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Modified EDF Goodness of Fit Tests for Logistic Distribution Under SRS and ERSS

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Abstract

The Extreme Ranked Set Sampling (ERSS), which was introduced by Samawi et al. (1996), is a modification of the Ranked Set Sampling (RSS) of McIntyre (1952). In this paper, we study the power of a set of modified Empirical Distribution Function (EDF) goodness of fit tests under both Simple Random Sampling (SRS) and ERSS. A simulation study is conducted to compare the power functions of these tests to SRS counterparts.

Keywords: Goodness of fit tests; Empirical distribution function; Power function; Logistic distribution; Ranked set sampling; Kolmogorov-Smirnov test.

1. INTRODUCTION

The Ranked Set Sampling (RSS) is a sampling technique which is very useful in various situations where visual ordering of sample units, with respect to the variable of interest, is less expensive in its quantification. McIntyre (1952) was the first to employ the visual ranking of sampling units to estimate the mean pasture and forage yields. Without any theoretical developments, he showed that the RSS is more efficient and cost-effective method than the Simple Random Sampling (SRS) technique. In general, the RSS produces a sample which is more informative than a simple random sample when estimating the mean of the population of interest. The RSS technique can be described as follows.

1. Select m random samples from the population of interest each of size m .
2. Detect from the i^{th} sample, using a visual inspection, the i^{th} order statistic and choose it for actual quantification, say, Y_i , $i = 1, ..., m$.
3. RSS is the set of the order statistics $Y_1, ..., Y_m$.
4. The steps (1)-(3) could be repeated r times to get more observations.

Takahasi and Wakimoto (1968) gave the theoretical setups for RSS. They showed that the mean of an RSS is the minimum variance unbiased estimator for the population mean. Dell and Clutter (1972) showed that the sample mean of an RSS remains unbiased and more efficient than the sample mean of an SRS even if ranking is imperfect. Since visual ranking has to be done with the experimenter's experience, then two factors may affect the efficiency of an RSS; the set size and ranking errors. In general, the larger the set size, the larger the efficiency of an RSS, while the larger the set size the more the difficulty in visual ranking and hence the larger the ranking error (Al-Saleh and Al-Omari, 2002). For this, several authors have modified the MacIntyre's RSS scheme to produce other sampling schemes which are more amenable to visual ranking than the RSS. Samawi et al. (1996) investigated Extreme Ranked Set Sample (ERSS), i.e. they quantified the smallest and the largest order statistics instead of detailed ranking. Muttlak (1997) introduced Median Ranked Set Sampling (MRSS) which consists of quantifying only the median in each set. Bhoj (1997) proposed a modification to the RSS and called it new ranked set sampling (NRSS). Al-Odat and Al-Saleh (2001) introduced the concept of varied set size RSS, which is called later Moving Extremes Ranked Set Sampling (MERSS).

It was very attractive for the statisticians to redevelop (when it is possible) the well-known statistical techniques under the RSS and its modifications. For more details about these developments (see Chen, 2000). In RSS literatures, we have noted that the researchers have not paid enough attention to goodness of fit tests when the sample is collected via the RSS techniques. Stockes and Sager (1988) studied the characteristics of an RSS. They also gave an unbiased estimator for the population distribution function based on the empirical distribution function of an RSS. Then, they proposed a Kolmogorov-Smirnov goodness of fit test based on the empirical distribution function of an RSS. Moreover, they derived the null distribution of their proposed test. Al-Subh et al. (2008a) have improved the power of the chi-square test for goodness of fit under RSS. They also conducted a simulation study for the power of chi-square test under SRS and RSS techniques. Al-Subh et al. (2008b) gave a comparison study for the power of a set of empirical distribution function goodness of fit tests for the logistic distribution under SRS and RSS.

In this paper, we propose a method to improve the power of empirical distribution function goodness of fit tests for logistic distribution under ERSS. We also conduct a simulation study to compare the power of each test under the ERSS and its SRS counterparts. The paper is organized as follows. In Section 2, we present a set of modified empirical distribution function (MEDF) goodness of fit tests in SRS. We also propose its RSS counterparts. We apply these test statistics for the logistic distribution in Section 3. In Section 4, we define two algorithms to calculate the percentage points and the power function at an alternative distribution. In Section 5, a simulation study is conducted to study the efficiency of these test statistics under RSS with its SRS counterpart. In Section 6, we state our conclusions.

2. MEDF GOODNESS OF FIT TESTS

Stephens (1974) gave a practical guide to goodness of fit tests using statistics based on the empirical distribution function (EDF). Green and Hegazy (1976) studied modified forms of the Kolmogorov-Smirnov D, Cramer-von Mises W^2 and Anderson-Darling A^2 goodness of fit tests. Stephens (1979) gave goodness of fit tests for logistic distribution based on an SRS. A comprehensive survey for goodness of fit tests based on SRS can be found in Stephens (1986).

In a goodness of fit test problem, the objective is to test a hypotheses:

$H_o : F(x) = F_o(x) \quad \forall x$, vs. $H_1 : F(x) \neq F_o(x)$ for some x,

where $F_o(x)$ is a known distribution function based on $X_1, X_2, ..., X_r$, a random sample from the distribution function $F(x)$. Under SRS, the following set of MEDF goodness of fit tests could be achieved to complete the mission:

a) Tests related to Kolmogorov statistic: D

$$D_1 = \max_{1 \le i \le r} \left| z_i - (i/r) \right|,$$

where $z_i = (x_i - \theta)/\sigma$, $i = 1, 2, ..., r$ and r is the sample size.

$$D_{11} = \max_{1 \le i \le r} \left| z_i - (i/(r+1)) \right|,$$

$$D_2 = \sum_{i=1}^{r} \left| z_i - (i/r) \right|,$$

$$D_{22} = \sum_{i=1}^{r} \left| z_i - ((i+0.5)/(r+1)) \right|,$$

$$D_3 = \sup_{1 \le i \le r} \left| z_i - (i/r) \right|,$$

$$D_4 = \sum_{i=1}^{r} \max\{ \left| (i/r) - z_i \right|, \left| ((i-1)/r) - z_i \right| \},$$

b) Tests related to Cramer-von Mises statistic: W^2

$$W_o = \sum_{i=1}^{r} \left[z_i - (2i-1)/2r \right]^2,$$

$$W_{11} = \sum_{i=1}^{r} \left[z_i - i/(r+1) \right]^2,$$

$$W_{21} = \sum_{i=1}^{r} \left[z_i - (2i-1)/2(r+1) \right]^2,$$

c) Tests related to Anderson-Darling statistic: A^2

$$aa_{21} = -r - r/(r+1)^2 \{ \sum_{i=1}^{r} [(2i-1)\ln z_i + (2i+1)\ln(1 - z_{r-i+1})] - \left[(2r+1)\ln z_r - \ln(1 - z_r) \right] \},$$

$$aa_{22} = -r - \{2r/(r+1)^2\} [\left\{ \sum_{i=1}^{r} i \left[\ln z_i + \ln(1 - z_{r+1-i}) \right] \right\} - \{r/(r+1)^2\} [0.25\{\ln z_1 + \ln(1 - z_r)\} + (r+0.75)\{\ln z_r + \ln(1 - z_1)\}],$$

$$aa_{12} = -(r+1) - \{1/(r+1)\}\left\{\sum_{i=1}^{r} 2i\left[\ln z_i + \ln\left(1 - z_{r+1-i}\right)\right]\right\}. \qquad (1)$$

It can be noted that testing the hypotheses $H_o : F(x) = F_o(x), \quad \forall x$ is equivalent to testing the hypotheses $H_o^* : G_i(y) = G_{io}(y), \quad \forall y$ vs. $H_1^* : G_i(y) \neq G_{io}(y)$ for some i, where $G_i(y), \, G_{io}(y)$ are the cdf's of the i^{th} order statistics of random samples each of size $2m-1$, chosen from $F(x)$ and $F_o(x)$, respectively. The reason of choosing an odd set size, rather than an even one, is to simplify the comparison with the MRSS. Furthermore, an even set size produces two middle values where their quantification is more expensive than that in case of an odd set size. According to Arnold et al. (1992), $G_i(y)$ and $G_{io}(y)$ have the following representations:

$$G_i(y) = \sum_{j=i}^{2m-1} \binom{2m-1}{j} [F(y)]^j [1 - F(y)]^{(2m-1)-j}$$

and

$$G_{io}(y) = \sum_{j=i}^{2m-1} \binom{2m-1}{j} [F_o(y)]^j [1 - F_o(y)]^{(2m-1)-j},$$

respectively. For example, in case of $m = 2, \; i = 1, \; 2$ and 3, the cdf's $G_i(y)'s$ and $G_{io}(y)'s$ are given by

$G_1(y) \; = 1 - [1 - F(y)]^3,$

$G_{1o}(y) = 1 - [1 - F_o(y)]^3.$

$G_2(y) \; = 3F^2(y)(1 - F(y)] + F^3(y),$

$\quad\quad = 3F^2(y) - 2F^3(y),$

$G_{2o}(y) = 3F_o^2(y) - 2F_o^3(y).$

and

$G_3(y) = F^3(y),$

$G_{3o}(y) = F_o^3(y).$

It is easy to show that the equation $G_i(y) = G_{i0}(y)$ has the unique solution $F(x) = F_o(x).$

If we apply ranked set sampling to collect the data using the i^{th} order statistic, then we may use the resulting data to build empirical distribution function goodness of fit tests for the hypotheses H_o^* vs. H_1^*. Let $Y_1, ..., Y_r$ be a random sample of size r selected via the i^{th} order statistic. Let T denotes a test in (1) and

T^* denotes its counterpart in RSS when testing H_o^* vs. H_1^* using the data $Y_1,...,Y_r$.

Attention is restricted to the case when $F_o(x) = (1+e^{-(x-\theta)/\sigma})^{-1}$, i.e., for the logistic distribution. Moreover, a simulation study is conducted to compare the power of the test T^* with that of the test T based on samples with the same size. The power of the T^* test can be calculated according to the equation

Power of $T^*(H) = P_H(T^* > d_\alpha)$, (2)

where H is a cdf under the alternative hypothesis H_1^*. Here d_α is the 100α percentage point of the distribution of T^* and H_o. We calculate the efficiency of T^* relative to T as the ratio of powers:

$$eff\ (T^*,\ T) = \frac{\text{power of } T^*}{\text{power of } T},$$

Hence, T^* is more powerful than T if $eff\ (T^*,\ T) > 1$.

3. TEST FOR LOGISTIC DISTRIBUTION

Let $Y_1,...,Y_r$ be as in the introduction. To test the hypothesis

$$H_o : F(x) = F_o(x)\ \ \forall x, \quad \text{vs.} \quad H_1 : F(x) \neq F_o(x),$$

for some x, it is equivalent to test

$H_o^* : G_i(y) = G_{io}(y),\ \ \forall y \quad \text{vs.} \quad H_1^* : G_i(y) \neq G_{io}(y)$

for some i, where

$$F_0(x) = (1+e^{-(x-\theta)/\sigma})^{-1},$$

$$G_i(y) = \sum_{j=i}^{2m-1} \binom{2m-1}{j} [F(y)]^j [1-F(y)]^{(2m-1)-j},$$

and

$$G_{io}(y) = \sum_{j=i}^{2m-1} \binom{2m-1}{j} [F_o(y)]^j [1-F_o(y)]^{(2m-1)-j}.$$

If θ and σ are unknown, then we may estimate them using their maximum likelihood estimator from $l(\theta,\ \sigma)$, the likelihood function of the data, i.e.,

$$l(\theta,\ \sigma) = \prod_{i=1}^{r} \frac{(2m-1)!}{(i-1)!(2m-1-i)!} [F_o(y_i;\theta,\sigma)]^{i-1} [1-F_o(y_i;\theta,\sigma)]^{2m-1-i} f_o(y_i;\theta,\sigma),$$

where

$$f_o(y;\theta,\sigma) = \frac{e^{-(y-\theta)/\sigma}}{\sigma(1+e^{-(y-\theta)/\sigma})^2}, \quad -\infty<y<\infty.$$

We perform a goodness of fit test for the hypotheses

$$H_o^* : G_i(y) = G_{io}(y) \quad \text{vs.} \quad H_1^* : G_i(y) \neq G_{io}(y).$$

using the tests given in (1), but using the data $Y_1,...,Y_r$.

4. ALGORITHM FOR POWER COMPARISON

Without loss of generality, we assume $\theta = 0,\ \sigma = 1$. To compare the powers of T^* and T ; the following algorithm is designed to calculate the percentage points:

1. Let $Y_1,...,Y_r$ be a random sample from $G_{i0}(x),\ i = 1,\ 2,\ 3.$
2. Find the EDF $F_r^*(x)$ as follows:

$$F_r^*(x) = \frac{1}{r}\sum_{j=1}^{r} I\left(Y_{(i)j} \leq x\right), \quad I\left(Y_{(i)j} \leq x\right) = \begin{cases} 1 & , Y_{(i)j} \leq x, \\ 0 & , \quad o.w., \end{cases} \tag{3}$$

3. Use $F_r^*(x)$ to calculate the value of T^* as in (1).
4. Repeat the steps (1-3) 10, 000 times to get $T_1^*,...,\ T_{10,000}^*$.
5. The percentage point d_α of T^* is given by the $(1-\alpha)100$ quantile of $T_1^*,...,\ T_{10,000}^*$

Secondly, to calculate the power of T^* at H, we need to use simulation. So, we design the following algorithm:

1. Let $Y_1,...,Y_r$ be a random sample from H, a distribution under H_1^*, $i = 1,\ 2,\ 3.$
2. Find the EDF $F_r^*(x)$ as in (3).
3. Calculate the value of T^* as in (1) but using the data $Y_1,...,Y_r$.
4. Repeat the steps (1) - (3), 10,000 times to get $T_1^*,...,\ T_{10,000}^*$.
5. Power of $T^*(H) \approx \dfrac{1}{10,000}\sum_{t=1}^{10,000} I\,(T_t^* > d_\alpha)$, where $I\,(.)$ stands for indicator function.

5. SIMULATION STUDY

The power of each test is approximated based on a Monte Carlo simulation of 10,000 iterations according to the algorithm of Section 4. We compared the efficiencies of the tests for different sample sizes: $r = 10,\ 20,\ 30,$ different set sizes: $m = 1,\ 2,\ 3,\ 4$ (m=1 means SRS case) and different alternative distributions: $Normal = N\,(0,\ 1)$, $Laplace = L\,(0,\ 1)$, $Lognormal = LN\,(0,\ 1)$, $Cauchy = C\,(0,\ 1)$, $StudentT = S\,(5)$, and $Uniform = U\,(0,\ 1)$. The Simulation results are presented in the Tables (1)-(3). For Uniform distribution, computations show that the efficiency of all tests equal one, except for case $r = 10$. For m=1, (SRS case, efficiency = 1), so these efficiencies are omitted. Since Table 1 and Table 3 are equivalent for symmetric distributions, we present only the results for asymmetric alternatives in Table 3.

From the simulation results given in Tables (1)-(3), the following remarks may be made:

1. The efficiencies in Table 1 and Table 3 are all greater than 1, which means that the MEDF tests under ERSS are more powerful than their counterparts in SRS.
2. From Table 1 and Table 2, we note that the efficiency increases as the distribution under the alternative hypothesis departs to asymmetry.
3. In general, it can be noted from Table 2 (median case) that the efficiencies of the modified tests are less than their counterpart in SRS case except for uniform distribution.
4. From Table 2 (median case), we note that the efficiencies of the modified tests are greater than their counterparts in SRS case for D_3 for Laplace distribution.

Table 1. The efficiency values of tests using RSS with respect to SRS for $r = 10,\ 20,\ 30$ and $m = 1,\ 2,\ 3,\ 4$ (using first order statistics).

H	T	Minimum , $\alpha = 0.05.$								
		$r = 10,$ m			$r = 20,$ m			$r = 30,$ m		
		2	3	4	2	3	4	2	3	4
$N(0,\ 1)$	D_1	3.31	11.45	18.33	6.85	11.19	11.73	5.17	5.91	5.92
	D_{11}	13.13	30.88	38.5	12.17	16.63	16.93	8.04	8.69	8.70
	D_2	4.4	26.87	50.87	11.44	21.27	22.2	5.52	6.49	6.49
	D_{22}	5.8	38.5	75.1	22.45	47.5	49.95	9.67	11.88	11.90
	D_3	9.34	20.42	24.61	9.40	12.16	12.35	5.87	6.25	6.25
	D_4	18	43.05	51	11.94	15.52	15.63	4.83	5.13	5.13
	WW_0	15.91	35.91	42.09	11.95	15.06	15.15	5.71	5.99	5.99
	WW_{11}	24.38	61.31	73.92	19.74	26.11	26.32	9.41	10	10
	WW_{21}	26.76	52.41	58	20.22	24.32	24.39	10.29	10.64	10.64
	aa_{21}	33.3	81.3	96.7	21.97	28.43	28.57	7.13	7.46	7.46
	aa_{22}	33	100.43	133.43	24.39	35.29	35.71	7.26	7.81	7.81
	aa_{12}	35	127	182	47.85	75.54	76.92	11.79	12.99	12.99
$L(0,\ 1)$	D_1	1.31	2.16	2.61	2.25	3.63	4.47	2.56	3.83	4.38
	D_{11}	5.86	9	10.86	4.37	6.55	7.78	4.30	6.10	6.67
	D_2	1.38	3.08	4.63	4.59	8.91	11.41	5.83	9.88	11.56
	D_{22}	1.30	3.48	5.30	5.91	12.39	16.35	8.61	15.58	18.48

	D_3	4.51	6.46	7.54	3.29	4.76	5.60	3.48	4.82	5.26
	D_4	8.25	13.6	16.5	7.02	11.09	13.23	6.73	9.97	10.94
	WW_0	7.13	11.58	13.67	5.81	8.80	10.48	5.33	7.78	8.48
	WW_{11}	9.53	16.18	19.65	8.71	13.71	16.4	7.43	11.18	12.27
	WW_{21}	11.62	17.76	21	10.24	15.21	17.29	9.98	13.79	14.94
	aa_{21}	11.8	21.53	27.67	9.97	17.48	21.10	9.94	15.09	17.02
	aa_{22}	9.58	19.5	26.5	9.21	18.21	23.67	8.5	14.36	16.70
	aa_{12}	10.4	23.1	31.8	12.59	25.71	33.53	11.06	19.18	22.27
$LN(0, 1)$	D_1	3.42	10.88	17.49	6.67	11.08	11.59	5.62	6.44	6.45
	D_{11}	12	26.93	33.85	10.58	14.43	14.69	8.18	8.84	8.85
	D_2	4.67	25	48.6	13.28	24.62	25.62	6.39	7.46	7.46
	D_{22}	7	40.89	80.89	25.28	52.83	55.5	11.45	14.07	14.08
	D_3	9.15	18.34	22.59	8.12	10.60	10.74	5.97	6.36	6.37
	D_4	17.55	39.95	48.05	11.41	15.03	15.15	5.03	5.35	5.35
	WW_0	16.26	35.04	41.65	11.11	14.19	14.29	6.10	6.41	6.41
	WW_{11}	24.79	60.08	73.15	17.90	24.15	24.39	9.88	10.53	10.53
	WW_{21}	33.43	62.5	67.5	18.24	21.65	21.74	11.08	11.49	11.49

	aa_{21}	36.78	88.78	107.11	16.78	21.59	21.74	7.39	7.75	7.75
	aa_{22}	36.17	117.83	154.67	19.88	29.85	30.27	8.05	8.70	8.70
	aa_{12}	32	121	181	39.6	65.2	66.6	12.57	13.89	13.89
$C(0,\ 1)$	D_1	3.38	7.52	11.96	5.06	12.02	16.67	7.39	16.08	19.47
	D_{11}	3.76	7.68	11.82	6.40	14.60	19.65	8.17	16.79	20.28
	D_2	2.47	5.26	8.62	3.43	8.81	14.21	3.98	11	15.65
	D_{22}	2.44	5.38	8.77	3.44	8.91	13.88	3.90	10.68	14.52
	D_3	2.91	6.26	10.19	5.46	13.71	19.49	6.91	15.79	19.94
	D_4	2.17	4.53	7.42	3.24	8.82	14.3	3.89	11.15	16.35
	WW_0	2.44	5.26	8.78	3.87	10.67	16.52	5.33	13.83	19.10
	WW_{11}	2.77	6.05	9.64	4.34	11.16	16.16	5.73	13.81	18.06
	WW_{21}	1.96	4.02	6.69	3.77	10.09	15.89	4.77	13.26	19.19
	aa_{21}	1.58	2.18	2.5	1.82	2.19	2.29	1.85	2.04	2.06
	aa_{22}	1.53	1.95	2.14	1.65	1.86	1.91	1.68	1.79	1.80
	aa_{12}	1.56	1.98	2.18	1.70	1.89	1.94	1.70	1.81	1.82
$U(0,\ 1)$	D_1	1	1	1	1	1	1	1	1	1
	D_{11}	1	1	1	1	1	1	1	1	1
	D_2	1.93	1.93	1.93	1	1	1	1	1	1

	D_{22}	11.63	11.63	11.63	1	1	1	1	1	1
	D_3	1	1	1	1	1	1	1	1	1
	D_4	1.01	1.03	1.01	1	1	1	1	1	1
	WW_0	1	1	1	1	1	1	1	1	1
	WW_{11}	1.01	1.01	1.01	1	1	1	1	1	1
	WW_{21}	1	1	1	1	1	1	1	1	1
	aa_{21}	1.01	1.01	1.01	1	1	1	1	1	1
	aa_{22}	1.03	1.03	1.01	1	1	1	1	1	1
	aa_{12}	3.56	3.56	3.56	1	1	1	1	1	1
$S(5)$	D_1	1.41	4.07	5.87	3.48	6.87	8.82	3.82	6.32	7.30
	D_{11}	4.63	10.54	13.58	6.47	11.11	13.41	5.65	8.65	9.56
	D_2	1.38	6.57	10.14	5.81	14.28	18.92	5.74	10.49	12.30
	D_{22}	1.33	7.67	12.33	7.28	20	27.4	7.96	15.77	18.75
	D_3	4.13	8.15	10.37	5.29	8.72	10.38	4.71	7	7.73
	D_4	6.46	15	19.18	8.12	14.35	17.20	5.88	9.08	9.91
	WW_0	6.06	13.68	17.39	7.56	13.19	15.61	6.01	8.91	9.69
	WW_{11}	6.96	17.17	22.21	9.95	18.13	21.62	8.37	12.94	14.16

	WW_{21}	8.73	17.57	21.47	10.93	17.31	20	9.80	13.82	14.65
	aa_{21}	9.53	23.32	31.26	12.21	22.18	26.61	9.17	14.10	15.22
	aa_{22}	7.87	22.8	32.13	11	22.75	29.04	8.24	14.10	15.92
	aa_{12}	8.82	30	42.55	16	35.17	44.89	10.51	18.42	20.87

Table 2. The efficiency values of tests using RSS with respect to SRS for $r = 10,\ 20,\ 30$ and $m = 1,\ 2,\ 3,\ 4$ (using median).

H	T	Median , $\alpha = 0.05.$								
		$r = 10,$ m			$r = 20,$ m			$r = 30,$ m		
		2	3	4	2	3	4	2	3	4
$N(0,\ 1)$	D_1	1	1.128	1.15	0.88	1	1.06	0.98	0.86	0.96
	D_{11}	1.286	1.238	1.24	0.85	0.95	0.9	1.07	0.93	0.97
	D_2	1	1.8	1.5	0.83	1.09	0.93	1.21	0.88	0.99
	D_{22}	0.75	1.625	1.13	0.81	1.1	1.1	1.29	0.82	1.02
	D_3	1.345	1.276	1.41	0.92	0.96	0.98	1.04	0.93	0.95
	D_4	1.5	1.286	1.5	1.05	1.16	1.1	1.02	0.85	0.89
	WW_0	1.389	1.222	1.39	1.1	1.16	1.12	1.11	0.92	0.9
	WW_{11}	1.4	1.3	1.5	1.09	1.09	1.09	1.16	0.88	0.89
	WW_{21}	1.846	1.308	1.54	0.95	0.88	0.93	1.08	0.94	0.81
	aa_{21}	1.714	1.143	1.57	0.87	0.82	0.82	1.09	0.81	0.74

	aa_{22}	1.4	1.2	1.2	0.94	0.94	0.87	1.12	0.83	0.78
	aa_{12}	1.5	1.5	1	1	0.93	0.93	1.14	0.77	0.73
$L(0, 1)$	D_1	0.833	1.125	1.02	1.3	1.06	1.24	1.39	1.4	1.46
	D_{11}	0.96	1.2	1.04	1.2	1.06	1.2	1.5	1.43	1.57
	D_2	0.583	0.708	0.71	1.03	1.06	1.25	2.33	2.45	3.27
	D_{22}	0.591	0.636	0.5	0.92	0.88	0.88	2.19	2.23	3.16
	D_3	1.773	2.364	1.95	1.34	1.12	1.33	1.54	1.49	1.58
	D_4	0.385	0.487	0.56	1.38	1.31	1.82	2.07	2.22	2.95
	WW_0	0.73	0.96	0.92	1.31	1.13	1.52	1.9	2.02	2.4
	WW_{11}	0.67	0.78	0.72	1.18	0.97	1.33	1.79	1.84	2.24
	WW_{21}	0.70	0.65	0.7	1.27	1.1	17.8	1.5	1.59	1.97
	aa_{21}	0.63	0.56	0.56	1.08	1	1.19	1.7	1.88	2.62
	aa_{22}	0.47	0.47	0.4	1.22	0.96	0.61	1.98	2.18	2.93
	aa_{12}	0.36	0.36	0.18	1.21	0.79	0.79	1.94	1.91	2.76
$LN(0, 1)$	D_1	0.84	1.07	1	0.99	1.06	1.04	0.89	0.98	0.91
	D_{11}	1	1.04	0.96	1	1.1	0.97	0.81	0.86	0.83
	D_2	0.70	1.15	1.08	1.14	1.21	1.29	0.87	0.96	0.84
	D_{22}	0.67	1.22	1	1.22	1.39	1.61	0.99	1.09	0.96

	D_3	1	1.08	0.97	1.01	1.07	0.94	0.83	0.87	0.82
	D_4	1	1.15	0.85	1.21	1.07	1.23	0.85	0.89	0.85
	WW_0	0.96	1.125	0.88	0.7	1.13	1.18	0.78	0.86	0.84
	WW_{11}	0.93	1.143	0.93	1.24	1.18	1.15	0.78	0.86	0.86
	WW_{21}	1	0.789	0.89	1.2	0.94	1.03	0.81	0.91	0.93
	aa_{21}	1	0.9	0.8	1.31	1	1	0.68	0.74	0.76
	aa_{22}	0.86	0.857	0.57	1.14	1.04	0.93	0.68	0.79	0.76
	aa_{12}	1	1	1	1	1.07	0.93	0.74	0.87	0.82
$C(0, 1)$	D_1	0.86	0.96	0.78	1	0.96	0.98	1	1	1.15
	D_{11}	0.69	0.714	0.65	0.79	0.89	0.89	0.93	0.91	0.98
	D_2	0.86	0.804	0.63	0.81	0.72	0.68	0.85	0.69	0.77
	D_{22}	0.77	0.705	0.56	0.79	0.67	0.64	0.79	0.61	0.65
	D_3	0.79	0.872	0.83	0.91	1.02	1.11	1.14	1.14	1.26
	D_4	0.76	0.655	0.55	0.82	0.69	0.71	0.94	0.81	0.91
	WW_0	0.74	0.796	0.63	0.88	0.78	0.82	1.05	1	1.1
	WW_{11}	0.69	0.692	0.52	0.78	0.69	0.67	0.91	0.82	0.84
	WW_{21}	0.76	0.694	0.69	0.82	0.76	0.8	0.95	0.91	0.84

	aa_{21}	0.36	0.203	0.14	0.33	0.15	0.11	0.34	0.15	0.09
	aa_{22}	0.42	0.223	0.13	0.41	0.18	0.11	0.41	0.16	0.1
	aa_{12}	0.41	0.211	0.12	0.4	0.17	0.11	0.4	0.15	0.1
$U(0,\ 1)$	D_1	1	1	1	1	1	1	1	1	1
	D_{11}	1	1	1	1	1	1	1	1	1
	D_2	2.33	2.433	2.44	1	1	1	1	1	1
	D_{22}	12.98	15.84	16.4	1	1	1	1	1	1
	D_3	1	1	1	1	1	1	1	1	1
	D_4	1.01	1.002	1	1	1	1	1	1	1
	WW_0	1	1	1	1	1	1	1	1	1
	WW_{11}	1.01	1.003	1	1	1	1	1	1	1
	WW_{21}	1	1	1	1	1	1	1	1	1
	aa_{21}	1	1	1	1	1	1	1	1	1
	aa_{22}	1.03	1.003	1	1	1	1	1	1	1
	aa_{12}	3.32	3.917	3.94	1	1	1	1	1	1
$S(5)$	D_1	0.88	0.976	1.02	0.97	1.01	0.84	1.01	0.94	1.02
	D_{11}	0.70	0.9	0.8	0.91	0.89	0.79	1.02	0.98	0.93
	D_2	0.76	0.857	0.9	1.03	1.06	0.97	1.11	1.03	1.03

	D_{22}	0.72	0.778	0.89	1	0.91	0.83	1.07	1	0.98
	D_3	0.73	0.951	0.8	1.01	1	0.86	1.08	0.99	0.95
	D_4	0.70	1.043	0.87	1.02	1.07	1.02	1.08	1.11	1.09
	WW_0	0.69	0.966	0.83	0.98	1	0.94	1.5	1.47	1.41
	WW_{11}	0.60	0.85	0.8	0.86	0.86	0.86	0.95	0.92	0.89
	WW_{21}	0.74	0.913	0.83	0.9	0.85	0.82	0.81	0.92	0.81
	aa_{21}	0.57	0.857	0.79	0.81	0.84	0.75	0.95	1.14	0.98
	aa_{22}	0.5	0.583	0.58	0.78	0.81	3.63	0.96	1.02	0.93
	aa_{12}	0.5	0.5	0.5	0.55	0.65	0.55	0.9	0.95	0.81

Table 3. The efficiency values of tests using RSS with respect to SRS for $r = 10, 20, 30$ and $m = 1, 2, 3, 4$ (using largest order statistics).

H	T	Maxima , $\alpha = 0.05$.								
		$r = 10$, m			$r = 20$, m			$r = 30$, m		
		2	3	4	2	3	4	2	3	4
$LN(0, 1)$	D_1	11.33	19.2	21.4	9.54	11.17	11.24	6.41	6.58	6.58
	D_{11}	13.25	30.58	38.21	10.57	14.41	14.69	7.72	8.47	8.47
	D_2	34	59.73	65.6	18.24	21.67	21.74	6.36	6.67	6.67
	D_{22}	43.5	80.45	89.36	40.45	49.8	50	12.39	13.16	13.16
	D_3	10.57	21.74	26.57	8.13	10.58	10.76	5.79	6.25	6.25

	D_4	19.89	45.22	53.61	9.24	11.95	12.05	5.16	5.52	5.52
	WW_0	18.7	40.9	48.2	9.72	12.26	12.35	5.88	6.21	6.21
	WW_{11}	26.92	66	79.67	15.79	21.09	21.28	9.62	10.31	10.31
	WW_{21}	5.25	27.63	49.5	10.51	19.61	20.39	9.15	10.75	10.75
	aa_{21}	18.13	71.13	107.75	11.06	20.17	20.81	6.41	7.41	7.41
	aa_{22}	38.67	119.17	155.17	18.89	27.39	27.78	7.41	8.06	8.06
	aa_{12}	86	323	454.5	34.333	54.44	55.56	11.96	13.33	13.33

6. CONCLUSION

In this paper, we have shown that the powers of a set of modified EDF goodness of fit tests can be much improved if the sample is collected via the ERSS. Moreover, modified EDF tests showed a very excellent power performance in comparison with their SRS counterparts. Although our study is limited to the logistic distribution under the null hypothesis, it can easily be extended to other distributions.

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