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A New Method for Complex Decision Making Based on TOPSIS for Complex Decision Making Problems with Fuzzy Data

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Abstract

The aim of this paper is to extend the TOPSIS method for decision-making problems with Fuzzy data. By this extension of TOPSIS method, an algorithm for determining the most preferable choice among all possible choices in the case of fuzzy data is presented. To illustrate the performance of the proposed algorithm, a decision-making problem is solved at the end of paper.

Keywords: TOPSIS; Fuzzy number; Ranking; Fuzzy distance

1 Introduction

Fuzzy set theory has been successfully applied and implemented in production management [3].The use of fuzzy set theory as methodology for modeling and analyzing decision systems is particularly interesting to researchers concerned with complex decision-making problems[2,3,6]. Decision making is an integral part of any business organization. The process involves selecting the best among several decisions through a proper evaluation of the parameters of each decision environment. The types of decisions can be classified into following categories:

1. Decision under certainty.

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2. Decision under risk.

3. Decision under uncertainty.

Most of decisions in the strategic level of management belong to categories (2) and (3). However, research on the application of fuzzy TOPSIS is lacking In this paper we want to extend one of the methods which is used to deciding under certainty to solving problems under uncertainty.
Hwang and Yoon proposed the TOPSIS (Techniques for Order Preference by Similarity to an Ideal Solution) method which is a multiple criteria method to identify solution from finite set of points. The basic principle is that the chosen points should have the "shortest" distance from the positive ideal and the "farthest" distance from the negative ideal solution.
In their TOPSIS model ,the measurement of weights and qualitative attributes did not consider the uncertainty associated with the mapping of human perception to a number[4]. The concept of applying fuzzy numbers to TOPSIS was first suggested by Negi and Chenand Hwang[6] ,but their fuzzy TOPSIS algorithms are incomplete. The main steps of multiple criteria-attribute (complex) decision-making are as following:

a) Establishing system evaluation criteria that relate system capabilities to goals.

b) Developing alternative systems for attaining the goals. (generating alternative)

c) Evaluating alternative in terms of criteria.(the values of the criterion functions)

d) Applying a normative multi-criteria analysis method.

e) Accepting one point as "optimal".

f) If the final solution is not accepted, gather new information and go into the next iteration of multi-criteria optimization.

Steps (a) and (e) are performed at the upper level, where decision makers have the central role, and the other are mostly engineering tasks. For steps d and a decision maker should express her/his idea about importance of criteria to determining weights of criteria. These weights do not have clear economic significance, but they match model with actual concepts of decision making.
By considering this fact that in many cases determining precisely the exact value of the attribute respect to criteria is difficult, their valuesare considered as fuzzy data. Therefore the concept of TOPSIS is extended to solving problems under uncertainty.

At the rest of this paper, in section 2, briefly introducing of fuzzy numbers is brought, in section 3, extended TOPSIS methodology is presented and finally, in section 4, a numerical example is solved.

2 Preliminaries

An arbitrary fuzzy number is represented by an ordered pair of functions $(U^l(r), U^u(r))$, $0 \leq r \leq 1$, which satisfy the following requirement:

1. $U^u(r)$ is a bounded left continuous non increasing function over [0,1].
2. $U^l(r)$ is a bounded left continuous non increasing function over [0,1].
3. $U^l(r) \leq U^u(r)$, $(0 \leq r \leq 1)$.

A crisp number α is simply represented by $U^l(r) = U^u(r) = \alpha$, $0 \leq r \leq 1$. By appropriate definitions the fuzzy numbers space $\{U^l(r), U^u(r)\}$ becomes a convex cone E which is then embedded isomorphically and isometrically in to a Banach space.
Let $\mathbb{D}^n$ denote the set of upper semi continues (U.S.C) normal fuzzy sets on $\mathbb{R}^n$ with compact support. That is, $U \in \mathbb{D}^n$, then $U : \mathbb{R}^n \to [0, 1]$ is U.S.C, $Supp(U) = \overline{\{x \in \mathbb{R}^n : \ U(x) > 0\}}$ is compact and there exists at least one $\varepsilon \in Supp(U)$ for which $U(\varepsilon) = 1$

2.1 Fuzzy number operations

1. $\tilde{x} = \tilde{y}$ if and only if $x^l(r) = y^l(r)$ and $x^u(r) = y^u(r)$ $(0 \leq r \leq 1)$.
2. $\tilde{x} + \tilde{y} = (x^l(r) + y^l(r), x^u(r) + y^u(r))$
3. $K\tilde{x} = \begin{cases} (kx^l(r), kx^u(r)) & k \geq 0 \\ (kx^u(r), kx^l(r)) & k \leq 0 \end{cases}$

Definition 2.1. *The distance of two fuzzy numbers $\tilde{x}$ and $\tilde{y}$ is defined as* $D(\tilde{x}, \tilde{y}) = (\int_0^1 (|x^l(r) - y^l(r)|^2 + |x^u(r) - y^u(r)|^2) dr)^{1/2}$

Definition 2.2. *The distance of two fuzzy number vector $U = (\tilde{u_1}, \cdots, \tilde{u_n})$ where $\tilde{u}_i = (u_i^l(r), u_i^u(r))$, $1 \leq i \leq n$, $0 \leq r \leq 1$ and $Z = (\tilde{z}_1, \cdots, \tilde{z}_n)$ where $\tilde{z}_i = (z_i^l(r), z_i^u(r))$, $1 \leq i \leq n$, $0 \leq r \leq 1$ is defined as follows:*

$$D(Z, U) = \left(\int_0^1 \sum_{i=1}^{n} ([u_i^l(r) - z_i^l(r)]^2 + [u_i^u(r) - z_i^u(r)]^2) dr \right)^{1/2}$$

and the metric $D(U,0)$ is defined as

$$D(U,0)=\left(\int_0^1\sum_{i=1}^{n}[(u_i^l(r))^2+(u_i^u(r))^2]dr\right)^{1/2}$$

3 TOPSIS method for decision making under uncertainly

Suppose $A_1,A_2,\cdots,A_m$ are m possible points which decision makers have to choose, $C_1,C_2,\cdots,C_n$ are criteria, $\tilde{x}_{ij}$ is the rating of point A_i with respect to criterion C_j and is not known exactly and only we know $\tilde{x_{ij}}=(x_{ij}^l(r),x_{ij}^u(r))$, $i=1,2,\cdots,m,\ j=1,2,...,n$ and $0\le r\le 1$.

This problem with fuzzy data can be expressed in matrix format as

	C_1	C_2	$\cdots$	C_n
A_1	$(x_{11}^l(r),x_{11}^u(r))$	$(x_{12}^l(r),x_{12}^u(r))$	$\cdots$	$(x_{1n}^l(r),x_{1n}^u(r))$
A_2	$(x_{21}^l(r),x_{21}^u(r))$	$(x_{22}^l(r),x_{22}^u(r))$	$\cdots$	$(x_{2n}^l(r),x_{2n}^u(r))$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
A_m	$(x_{m1}^l(r),x_{m1}^u(r))$	$(x_{m2}^l(r),x_{m2}^u(r))$	$\cdots$	$(x_{mn}^l(r),x_{mn}^u(r))$

where w_j is the weight of criterion C_j.

3.1 the proposed algorithmic method

A systematic approach to extend the TOPSIS to use fuzzy data is proposed in this section.

The normalized values $\bar{n}_{ij}^l(r)$and $\bar{n}_{ij}^u(r)$ are calculated as

$$\bar{n}_{ij}^l(r)=x_{ij}^l(r)\Big/(\int_0^1\sum_{i=1}^{m}[(x_{ij}^l(r))^2+(x_{ij}^u(r))^2]dr)^{1/2}$$
$$\bar{n}_{ij}^u(r)=x_{ij}^u(r)\Big/(\int_0^1\sum_{i=1}^{m}[(x_{ij}^l(r))^2+(x_{ij}^u(r))^2]dr)^{1/2}\ ;$$

where $0\le r\le 1,\ i=1,\cdots,m,\ j=1,\cdots,n$. The normalization method mentioned above is to preserve the property that ranges of normalized fuzzy numbers belongs to [0,1].

The weighted normalized fuzzy numbers decision matrix to notify the different importance of each criterion is constructed as

$$\bar{V}_{ij}^l(r)=W_j\bar{n}_{ij}^l(r),i=1,...,m,j=1,...,n,0\le r\le 1$$

$$\bar{V}_{ij}^u(r)=W_j\bar{n}_{ij}^u(r),i=1,...,m,j=1,...,n,0\le r\le 1$$

Where W_j is the weighted of the j-th attribute or criterion and $\sum W_j = 1$. Then positive ideal solution and negative solution are defined as

$$\bar{A}^+ = (\bar{V}_1^+, \cdots, \bar{V}_n^+) = \left((\max_i \bar{V}_{ij}^u(0)|j \in I), (\min_i \bar{V}_{ij}^l(0)|j \in J)\right)$$
$$\bar{A}^- = (\bar{V}_1^-, \cdots, \bar{V}_n^-) = \left((\min_i \bar{V}_{ij}^l(0)|j \in I), (\max_i \bar{V}_{ij}^u(0)|j \in J)\right);$$

since if $r = 0$ then the fuzziness value is more than other cases.
Where I and J are associated respectively with benefit and cost criteria. After positive and negative ideal solution are considered as two fuzzy vector with fuzzy number as follows:

$$\tilde{A}^+ = (\bar{A}^{+^l}(r), \bar{A}^{+^u}(r)), \quad 0 \leq r \leq 1$$

where

$$\bar{A}^{+^l}(r) = (\bar{V}_1^{+^l}(r), \bar{V}_2^{+^l}(r), \cdots, \bar{V}_n^{+^l}(r)), \quad 0 \leq r \leq 1$$

which

$$\bar{V}_i^{+^l}(r) = \varepsilon(r-1) + \bar{V}_i^+, \quad i = 1, \cdots, n;$$

and

$$\bar{A}^{+^u}(r) = (\bar{V}_1^{+^u}(r), \bar{V}_2^{+^u}(r), \cdots, \bar{V}_n^{+^u}(r)), \quad 0 \leq r \leq 1$$

which

$$\bar{V}_i^{+^u}(r) = \varepsilon(1-r) + \bar{V}_i^+, \quad i = 1, \cdots, n;$$

$\tilde{A}^-$ is defined in a similar way.

The separation of each point from positive and negative ideal solution calculated by using the n-th dimensional Euclidean distance as

$$\bar{d}_i^{\,+} = \left(\int_0^1 (\sum_{j\in I}[\bar{v}_{ij}^l(r) - \bar{V}_j^{+^l}(r)]^2 + \sum_{j\in J}[\bar{v}_{ij}^u(r) - \bar{V}_j^{+^u}(r)]^2) dr\right)^{1/2}$$
$$\bar{d}_i^{\,-} = \left(\int_0^1 (\sum_{j\in I}[\bar{v}_{ij}^u(r) - \bar{V}_j^{-^u}(r)]^2 + \sum_{j\in J}[\bar{v}_{ij}^l(r) - \bar{V}_j^{-^l}(r)]^2) dr\right)^{1/2}$$

where $i = 1, \cdots, m$.

A relative closeness coefficient ($\bar{R}_i$) is defined to determine the ranking order of all points. To determining the relative closeness coefficient $\bar{d}_i^+$ and $\bar{d}_i^-$ of each point A_j are calculated, then

$$\bar{R}_i = \bar{d}_i^- / (\bar{d}_i^+ + \bar{d}_i^-); \quad i = 1, \cdots, m$$

4 Numerical example

A hypothetical complex decision-making problem is applied to demonstrate the computational process of the proposed fuzzy TOPSIS model.
Assume that a compony needs to select a consumer as the best one.After an

initial screening ,four alternative are chosen for further evaluation .
Consider $V_{ij}(0)$ as below :

$$
\begin{array}{l}
V_{11} = (0.4433,\ 0.7500,\ 0.7500,\ 0.9344,\ 0.0400,\ 0.2667,\ 0.0089,\ 0.1933) \\
V_{12} = (0.1100,\ 0.2833,\ 0.2833,\ 0.5367,\ 0.0400,\ 0.1333,\ 0.0400,\ 0.2933) \\
V_{13} = (0.2078,\ 0.4300,\ 0.4344,\ 0.7156,\ 0.0400,\ 0.1867,\ 0.0333,\ 0.3144) \\
V_{14} = (0.5289,\ 0.6800,\ 0.6800,\ 0.7304,\ 0.0000,\ 0.1511,\ 0.0000,\ 0.0500) \\
V_{15} = (0.2333,\ 0.4333,\ 0.4333,\ 0.6333,\ 0.0000,\ 0.2000,\ 0.0000,\ 0.2000) \\
V_{21} = (0.4433,\ 0.7500,\ 0.7500,\ 0.9344,\ 0.0400,\ 0.2667,\ 0.0089,\ 0.1933) \\
V_{22} = (0.1344,\ 0.3211,\ 0.3211,\ 0.5878,\ 0.0400,\ 0.1467,\ 0.0400,\ 0.3067) \\
V_{23} = (0.1833,\ 0.3967,\ 0.3967,\ 0.6644,\ 0.0400,\ 0.1733,\ 0.0333,\ 0.3011) \\
V_{24} = (0.6026,\ 0.7747,\ 0.7747,\ 0.8321,\ 0.0000,\ 0.1722,\ 0.0000,\ 0.0574) \\
V_{25} = (0.1795,\ 0.3333,\ 0.3333,\ 0.4872,\ 0.0000,\ 0.1538,\ 0.0000,\ 0.1538) \\
V_{31} = (0.1633,\ 0.3900,\ 0.3900,\ 0.6122,\ 0.0400,\ 0.1867,\ 0.0133,\ 0.2356) \\
V_{32} = (0.1344,\ 0.3211,\ 0.3211,\ 0.5878,\ 0.0400,\ 0.1467,\ 0.0400,\ 0.3067) \\
V_{33} = (0.1100,\ 0.2833,\ 0.2833,\ 0.5367,\ 0.0400,\ 0.1333,\ 0.0400,\ 0.2933) \\
V_{34} = (0.5471,\ 0.7034,\ 0.7034,\ 0.7555,\ 0.0000,\ 0.1563,\ 0.0000,\ 0.0521) \\
V_{35} = (0.1496,\ 0.2778,\ 0.2778,\ 0.4060,\ 0.0000,\ 0.1282,\ 0.0000,\ 0.1282) \\
V_{41} = (0.3967,\ 0.6900,\ 0.6900,\ 0.9022,\ 0.0400,\ 0.2533,\ 0.0111,\ 0.2233) \\
V_{42} = (0.2078,\ 0.4344,\ 0.4344,\ 0.7156,\ 0.0400,\ 0.1867,\ 0.0333,\ 0.3144) \\
V_{43} = (0.1833,\ 0.3967,\ 0.3967,\ 0.6644,\ 0.0400,\ 0.1733,\ 0.0333,\ 0.3011) \\
V_{44} = (0.7000,\ 0.9000,\ 0.9000,\ 0.9667,\ 0.0000,\ 0.2000,\ 0.0000,\ 0.0667) \\
V_{45} = (0.2094,\ 0.3889,\ 0.3889,\ 0.5684,\ 0.0000,\ 0.1795,\ 0.0000,\ 0.1795)
\end{array}
$$

by Equations the ideal fuzzy solution and negative ideal can be obtained and the closeness coefficients is :

A_1	0.5890	1.1417	0.6597
A_2	0.6984	1.1309	0.6182
A_3	1.6119	0.1464	0.0833
A_4	0.2578	1.5473	0.8572

the closeness in above table make it obvious that the ranking order for the four alternative is A_4, A_1, A_2 and A_3.Therefore, the decision- makers can recommend A_4 as the best costumer.

5 Conclusion

In this paper, a new method when the availability of information for decision environment is not exact is presented.
Under such situation The Fuzzy TOPSIS method by considering the estimate

data as fuzzy data help decision-makers to rank their choices. The interpretation of some factors as weight as economical component is not clear in general but in the numerical example interpretation are presented just for similar situation.

References

[1] S.J.Chen,C.L.Hwang ,1992,Fuzzy Multiple Attribute Decision Making:Methodes and Applications.

[2] TA-ChangChu,2001,fuzzy TOPSIS under group decision,Internation jornal of uncertainty,10,pp.687-701

[3] P.AnandRaj,D.Nagesh Kumard, 1999 , Ranking alternatives with weights using maximizing set and minimizinf set, Fuzzy Sets and Systems ,105,pp.365-375

[4] Makridakis S, Wheelwright SC, McGee VE. Forecasting: methods and applications. New York: Wiley, 1983.

[5] Yeh CH, Willis RJ, Deng H, Pan H. Task oriented weighting in multi-criteria analysis. European Journal of Operational Research 1999;119/1:in press.

[6] Hwang CL, Yoon KS. Multiple attribute decision making: methods and applications. New York: Springer, 1981.

[7] Zeleny M. Multiple criteria decision making. New York: McGraw-Hill, 1982.

[8] Fischer GW. Range sensitivity of attribute weights in multiattribute value model. Organizational Behavior and Human Decision Processes (1995;62:252)66.

[9] Triantaphyllou E, Sanchez A. A sensitivity analysis approach for some deterministic multi-criteria decision making methods. Decision Sciences (1997;28/1:151)94.

[10] Charnes A, Cooper WW, Rhodes E. Measuring the efciency of decision making units. European Journal Operational Research (1978;2:429)44.

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