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# An Algorithm for the Calculation of Progress or Regress via TOPSIS and Malmquist Productivity Index

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**Abstract**

In this paper, the TOPSIS method is used for two times $t$ and $t+1$. In order to calculate the progress or regress via Malmquist index, the positive and negative ideal and negative ideals at time $t$ and $t+1$ are calculated first, then we consider the distance all of the possible alternatives among by positive ideal and negative ideal at time periods of $t$ and $t+1$. Then we calculate the distance of all the $A_j$ for the positive ideal and negative ideal for the other time period. Then we use the Malmquist productivity index to calculate the progress or regress of all $A_j$.

**Keywords:** Topsis, Positive ideal solution, Nagative ideal solution, Malmquist index, Progress, Regress

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Table 1: Matrix format a MCDM problem

| | $c_1$ | $c_2$ | . | . | . | $c_n$ |
|---|---|---|---|---|---|---|
| $A_1$ | $x_{11}$ | $x_{12}$ | . | . | . | $x_{1n}$ |
| $A_2$ | $x_{21}$ | $x_{22}$ | . | . | . | $x_{2n}$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| $A_m$ | $x_{m1}$ | $x_{m2}$ | . | . | . | $x_{mn}$ |

# 1 Introduction

Decision-making problem is the process of finding the best option among all of the feasible alternatives. For many problems, the decision-maker wants to solve a multiple criteria decision making (MCDM) problem. In the MCDM, the managerial level defines the goals and chooses the final optimal alternative. The multi-criteria nature of decisions is emphasized at this managerial level, at which public officials called decision makers have the power to accept or reject the solution proposed by engineering level [1].
In classical MCDM methods, the rating and the weights of the criteria are known precisely [2,3]. A survey of the methods has been presented in Hwang and Yoon [3]. Technique for order performance by similarity to ideal solution(TOPSIS) [4], one of the known classical $MCDM$ methods, was first developed by Hwang and Yoon [3] for solving a MCDM problem.

# 2 TOPSIS method

A MCDM problem can be concisely expressed in the matrix format as

$$w = [w_1, \ldots, w_n]$$

Where $A_1, \ldots, A_m$ are the possible alternatives among which decision makers have to choose; $c_1, \ldots, c_n$, are the criteria with alternative performance which are measured; $x_{ij}$ is the rating of alternative $A_i$ with respect to criterion $c_j$; $w_j$ is the weight of criterion $c_j$.
TOPSIS method is presented in Chen and Hwang [1], in reference to Hwang and Yoon [2]. TOPSIS is a multiple criteria method to identify the solution from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the

Table 2: Matrix format in time t

| t | $c_1$ | $c_2$ | . | . | . | $c_n$ |
|---|---|---|---|---|---|---|
| $A_1$ | $x_{11}^t$ | $x_{12}^t$ | . | . | . | $x_{1n}^t$ |
| $A_2$ | $x_{21}^t$ | $x_{22}^t$ | . | . | . | $x_{2n}^t$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| $A_m$ | $x_{m1}^t$ | $x_{m2}^t$ | . | . | . | $x_{mn}^t$ |

Table 3: Matrix format in time t+1

| t | $c_1$ | $c_2$ | . | . | . | $c_n$ |
|---|---|---|---|---|---|---|
| $A_1$ | $x_{11}^{t+1}$ | $x_{12}^{t+1}$ | . | . | . | $x_{1n}^{t+1}$ |
| $A_2$ | $x_{21}^{t+1}$ | $x_{22}^{t+1}$ | . | . | . | $x_{2n}^{t+1}$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| $A_m$ | $x_{m1}^{t+1}$ | $x_{m2}^{t+1}$ | . | . | . | $x_{mn}^{t+1}$ |

farthest distance from the negative ideal solution. The procedure of TOPSIS can be expressed in a set of steps:

1. First we calculate the normalized decision matrix.

2. Then we calculate the weighty normalized decision matrix.

3. After that we calculate the positive and negative ideals.

4. We calculate distance of all DMs by the positive and negative ideals.

# 3 Calculate progress or regress via TOPSIS and Malmquist Index

Suppose we have matrix (1) in two times of $t$ and $t+1$ as follows: Now we can calculate the normalized decision matrices (2) and (3). The normalized value of $n_{ij}$ is calculated as Where $n_{ij}^t$ and $n_{ij}^{t+1}$ are defined by (1) and (2) as follows:

$$n_{ij}^t = \frac{x_{ij}^t}{\sqrt{\sum_{i=1}^m (x_{ij}^t)^2}} \qquad j = 1, \ldots, n \qquad i = 1, \ldots, m \tag{1}$$

Table 4: Normalized matrix in time t

| t | $c_1$ | $c_2$ | . | . | . | $c_n$ |
|---|---|---|---|---|---|---|
| $A_1$ | $n_{11}^t$ | $n_{12}^t$ | . | . | . | $n_{1n}^t$ |
| $A_2$ | $n_{21}^t$ | $n_{22}^t$ | . | . | . | $n_{2n}^t$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| $A_m$ | $n_{m1}^t$ | $n_{m2}^t$ | . | . | . | $n_{mn}^t$ |

Table 5: Normalized matrix in time t+1

| t+1 | $c_1$ | $c_2$ | . | . | . | $c_n$ |
|---|---|---|---|---|---|---|
| $A_1$ | $n_{11}^{t+1}$ | $n_{12}^{t+1}$ | . | . | . | $n_{1n}^{t+1}$ |
| $A_2$ | $n_{21}^{t+1}$ | $n_{22}^{t+1}$ | . | . | . | $n_{2n}^{t+1}$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| $A_m$ | $n_{m1}^{t+1}$ | $n_{m2}^{t+1}$ | . | . | . | $n_{mn}^{t+1}$ |

$$n_{ij}^{t+1} = \frac{x_{ij}^{t+1}}{\sqrt{\sum_{i=1}^{m}(x_{ij}^{t+1})^2}} \qquad j = 1, \dots, n \qquad i = 1, \dots, m \tag{2}$$

Now consider the normalized matrices (4) and (5). Then we calculate the weighty normalized decision matrices (6) and (7) by relations (3) and (4) as follows: where $v_{ij}^t$ and $v_{ij}^{t+1}$ are

$$v_{ij}^t = w_i^t n_{ij}^t \qquad j = 1, \dots, m, i = 1, \dots, n \qquad \sum_{i=1}^{n} w_i^t = 1 \tag{3}$$

Table 6: Weighty Normalized matrix in time t

| t | $c_1$ | $c_2$ | . | . | . | $c_n$ |
|---|---|---|---|---|---|---|
| $A_1$ | $v_{11}^t$ | $v_{12}^t$ | . | . | . | $v_{1n}^t$ |
| $A_2$ | $v_{21}^t$ | $v_{22}^t$ | . | . | . | $v_{2n}^t$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| $A_m$ | $v_{m1}^t$ | $v_{m2}^t$ | . | . | . | $v_{mn}^t$ |

Table 7: Weighty Normalized matrix in time t+1

| t+1 | $c_1$ | $c_2$ | . . . | $c_n$ |
|---|---|---|---|---|
| $A_1$ | $v_{11}^{t+1}$ | $v_{12}^{t+1}$ | . . . | $v_{1n}^{t+1}$ |
| $A_2$ | $v_{21}^{t+1}$ | $v_{22}^{t+1}$ | . . . | $v_{2n}^{t+1}$ |
| . | . | . | . . . | . |
| . | . | . | . . . | . |
| . | . | . | . . . | . |
| $A_m$ | $v_{m1}^{t+1}$ | $v_{m2}^{t+1}$ | . . . | $v_{mn}^{t+1}$ |

$$v_{ij}^{t+1} = w_i^{t+1} n_{ij}^{t+1} \qquad j = 1, \dots, m, i = 1, \dots, n \qquad \sum_{i=1}^{n} w_i^{t+1} = 1 \tag{4}$$

Here we have the positive ideal and the negative ideal at times $t$ and $t+1$ as follows

$$A^t = \{v_1^{+t}, \dots, v_n^{+t}\} = \{(\max_j v_{ij}^t \mid i \in I), (\min_j v_{ij}^t \mid i \in J)\}$$

$$B^t = \{v_1^{-t}, \dots, v_n^{-t}\} = \{(\min_j v_{ij}^t \mid i \in I), (\max_j v_{ij}^t \mid i \in J)\}$$

where $A^t$ and $B^t$ represent the positive ideal and the negative ideal at time $t$ respectively. I is associated with the benefit criteria and J is associated with the cost criteria. Similarly, at time $t+1$ we have

$$A^{(t+1)} = \{v_1^{+(t+1)}, \dots, v_n^{+(t+1)}\} = \{(\max_j v_{ij}^{(t+1)} \mid i \in I), (\min_j v_{ij}^{(t+1)} \mid i \in J)\}$$

$$B^{(t+1)} = \{v_1^{-(t+1)}, \dots, v_n^{-(t+1)}\} = \{(\min_j v_{ij}^{(t+1)} \mid i \in I), (\max_j v_{ij}^{(t+1)} \mid i \in J)\}$$

where $A^{(t+1)}$ and $B^{(t+1)}$ represent the positive ideal and the negative ideal at time $(t+1)$ respectively.

Now we can calculate the separation measuring, using the n-dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as

$$p_j{}^{(t)(t)} = \left\{ \sum_{j=1}^{n} (v_{ij}^t - v_i^{+t})^2 \right\}^{\frac{1}{2}}, \qquad j = 1, \dots, m \tag{5}$$

$$n_j{}^{(t)(t)} = \left\{ \sum_{j=1}^{n} (v_{ij}^{t} - v_i^{-t})^2 \right\}^{\frac{1}{2}}, \qquad j = 1, \ldots, m \tag{6}$$

where $p_j{}^{(t)(t)}$ and $n_j{}^{(t)(t)}$ represent the distance to the positive ideal and the negative ideal respectively, at time t.

$$p_j{}^{(t+1)(t+1)} = \left\{ \sum_{j=1}^{n} (v_{ij}^{t+1} - v_i^{+t+1})^2 \right\}^{\frac{1}{2}}, \qquad j = 1, \ldots, m \tag{7}$$

$$n_j{}^{(t+1)(t+1)} = \left\{ \sum_{j=1}^{n} (v_{ij}^{t+1} - v_i^{-t+1})^2 \right\}^{\frac{1}{2}}, \qquad j = 1, \ldots, m \tag{8}$$

where $p_j{}^{(t+1)(t+1)}$ and $n_j{}^{(t+1)(t+1)}$ represent the distance to the positive ideal and the negative ideal respectively, at time t+1.
Now we have

$$p_j{}^{(t)(t+1)} = \left\{ \sum_{j=1}^{n} (v_{ij}^{t} - v_i^{+t+1})^2 \right\}^{\frac{1}{2}}, \qquad j = 1, \ldots, m \tag{9}$$

$$n_j{}^{(t)(t+1)} = \left\{ \sum_{j=1}^{n} (v_{ij}^{t} - v_i^{-t+1})^2 \right\}^{\frac{1}{2}}, \qquad j = 1, \ldots, m \tag{10}$$

and

$$p_j{}^{(t+1)(t)} = \left\{ \sum_{j=1}^{n} (v_{ij}^{t+1} - v_i^{+t})^2 \right\}^{\frac{1}{2}}, \qquad j = 1, \ldots, m \tag{11}$$

$$n_j{}^{(t+1)(t)} = \left\{ \sum_{j=1}^{n} (v_{ij}^{t+1} - v_i^{-t})^2 \right\}^{\frac{1}{2}}, \qquad j = 1, \ldots, m \tag{12}$$

Where $p_j^{(t)(t+1)}$ is distance $A_j$ at time t from the positive ideal at time t+1, Similarly, we define the $p_j^{(t)(t+1)}$, and $n_j^{(t)(t+1)}$ are distance $A_j$ at time t from the negative ideal in time t+1.
But if the distance of the positive ideal is less and the distance of the negative ideal is more,decision-making is better. Then $M^+$ and $M^-$ are the positive ideal and the negative ideal can be defined as

$$M^+ = \frac{p_j{}^{(t+1)(t+1)}}{p_j{}^{(t)(t)}} \left[ \frac{p_j{}^{(t+1)(t+1)}}{p_j{}^{(t+1)(t+1)}} \cdot \frac{p_j{}^{(t)(t+1)}}{p_j{}^{(t+1)(t)}} \right]^{\frac{1}{2}} \tag{13}$$

$$M^+ = \left[ \frac{p_j{}^{(t+1)(t+1)}}{p_j{}^{(t)(t+1)}} \cdot \frac{p_j{}^{(t+1)(t)}}{p_j{}^{(t)(t)}} \right]^{\frac{1}{2}} \tag{14}$$

$$M^- = \frac{n_j{}^{(t)(t)}}{n_j{}^{(t+1)(t+1)}} \left[ \frac{n_j{}^{(t+1)(t+1)}}{n_j{}^{(t)(t+1)}} \cdot \frac{n_j{}^{(t+1)(t)}}{n_j{}^{(t)(t)}} \right]^{\frac{1}{2}} \tag{15}$$

$$M^- = \left[ \frac{n_j{}^{(t)(t+1)}}{n_j{}^{(t+1)(t+1)}} \cdot \frac{n_j{}^{(t)(t)}}{n_j{}^{(t+1)(t)}} \right]^{\frac{1}{2}} \tag{16}$$

Now by (20)and(21) we have

1. If $M^+ > M^-$, there is progress.
2. If $M^+ < M^-$, there is regress.
3. If $M^+ = M^-$, we don't have anything.

## 4 Example

In this section, we work out a numerical example to illustrate the TOPSIS method for decision-making problems via Malmquist productivity index in time t and t+1.

| t | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ |
|---|---|---|---|---|---|---|---|---|
| $c_1$ | 2 | 3 | 5 | 1 | 4 | 8 | 2 | 5 |
| $c_2$ | 1 | 3 | 4 | 6 | 7 | 5 | 9 | 6 |

And

| t+1 | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ |
|---|---|---|---|---|---|---|---|---|
| $c_1$ | 2 | 3 | 4 | 2 | 3 | 6 | 2 | 5 |
| $c_2$ | 1 | 3 | 4 | 6 | 7 | 5 | 9 | 6 |

In this example, $A_1, \ldots, A_8$ are projects at time $t$ and $t+1$. $C_1$ is the cost and $C_2$ is the benefit.

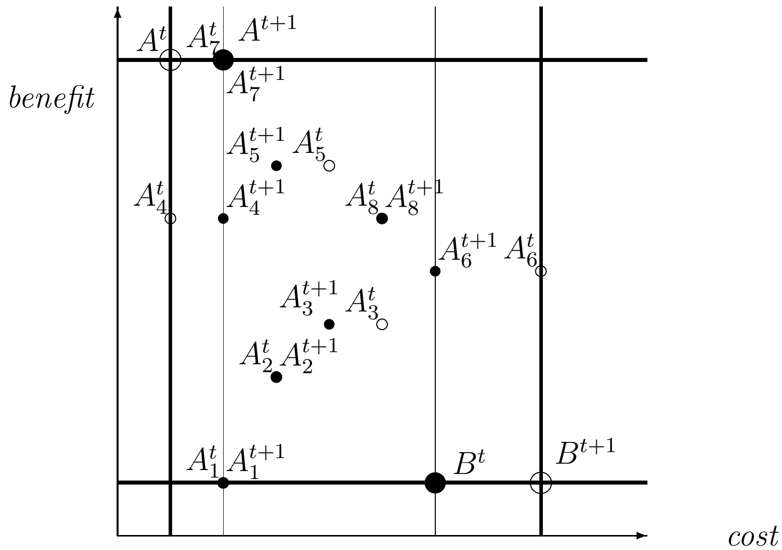


Fig. 1. The ideal negative and positive at time t and t+1

where the solid circle defined $A_j, j = 1, \ldots, m$ at time t and the circle defined $A_j, j = 1, \ldots, m$ at time $t+1$.

| $c_1$ | $c_2$ | $n_{ij}(c_1)$ | $n_{ij}(c_2)$ | ${p_j}^{(t)(t)}$ | ${n_j}^{(t)(t)}$ | ${p_j}^{(t)(t+1)}$ | ${n_j}^{(t)(t+1)}$ |
|---|---|---|---|---|---|---|---|
| 2 | 1 | 0.164399 | 0.0628669 | 0.129969 | 0.247453 | 0.2368254 | 0.208075 |
| 3 | 3 | 0.246598 | 0.565825 | 0.131583 | 0.2095668 | 0.217468 | 0.169546 |
| 5 | 4 | 0.140997 | 0.251478 | 0.175578 | 0.148187 | 0.20585 | 0.110979 |
| 1 | 6 | 0.082199 | 0.377217 | 0143849 | 0.287698 | 0.262874 | 0.249133 |
| 4 | 7 | 0.328798 | 0.440086 | 0.148187 | 0.175578 | 0.206693 | .0135719 |
| 8 | 5 | 0.657596 | 0.314347 | 0.287698 | 0.143849 | 0.258204 | 0.139083 |
| 2 | 9 | 0.164399 | 0.565825 | 0.129969 | 0.247453 | 0.236825 | 0.208075 |
| 5 | 6 | 0.410997 | 0.377217 | 0.175578 | 0.148187 | 0.20585 | 0.110979 |

where $A^t = (0.0411, 0.164399)$ and $B^t = (0.328798, 0.02055)$ are the positive ideal and negative ideal respectively.

| $c_1$ | $c_2$ | $n_{ij}(c_1)$ | $n_{ij}(c_2)$ | $p_j^{(t+1)(t+1)}$ | $n_j^{(t+1)(t+1)}$ | $p_j^{(t+1)(t)}$ | $n_j^{(t+1)(t)}$ |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 0.193347 | 0.123325 | 0.215819 | 0.19579 | 0.116804 | 0.235737 |
| 3 | 4 | 0.290021 | 0.246651 | 0.161557 | 0.171998 | 0.111734 | 0.210572 |
| 4 | 1 | 0.386695 | 0.061663 | 0.264919 | 0.096674 | 0.202533 | 0.13584 |
| 2 | 5 | 0.193347 | 0.308313 | 0.123325 | 0.22933 | 0.05651 | 0.267829 |
| 3 | 8 | 0.290021 | 0.493301 | 0.057333 | 0.260012 | 0.132524 | 0.291375 |
| 6 | 6 | 0.580042 | 0.369976 | 0.214332 | 0.154157 | 0.249771 | 0.168948 |
| 2 | 9 | 0.193347 | 0.554964 | 0.096674 | 0.3134 | 0.126001 | 0.34626 |
| 5 | 6 | 0.483368 | 0.369976 | 0.171998 | 0.161557 | 0.201638 | 0.186088 |

where $A^{t+1} = (0.096674, 0.277482)$ and $B^{t+1} = (0.290021, 0.030831)$ are the positive ideal and negative ideal respectively.

| | $M^+$ | $M^-$ |
|---|---|---|
| 1 | 0.904982193 | 1.056202483 |
| 2 | 0.733530519 | 0.990476666 |
| 3 | 1.213492469 | 1.11906877 |
| 4 | .429300085 | 1.0080251343 |
| 5 | .49805999 | 0.560831676 |
| 6 | .848930811 | 0.876460394 |
| 7 | 0 | 0.688819 |
| 8 | 0.82746141 | 0.739611035 |

Here $A_1$, $A_2$, $A_4$,$A_5$,$A_6$,$A_7$ have regress and $A_3$, $A_8$ have progress.

# 5 Conclusion

Topsis is a multiple criteria method to identify solution from a finite set of alternative based on simultaneous minimization of distance from an ideal point and maximization of distance from a nadir point, and in this paper we calculate progress or regress by Malmquist index via Topsis method.

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