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What Helicity Can Tell Us about Solar Magnetic Fields

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Abstract. Concept of magnetic/current helicity was introduced to solar physics about 15 years ago. Earlier studies led to discovery of such fundamental properties as hemispheric helicity rule, and role of helicity in magnetic reconnection and solar eruptions. Later, the concept was successfully applied in studies of different solar processes from solar dynamo to flare and CME phenomena. Although no silver bullet, helicity has proven to be a very useful "tool" in answering many still-puzzling questions about origin and evolution of solar magnetic fields. I present an overview of some helicity studies and briefly analyze their findings.

Key words. Sun—magnetic fields—helicity.

"If you are after good publicity,
you should not speak about current helicity."

From a poem written by Jan Stenflo

1. Knots and bolts

The concept of helicity has its origin in the knot theory. Since its development, it has been successfully used in different fields of mathematics, plasma physics, and more recently, in solar physics and astrophysics.

To demonstrate the usefulness of helicity concept in solving topological problems, let us consider a seemingly complex object – a tangled knot (Fig. 1a). Is this knot topologically similar to a simple O-ring? One approach would be to untangle the knot and demonstrate that indeed it can be transformed to a ring. This direct approach, however, might not be always practical especially if one has no physical access to an object. Alternatively, one can calculate helicity of the knot; if helicity $H = 0$, then the knot is topologically the same object as O-ring. For the purpose of this example, helicity can be calculated by the means of "helicity meter" (Fig. 1, right panel). In this exercise, a tread forming the knot should be followed in one (arbitrary selected) direction and every crossing should be counted as $+1$ or -1. When applying helicity meter, the top arrow should always be aligned with thread in the direction it is followed. Orientation of lower arrow will determine sign of helicity: positive $+1$ or negative -1. In the course of this exercise, one can find that the knot on Fig. 1 has equal number of positive and negative crossings, and hence, zero helicity, similar to ring.

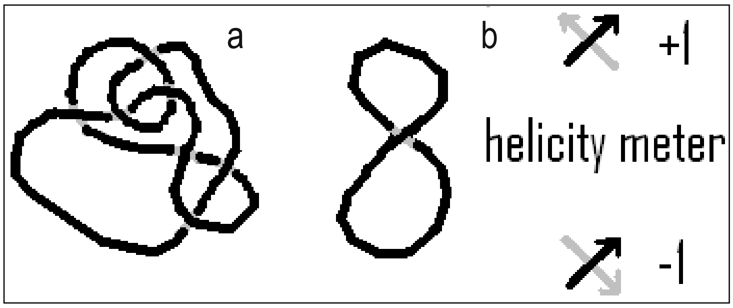

Figure 1. Knot **(a)** and figure-8, **(b)** as two examples of topological objects, which can be transformed to a simple O-ring. Right panel shows helicity meter that can be used to determine helicity of two objects shown to the left. Numbers next to helicity meter show corresponding helicity.

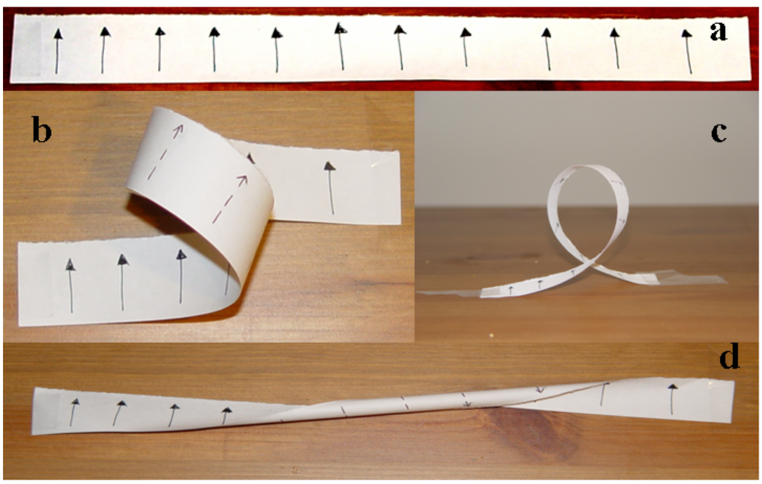

Figure 2. A ribbon **(a)** demonstration of two components of helicity: writhe (**b** and **c**) and twist **(d)**.

However, counting crossings to determine helicity may be misleading. For example, folding O-ring on itself into figure-8, will not change helicity. However, helicity meter applied to Fig. 1(b) will yield, negative helicity, $H = -1$, contrary to the fact that figure-8 can easily be converted to O-ring. This is because helicity may have several components, while "helicity meter" can only be used to find one component.

Figure 2 demonstrates two components of helicity: twist, T_w and writhe, W_r. Wrapping a straight paper ribbon (Fig. 2a) around itself creates writhe (Fig. 2b and c), while pulling two ends of ribbon apart, transforms writhe to internal twist (Fig. 2d). Total helicity, H, of this ribbon, shown in Fig. 2(b–d) is $H = T_w + W_r$. Figure 2(c) corresponds to $T_w = 0$, $H = W_r = -1$ (using helicity meter). Figure 2(d) corresponds to $W_r = 0$, $H = T_w = -1$, and hence, left-hand twist corresponds to negative helicity.

Mathematically, helicity is defined as a dot product of a vector and its curl, integrated over a closed volume. Thus, for example, magnetic helicity, $H_m = \int \mathbf{A} \cdot \nabla \times \mathbf{A} dD$, where D is closed volume, and $\mathbf{A}$ is vector potential of magnetic induction $\mathbf{B} = \nabla \times \mathbf{A}$. Similarly, one can define current helicity, $H_c = \int \mathbf{B} \cdot \nabla \times \mathbf{B} dD$, and kinetic helicity, $H_k = \int \mathbf{V} \cdot \nabla \times \mathbf{V} dD$, where $\mathbf{V}$ is velocity of flows. Assuming that magnetic field is represented by a thin flux tube model with flux Φ, one can show that magnetic helicity, $H_m = (2\pi)^{-1}\Phi^2(T_w + W_r)$ (Longcope and Klapper 1997), similar to our pictorial example on Fig. 2.

In a restrictive case of linear force-free field, $\nabla \times \mathbf{B} = \alpha \mathbf{B}$, $\alpha = \text{const}$, magnetic vector potential $\mathbf{A}$ can be expressed as $\mathbf{A} = \alpha^{-1}\mathbf{B} + \nabla\psi$, where ψ is an arbitrary scalar function. Then, magnetic helicity can be written as

$$H_m = \int \mathbf{A} \cdot \mathbf{B} = \int (\alpha^{-1}\mathbf{B} + \nabla\psi)\mathbf{B} dD = \int \alpha^{-1} B^2 dD + \int \nabla\psi B dD. \quad (1)$$

For a closed volume, magnetic field does not cross the volume boundary, which results in additional condition $\mathbf{n} \cdot \mathbf{B} = 0$. Under this condition, the second integral goes to zero, and magnetic helicity can be expressed in terms of α-coefficient, and magnetic energy, E_m:

$$H_m = 2\mu\alpha^{-1} E_m, \quad (2)$$

where μ is magnetic permeability in vacuum, and $\alpha = \text{const}$.

Magnetic helicity has several properties making it an important parameter to study. Due to inverse cascading, magnetic helicity conserves better than magnetic energy, and it plays an important role in (or contains important information about) such processes as magnetic reconnection, dynamo, and stability of magnetic fields (see individual articles in Brown *et al.* 1999; Büchner and Pevtsov 2003).

2. Observations of helicity

To determine H_m requires the knowledge of magnetic field and its vector potential throughout the closed volume encompassing this magnetic field. On the Sun, however, magnetic field is only measured in a few levels (typically one or two) currently restricted to the photosphere and low chromosphere. The detailed distribution of subphotospheric magnetic field is unknown. Thus, to derive even limited information about helicity requires additional assumptions.

In the framework of force free field model, α coefficient has the same sign as magnetic helicity (see, equation 2). This sign correlation encouraged use of α as helicity proxy (e.g., Pevtsov *et al.* 1995). Using vector magnetic observations in a single level in solar atmosphere, one can determine α either by fitting linear force free field to the observed (transverse) field (so called, α_{best}), or by computing vertical, z, component of α for each pixel in magnetogram and averaging it ($\langle\alpha_z\rangle$). Burnette *et al.* (2004) found a reasonably good correlation between α_{best} and $\langle\alpha_z\rangle$. Alternatively, one can also compute z-component of current helicity density, $h_c = (B \cdot J)_z = \mu B_z \cdot J_z$, where J is electric current density. Although current helicity is not a conserved quantity (unlike magnetic helicity), it has the same sign as H_m. Two different helicity proxies based on current helicity were used: a fractional imbalance of h_c (percentage of pixels of one sign of h_c in a given magnetogram) and area-averaged h_c $\langle h_c\rangle$ (e.g., Abramenko *et al.* 1996; Bao and Zhang 1998).

It has also been shown that sign of helicity may be inferred from the morphological patterns observed in various solar phenomena. For example, spiral pattern of filaments forming sunspot (super-) penumbra may be interpreted as left-hand (LH) or right-hand (RH) twist. Orientation of barbs in chromospheric filaments indicates two distinct types of filaments: dextral and sinistal. Chirality of filaments is thought to be associated with the handedness of their magnetic fields. Shape of sigmoidal coronal loops can be directly related to $\alpha > 0$ (S-shape) or $\alpha < 0$ (N-shape) magnetic field. A relationship between these patterns and magnetic field helicity proxy α has been established at least in a framework of linear force free field model (see, individual articles in Brown *et al.* 1999).

One can also compute change of helicity relative to a reference, e.g., potential field, so called relative helicity (e.g., Chae 2001 and references therein).

3. Hemispheric helicity rule

All helicity proxies discussed reveal a common tendency that became known as the hemispheric helicity rule. Solar magnetic fields in the Northern Hemisphere have negative sign of helicity, while the fields in the Southern Hemisphere possess positive helicity. The hemispheric helicity rule was found in the magnetic fields at different spatial scales, from network magnetic fields, sunspots, large-scale fields, and magnetic clouds, from the photosphere, through the chromosphere, corona, and solar wind (Brown *et al.* 1999). Typically, about 70–80% of solar features (e.g., active regions) follow this hemispheric dependency. A pictorial summary of the hemispheric helicity rule can be found in Pevtsov and Balasubramaniam (2003).

Several mechanisms were considered as potential origin of this rule including solar differential rotation, direct action of the Coriolis force, solar dynamo, and turbulent convection in upper portion of the convection zone (Σ-effect). A summary of these mechanisms can be found in Longcope *et al.* (1999) and Longcope & Pevtsov (2003). Similar to Joy's law that describes active region tilt relative to equator, the hemispheric helicity rule shows significant scatter, suggesting that turbulence in the convection zone may have some effect. It now appears that interaction between magnetic field and turbulent convection in upper portion of the convection zone (so called Σ-effect) is the leading mechanism behind the hemispheric helicity rule. The mechanism was originally proposed by Longcope *et al.* (1998), and more recent studies found additional support to it (e.g., Nandy 2006).

Both historic and more recent studies suggest that the hemispheric helicity rule does not change from one solar cycle to the other. However, some researchers suggested that the rule might change its sign in some periods of solar cycle. Thus, for example, Bao *et al.* (2000) found reverse sign of the rule for h_c at the beginning of Cycle 23. On the other hand, $\alpha_{\rm best}$ helicity proxy showed no change in the hemispheric helicity rule for the same period (Pevtsov *et al.* 2001). Hagino & Sakurai (2002) found indication that some periods of solar activity cycle disobey the helicity rule. Choudhuri *et al.* (2004) suggested a theoretical model that predicts deviations from the hemispheric helicity rule at the beginning of each solar cycle. On the other hand, Pevtsov *et al.* (2003a) pointed out lack of consistency between different magnetographs in respect to years when the hemispheric rule reverses its sign. They suggested that due to significant scatter in the data, years with low sunspot activity (e.g., beginning of solar cycle) may show a deviation from the rule simply because of insufficient statistical sample. One should also mention a recent study by Zhang (2006) suggesting that helicity of umbral and penumbral fields may have opposite sign, and different solar cycle behavior. Thus, the question of cycle dependence of the hemispheric helicity rule remains open (see, Pevtsov *et al.* 2008).

4. Helicity transport

Whatever the mechanism that generates helicity on the Sun, it operates continuously. Thus, helicity should be removed from the Sun at approximately the same rate as it is created. Accumulation of helicity would make it difficult for solar dynamo to operate – a problem referred to as dynamo quenching (e.g., Brandenburg and Sandin 2004). The prime way for the Sun to shed helicity is via coronal mass ejections (CMEs), although some helicity may also be dissipated "on site" by reconnecting different flux systems that have opposite signs of helicity. These opposite helicity reconnection events are likely to release significant amounts of energy and play role in (some) solar flares (e.g., Kusano *et al.* 2004).

Exactly, how much helicity is removed by CMEs and is there sufficient helicity in solar magnetic fields to account for that amount?

Lepping *et al.* (1990) fitted 18 magnetic clouds (MCs) to a linear force-free field (LFFF) flux tube model (typical $\alpha \approx 10^{-10}\,\mathrm{m}^{-1}$, $B_0 = 2 \times 10^{-4}$ Gauss). In the framework of LFFF model, helicity of MC can be expressed as $H_{MC} = \alpha^{-1} L \Phi^2$, where L is total length of flux tube and Φ is its magnetic flux. The exact length of MC flux tube can be estimated to be between twice the distance to the Sun and an arch with its two footpoint anchored at the Sun. With these uncertainties, H_{MC} was found to be about 5×10^{42} Mx^2. Demoulin *et al.* (2002) studied a long-term evolution of relative helicity in the active region NOAA AR7978. The region produced 26 CMEs over the period of one solar rotation, shedding $\approx 5.2 \times 10^{43}$ Mx^2 of magnetic helicity. However, magnetic field of the region persisted over five solar rotations continuing to produce CMEs and MCs. It was estimated that the total helicity ejected by MCs often exceeded coronal helicity of this source active region; and that differential rotation could not effectively replenish helicity lost to an MC.

Total magnetic helicity of active regions can be estimated using equation (2). For linear force-free field magnetic energy can be derived using Virial theorem on the basis of a single magnetogram:

$$E_m = \mu^{-1} \int (x B_x + y B_y) B_z dx dy, \tag{3}$$

where x, y are Cartesian coordinates, and $B_{x,y,z}$ denote two horizontal and the vertical component of magnetic field. This approach was tried on about 160 active regions observed in a chromospheric spectral line using National Solar Observatory at Kitt Peak full disk longitudinal magnetograph (Jones *et al.* 1992). Magnetic field of selected active regions was fitted by a set of LFFF models, and the resulting field lines were fitted to coronal images observed by EIT/SOHO in 195Å (Updike & Pevtsov 2002). Resulting distribution of helicity of active regions is shown in Fig. 3. Mean helicity of active regions' magnetic field is about 1.7×10^{43} Mx^2, about three times larger than a typical helicity of a magnetic cloud.

Thus, it is clear that magnetic field of active regions has sufficient amount of helicity to support continuous helicity removal by CMEs. On the other hand, several studies

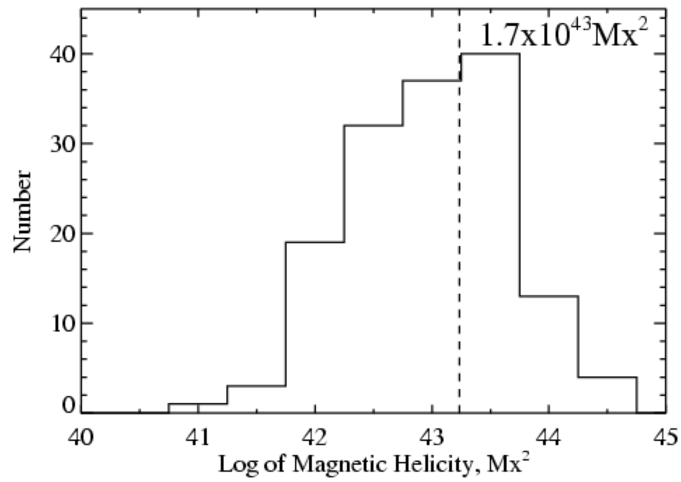

Figure 3. Distribution of magnetic helicity of active regions computed under the assumption of linear force-free field. Vertical dashed line shows mean of the distribution.

show that observed surface motions are insufficient to produce the required amount of helicity. The same studies suggest that helicity observed in the photosphere and corona originates below the surface. This raises that question, how helicity generated by subphotospheric processes is transported to the corona?

Longcope and Welsch (2000) considered evolution of twisted flux tubes as they rise through the photosphere to the corona. As flux tube crosses into the corona, it expands. This expansion disrupts torque balance between narrow and wide portions of the tube, and hence, some twist should be redistributed to the expanded portion of the tube. These authors predicted a particular behavior of twist depending on the rate the flux tube emerges. Pevtsov *et al.* (2003b) studied evolution of helicity proxy α in several emerging active regions, and found it to be consistent with Longcope and Welsch (2000) predictions. Fitting the model to the observations yields that subphotospheric twist will propagate from about 10 Mm to the corona in about one day. This suggests a possible mechanism of helicity transport from below the surface to the corona and (via CMEs) to the interplanetary space. When CME removes helicity from the corona above active region, helicity from the subphotospheric part of active region will replenish corona, and within a day the region will be able to produce another CME. Helicity will be transported by torsional waves, which may have very little observational signature in transverse displacements in the photosphere, which might explain the lack of significant horizontal motions that could be responsible/related to helicity buildup. Chae *et al.* (2003) considered a somewhat similar model of helicity pumping from narrow to wide portion of an expanded flux tube.

5. Some conclusions

Significant developments in the field of helicity makes it impossible to provide a comprehensive review of all aspects of helicity studies in solar physics in one short article. From the topics that we had visited, however, one can make the following conclusions:

- Magnetic field on the Sun exhibits the hemispheric helicity rule. Deviations from the rule were observed to occur in some (early) years of solar cycle, but lack of consistency between the instruments calls for additional studies to confirm if the rule may vary with the solar activity cycle.
- Helicity of strong (active region) magnetic fields observed in the photosphere is probably generated in mid-upper convection zone. Pevtsov & Longcope (2001) observations also suggest that helicity of weak (network) field may originate in the convection zone; surface dynamo only recycles fields generated below the surface. These observational results need to be verified using high resolution magnetograms from Hinode.
- It is suggested that helicity is constantly transported to the corona via torsional motions. In this scenario, magnetic field below the surface may serve as an “untapped” reservoir of helicity, which can replenish corona within a day after a major CME eruption removes helicity from the Sun.

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