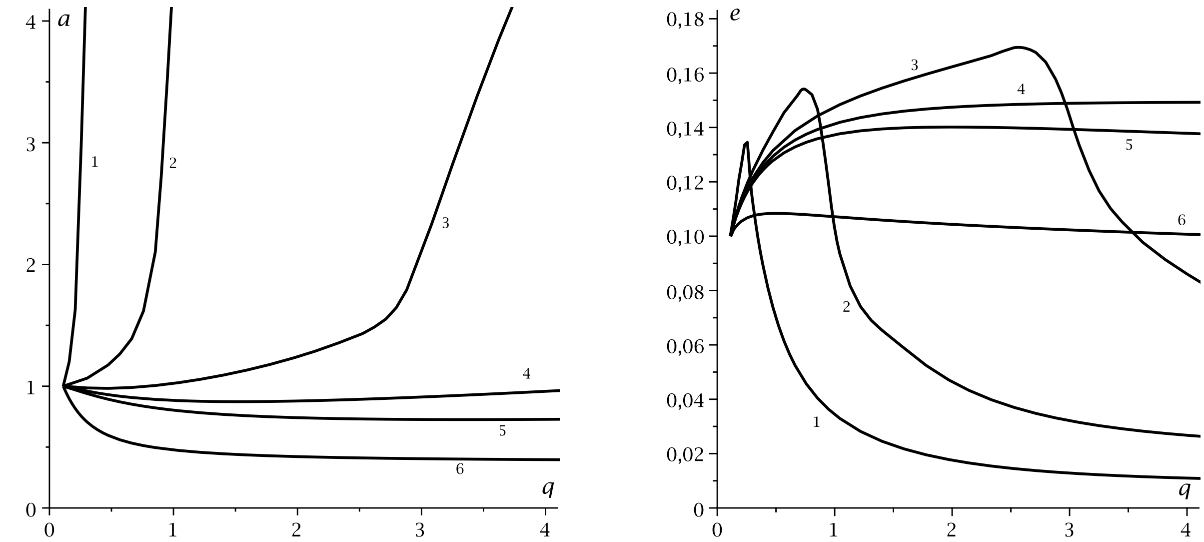
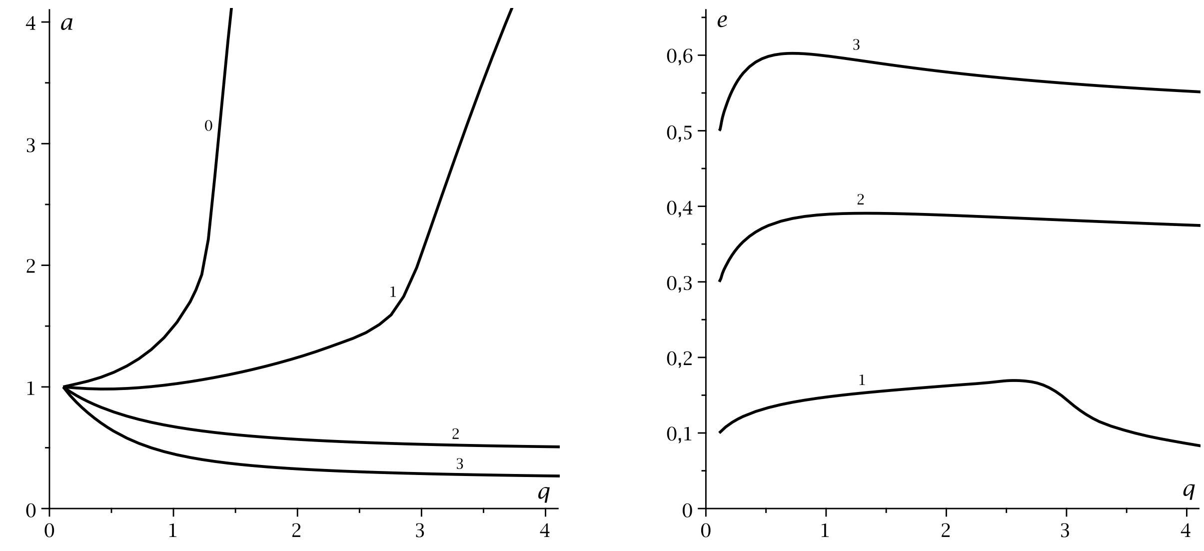
REFERENCES

1. A.Kruszevski, Adv. Astron. And Astrophys. **4**, 233 (1966).
2. J.Hadjidemetriou, Astrophys. And Space Sci. **3**, 330 (1969).
3. S.L.Piotrovski, Astron.J. (russian), **44**, 241 (1967).
4. S.S.Huang, Ann. Rev. Of Astron. And Astroph. **4**, 35 (1966).
5. B.Paczynski, Acta Astron. **16**, 231 (1966).
6. I.W.Mestschersky, Astron. Nachr. **159**, 229 (1902).
7. T.Levy-Civita, Rendiconti della reale Acad. Lincei. **8**, 12, 621 (1928).
8. L.G.Lukyanov, Letter in Astron. (russian), **31**, 8, 628 (2005).
9. L.G.Lukyanov, Astron.J. (russian), **85**, 755 (2008).
10. G.N.Douboshin, The celestial mechanics. The basic problems and methods (Moscow, 1968).
11. L.G.Lukyanov, Astron.J. (russian), **82**, 1137 (2005).
12. J.F.Sepinsky, B.Willems, V.Kalogera, F.A.Rasio, The Astrophys. J., **667** 1170 - 1184 (2007).



(a) (b)

Figure 1: The diagrams of functions: (a) — $a(q)$ and (b) — $e(q)$ at $a_0 = 1$ and $e_0 = 0.1$ for various P_0. Notations: 1 — $P_0 = 0.06$, 2 — $P_0 = 0.073$, 3 — $P_0 = 0.075$, 4 — $P_0 = 0.0758$, 5 — $P_0 = 0.0765$, 6 — $P_0 = 0.1$.



(a) (b)

Figure 2: The diagrams of functions: (a) — $a(q)$ and (b) — $e(q)$ at $a_0 = 1$ and $P_0 = 0.075$ for various e_0. Notations: 0 — $e_0 = 0$, 1 — $e_0 = 0.1$, 2 — $e_0 = 0.3$, 3 — $E_0 = 0.5$.

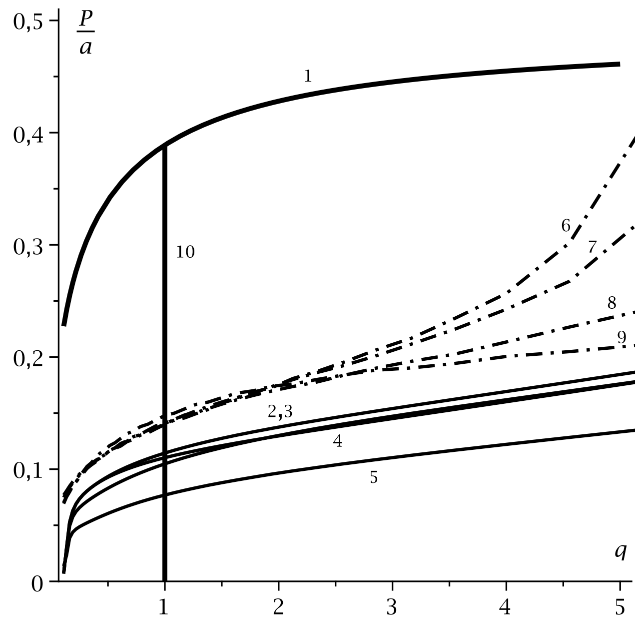


Figure 3: Characteristic curves on a plane $\left(q, \dfrac{P}{a}\right)$. Notations: 1 — the border of formation of the contact system, solid curves 2, 3, 4, 5 — the border of accreted disk formations for values $e_0 = 0$, $e_0 = 0.1$, $e_0 = 0.3$, $e_0 = 0.5$ accordingly, doted and dashed curves 6, 7, 8, 9 — the border of change of a sign of derivative a' for the same values e_0 accordingly, vertical direct line 10 — the border of change sign by derivative a' under the Paczynski-Huang formula.

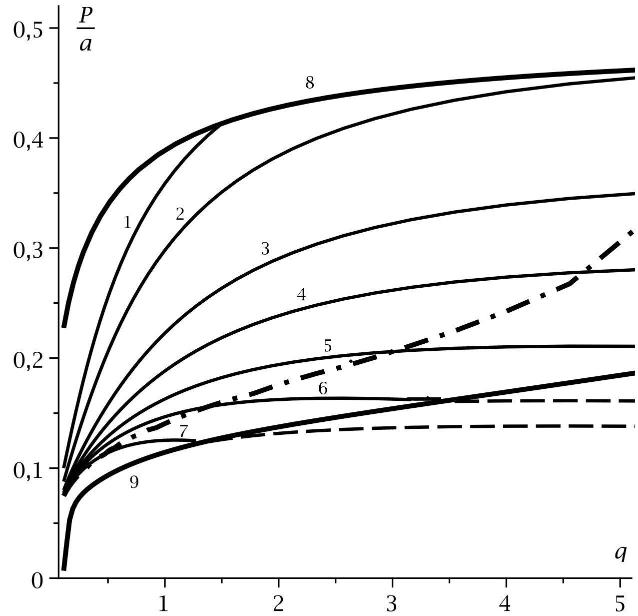
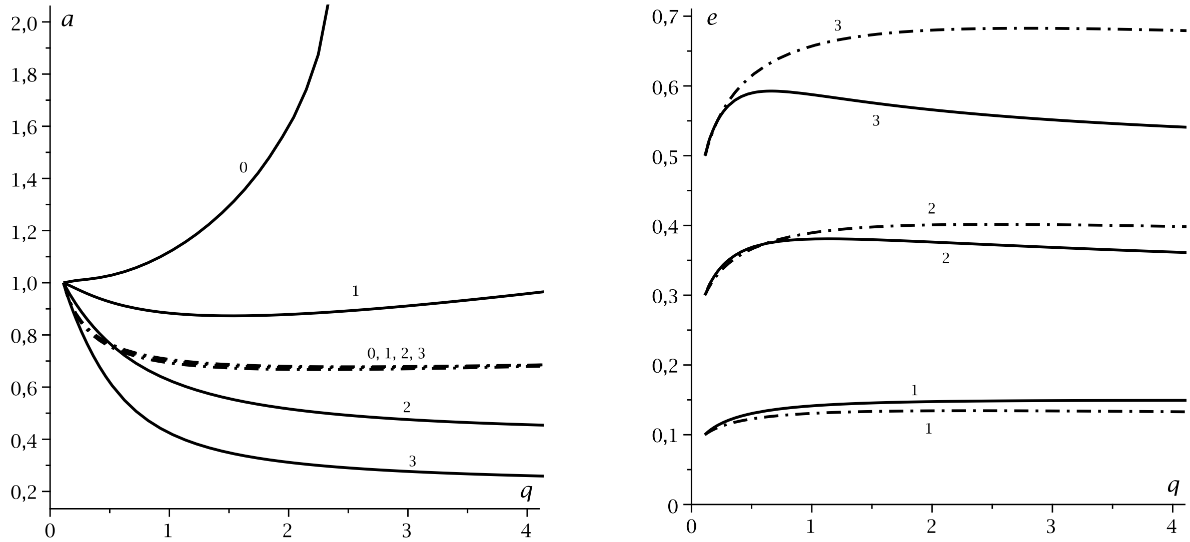
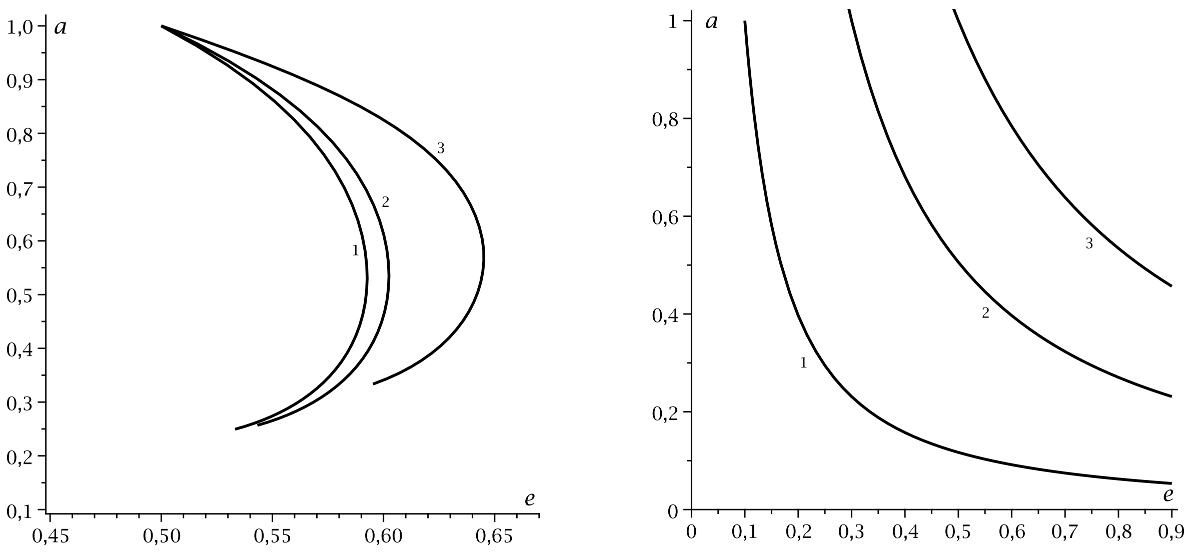


Figure 4: Dependence of the ratio $\dfrac{P}{a}$ from q for different trajectories of motion of stars at $a_0 = 1$ and $e_0 = 0.1$. Notations: 1 — $P_0 = 0.1$, 2 — $P_0 = 0.0878$, 3 — $P_0 = 0.08$, 4 — $P_0 = 0.0778$, 5 — $P_0 = 0.0765$, 6 — $P_0 = 0.0758$, 7 — $P_0 = 0.075$, 8 — the border of contact system, 9 — the border of accreted disk. Dashed curves — continuation the appropriate curves after formation of accreted disk, doted end dashed curve — the border of change of sign of derivative a'.



(a) (b)

Figure 5: The diagrams of functions: (a) — $a(q)$ and (b) — $e(q)$ for $a_0 = 1$ and $P_0 = 0.0758$. Notations: 0 — $e_0 = 0$, 1 — $e_0 = 0.1$, 2 — $e_0 = 0.3$, 3 — $e_0 = 0.5$. Solid curves correspond to absence of accreted disk, and dot end dash curve — to its presence.



(a) Without of accreted disk at $a_0 = 1$ and $e_0 = 0.5$. Notations: 1 — $P_0 = 0.0758$, 2 — $P_0 = 0.075$, 3 — $P_0 = 0.073$.

(b) With accreted disk. Notations: 1 — $e_0 = 0.1$, 2 — $e_0 = 0.3$, 3 — $e_0 = 0.5$.

Figure 6: Dependencies between a and e along trajectories.